DEANSHIP OF SCIENTIFIC RESEARCH

JUNIOR FACULTY RESEARCH PROJECT NO: …………………………

Title of Proposal:

Tilting and Cotilting Modules over Commutative Rings

Duration of Project (in months) : 12 months + 1 month in summer
Proposed Starting Date : February 1, 2005
Ending Date : April 15, 2006
Total Project Cost (SR) : 33,567 SR

Submitted by:

Dr. Jawad Abuihlail, Assistant Professor, Mathematical Sciences (   )

Senior consultant:

Prof. Salah-Eddine Kabbaj, Professor, Mathematical Sciences (   )

Date: Tuesday, November 23, 2004

APPROVALS:

Chairman: ………………………………… **Date:** ………………………
Department of Mathematical Sciences

Chairman,
Research Committee: ……………………… **Date:** ……..………………

Vice Rector for Graduate Studies
and Scientific Research: …………………….. **Date:** ………………………….

عمادة البحث العلمي

مشروع بحث رقم :............................

عنوان البحث : المعايير المائلة و المعايير المائلة الخلفية على الحلقات التبديلية

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مقدم من :

د. جواد يونس أبو اهليل، أستاذ مساعد، العلوم الرياضية (   )

المشرف:

أ. د. صلاح الدين قباج، أستاذ، العلوم الرياضية (   )

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تصديق

رئيس قسم الرياضيات: -------------------------- التاريخ: -------------------------------

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1. Abstract

An arbitrary module T over an arbitrary ring R is said to be *tilting* (*cotilting*) iff $\mathrm{Gen}(T) = \{M \mid \mathrm{Ext}^1_R(T,M) = 0\}$ ($\mathrm{Cog}(T) = \{M \mid \mathrm{Ext}^1_R(M,T) = 0\}$), where $\mathrm{Gen}(T)$ ($\mathrm{Cog}(T)$) is the class of T-generated (T-cogenerated) R-modules. The interest in tilting (cotilting) modules stems from the fact that they allow generalizations of Morita equivalences and dualities: projective generators are tilting and injective cogenerators are cotilting.

Although there is a wide literature of tilting modules over arbitrary rings, several aspects of the theory of infinitely generated tilting (cotilting) modules over *commutative* base rings are still not well understood. Till now we have a complete characterization only for tilting Abelian groups [GT00] and tilting modules over *small* Dedekind domains [TW02]. One of the main **<u>open problems</u>** in this field of research is to obtain characterizations of tilting (cotilting) modules over other classes of commutative rings and structures (e.g. Prüfer domains, Valuation domains, pullbacks etc.)

Our main objective in this project is undertake the necessary preparations for working on the topic mentioned through studying a number of *recent papers* on tiling (cotilting) modules over commutative base rings in addition to a book on related topics and exposing main results in the "Commutative Algebra Seminar" organized by the senior consultant. The expected outcome of the project is the formulation of a number of problems and conjectures to be worked out in future research projects.

<u>2000 Mathematics Subject Classifications:</u> 13C05, 13D07, 13F05, 13F30

<u>Key Words:</u> Tilting & Cotilting Modules, Torsion & Cotorsion Theories, Projective and Injective Dimension, Ext-Functor, Tor-Functor - Prüfer Domains, Valuation Domains.

<u>Duration of the Project:</u> 12 months + one month in summer

<u>Cost of the Project:</u> 33,567 SR

2. INTRODUCTION

Throughout, denote with R a commutative ring with $1_R \neq 0_R$, with R-Mod the category of R-modules and with Ext_R^i, $i \geq 1$ (Tor_i^R, $i \geq 1$) the ith Ext-functor (ith Tor-Functor) in R-Mod. We assume familiarity with the foundations of the theory of "Rings and Modules" and "Homological Algebra".

For an R-module T, let

Gen(T) : = the class of T-generated R-modules;

Pres(T) : = the class of T-presented R-modules;

Add(T) : = the class of R-direct summands of direct sums of copies of T;

σ[T] : = the full subcategory of R-submodules of T-generated R-modules;

$Cog(T)$: = the class of T-cogenerated R-modules;

Cop(T) : = the class of T-copresented R-modules;

$\mathrm{Pr}od(T)$: = the class of R-direct summands of direct products of copies of T;

π[T] : = the full subcategory of all factor modules of T-cogenerated R-modules.

For a given class $\mathfrak{m}$ of R-modules set

$${}^{\perp}\mathfrak{m} := \bigcap_{M \in \mathfrak{m}} \{X \in \text{R-Mod} : \mathrm{Ext}_R^i(X, M) = 0\ \forall\ i \geq 1\};$$

$${}^{\neg}\mathfrak{m} := \bigcap_{M \in \mathfrak{m}} \{X \in \text{R-Mod} : \mathrm{Tor}_i^R(X, M) = 0\ \forall\ i \geq 1\};$$

$$\mathfrak{m}^{\perp} := \bigcap_{M \in \mathfrak{m}} \{X \in \text{R-Mod} : \mathrm{Ext}_R^i(M, X) = 0\ \forall\ i \geq 1\};$$

$$\mathfrak{m}^{\neg} := \bigcap_{M \in \mathfrak{m}} \{X \in \text{R-Mod} : \mathrm{Tor}_i^R(M, X) = 0\ \forall\ i \geq 1\}.$$

A (finitely generated) R-module T is said to be a (*classical*) *tilting module* provided $\mathrm{Gen(T)} = {}^{\perp}\mathrm{T}$; equivalently iff σ[T] = R-Mod, $\mathrm{Gen(T)} = \mathrm{Pres(T)}$ and T is Gen(T)-projective (e.g. [Wis98]). A (finitely generated) R-module is said to be a (*classical*) *cotilting R-module* iff $\mathrm{Cog(T)} = \mathrm{T}^{\perp}$; equivalently π[T] = R-Mod, $\mathrm{Cog(T)} = \mathrm{Cop(T)}$ and T is Cog(T)-injective (e.g. [Wis]).

Any projective generator (injective cogenerator) in R-Mod is a tilting (cotilting) R-module. In fact, the interest in tilting (cotilting) modules stems mainly from this fact, as they allow generalization of the classical Morita equivalences and dualities as well as in the study of the representation theory of Artin algebras. Examples of infinitely generated tilting modules are $\mathbb{Q}/\mathbb{Z}$ in the category of torsion $\mathbb{Z}$-modules, $\mathbb{Q} \oplus \mathbb{Q}/\mathbb{Z}$ in the category of $\mathbb{Z}$-modules and Fuchs' divisible module ∂_{R} over an arbitrary integral domain R [Fuc84].

Although there is a wide literature on *Tilting Theory*, structure theorems for infinitely generated tilting (cotilting) modules over *commutative* base rings are far from being known. The only complete characterization of tilting modules we have till now are in the case of Abelian groups (i.e. $\mathbb{Z}$-modules) by R. Göbel and J. Trlifaj [GT00] and in the case of *small* Dedekind domains by J. Trlifaj and S. Wallutis in [TW02] (both assuming *Gödel's Axiom of Constructability,* V = L).

An **open problem** in this area of research is still the characterization of all tilting (cotilting) modules over other classes of commutative base rings and special structures (e.g. Prüfer domains, Valuation rings, pullbacks etc.) as well as to extend the characterizations of tilting modules over small Dedekind rings to *n-tilting modules* (see [Baz04b] for the definition).

The *main goal* of this project is to prepare the junior researcher to attack the open problem highlighted above as well as to work on related topics in the theory of tilting and cotilting modules over commutative base rings. To achieve this, an intensive study is to be undertaken of the **six core papers** on this topic surveyed in the *Literature Review* below in addition to one book on related topics. As an outcome of this project, the junior researcher is expected to raise and formulate (with the help of the senior consultant) some new problems and conjectures about this topic that will serve as a basis for future research projects.

3. LITERATURE REVIEW

Classical Tilting (*cotilting*) *modules* were first introduced in a rather restrictive way by S. Brenner & M. Butler in [BB80] for finite dimensional algebras over base fields. Their theory was generalized and extensively developed by D. Happel & C. Ringel [HR82] among others.

An R-module was defined to be (*classical*) *tilting* iff the following conditions hold:

(1) There are $T_1,\ T_2 \in \mathrm{Add}(T)$ and an exact sequence $0 \to R \to T_1 \to T_2 \to 0$.

(2) $\mathrm{Ext}^1_R(T, T^{(\Lambda)}) = 0$ for any set Λ;

(3) $\mathrm{proj.dim.}(T)\ \leq\ 1$;

(4) T is finitely generated.

An R-module was defined to be a (*classical*) *cotilting module* iff the following dual conditions of the four conditions above hold:

(1') There are $T_1,\ T_2 \in \mathrm{Prod}(T)$ and an exact sequence $0 \to T_1 \to T_2 \to R \to 0$.

(2') $\mathrm{Ext}^1_R(T^{\Lambda}, T) = 0$ for any set Λ;

(3') $\mathrm{inj.dim.}(T)\ \leq\ 1$;

(4') T is finitely generated.

A tilting (cotilting) R-module C is said to be of *finite type* (*cofinite type*) provided there exists a set $\mathfrak{S}$ of finitely presented R-modules with proj.dim.≤ 1 (inj.dim.≤ 1) such that $C^{\perp} = \mathfrak{S}^{\perp}$ ($^{\perp}C = \mathfrak{S}^{\neg}$).

Generalizations of this concept were given in recent years by considering, for instance, arbitrary base rings, and/or infinitely generated modules, and/or tilting (cotilting) modules of finite projective (injective) dimensions, called n-tilting (n-cotilting) modules.

There is a wide literature on tilting (cotilting) modules over arbitrary base rings; however we restrict ourselves in the following brief literature review *mainly* to a number of *recent* papers dealing with the following task: *Provide structure theorems for tilting (cotilting) modules over special classes of integral domains.*

An *infinitely generated* tilting module over an arbitrary commutative integral domain R is due to L. Fuchs [Fuc84]: ∂_R is the right R-module generated by the set of all k-tuples $(r_1,\ldots,r_k)$ of non-zero elements r_i of R, for $k \geq 0$, with defining relations $(r_1,\ldots,r_k)r_k = (r_1,\ldots,r_{k-1})$, $k \geq 1$. The right module ∂_R was shown to be tilting by D. Happel and C. Ringel [HR82] and was investigated later intensively by A. Facchini in [Fac87] & [Fac88]. After associating a left ideal I of the endomorphism ring E := $\mathrm{End}(\partial_R)$ and defining, in a natural way, the classes of I-divisible, I- reduced and I-cotorsion right E-modules, Facchini showed mainly that: "*The functors* $\mathrm{Hom}_R(\partial,-)$ *and* $-\otimes_E \partial$ *give equivalence between the full subcategory of all divisible right* R-*modules and the full subcategory of all* I-*reduced* I-*cotorsion right* E-*modules. The functors* $\mathrm{Ext}^1_R(\partial_R,-)$ *and* $\mathrm{Tor}^E_1(-,\partial_R)$ *give equivalence between the full subcategory of all reduced* R-*modules and the full subcategory of all* I-*divisible* I-*cotorsion* E-*modules*" [Fac87, Page 2237].

In [CW95], R. Colpi and J. Trlifaj relaxed the fourth condition in the original definition of tilting modules with the aim to extend some aspects of the theory of *classical tilting modules* to (not necessarily finitely generated) tilting modules. In particular, Proposition 1.3 provides a characterization of a tilting R-module as an arbitrary R-module for which $\mathrm{Gen(T)} = \{M \mid \mathrm{Ext}^1_R(T,M) = 0\}$. Dually, the original notion of classical cotilting modules was extended to arbitrary cotilting modules providing a general definition valid for a (possibly infinitely generated) cotilting R-module, namely $\mathrm{Cog(T)} = \{\mathrm{M} \mid \mathrm{Ext}^1_{\mathrm{R}}(\mathrm{M,T}) = 0\}$ (e.g. [CF00]).

R. Göbel and J. Trlifaj described in [GT00] *all* tilting Abelian groups (under *Gödel's Axiom of Constructability,* V = L): an Abelian group is tilting if and only if it's of the form $\bigoplus_{p\in\mathbb{P}} \mathbb{Z}_{p^\infty}^{(\alpha_p)} \oplus \mathrm{R}_{\mathbb{P}}^{(\beta)}$, where $\mathbb{P}$ is a subset of the set of all primes of $\mathbb{Z}$, α_p, β are non-zero cardinals and $\mathrm{R}_{\mathbb{P}} \subseteq \mathbb{Q}$ is the subring generated by $\mathbb{Z}$ and $\{p^{-1} \mid p \in \mathbb{P}\}$.

In [TW02], J. Trlifaj and S. Wallutis extended in a natural way these characterizations of tilting $\mathbb{Z}$-modules to modules over *small Dedekind domains* (those with cardinality at most 2^ω and with countable prime spectrum). The main result, Theorem 12, states that: *assuming Gödel's Axiom of Constructability (V = L), the class of all tilting modules over a small Dedekind domain consists precisely of those modules of the form* $\mathrm{T} = \bigoplus_{p\in\mathrm{P}} \mathrm{E}(\mathrm{R}/p)^{(\alpha_p)} \oplus \mathrm{T}'$, *where* $0 \neq \mathrm{P} \subseteq \mathit{Spec}(R)$, α_{P} *are non-zero cardinals for all* $p \in \mathrm{P}$, $\mathrm{E}(\mathrm{R}/p)$ *is the injective hull of* R/p *and T' is a projective generator over* $\mathrm{R}_{(\mathrm{P})} := \bigcap\{\mathrm{R}_p \mid p \in \mathrm{Spec(R)}\backslash\mathrm{P}\}$.(An <u>open problem</u> pointed out by the authors is whether this characterization of tilting R-modules remains true in ZFC (Zermelo-Fraenkel + Axiom of Choice).

Cotorsion theories were introduced by L. Salce [Sal79] in the context of Abelian groups. pair $(\mathfrak{F},\mathfrak{C})$ of classes of R-modules is said to be a *cotorsion theory* iff $\mathfrak{F} = \bigcap_{\mathrm{M}\in\mathfrak{C}} {}^{\perp}M$ *and* $\mathfrak{C} = \mathfrak{F}^{\perp}$. Every class $\mathfrak{M}$ of R-modules cogenerates a cotorsion theory, namely $({}^{\perp}\mathfrak{M}, ({}^{\perp}\mathfrak{M})^{\perp})$. In [BS01], S. Bazzoni and L. Salce investigated cotorsion theories over valuation domains. In particular they showed that under suitable conditions on a valuation domain R with quotient field Q (and assuming V = L), the cotorsion theory generated by the *divisible* R-module Q/R coincides with the cotorsion theory cogenerated by Fuchs' *tilting* module ∂_R .

In a recent paper [Sal04], L. Salce investigated the structure of tilting modules over a valuation domain R. In particular he proved that the *Fuchs-Salce divisible* S-*modules* δ_S are canonical generators for the tilting torsion classes over R, assuming V = L and that the cardinality of the pure-injective hull of R is at most $2^{\aleph}$ when the tilting generator has uncountable rank.

In a number of recent preprints and conference talks (e.g. [Baz04], [Baz]), S. Bazzoni considered tilting (cotilting) modules over commutative base rings. The main result of [Baz04a] states that n-cotilting modules are pure injective in case the base ring R is a Prüfer domain or is an arbitrary countable commutative base ring (recall that 1-cotilting modules over any base ring are pure injective by [Baz03, Theorem 2.8]).

In [Baz], S. Bazzoni showed that the study of cotilting modules over a commutative ring (Prüfer domain) R can be restricted to the local case (valuation domains): Theorem 2.4 states that *an (n-)cotilting module C over a commutative ring (Prüfer domain) is equivalent to* $\prod_{m \in Max(R)} C^m$ *, where* C^m *is the* R_m*-module* $Hom_R(R_m, C)$. In Proposition 4.3, S. Bazzoni characterizes cotilting modules over valuation domains and proves, in Corollary 4.6, that *all R-modules are cotilting if and only if the valuation domain R is strongly discrete. If R is a Prüfer domain then Theorem 6.10 (Corollary 7.5) shows that every cotilting (n-tilting) module is equivalent to a direct product of indecomposable pure injective modules (is of finite type).*

4. Objectives and Description of the Proposed Research

We plan to intensively study ***six core papers*** on the topic of "Tilting and Cotilting Modules over Commutative Rings" to come up with a number of problems and conjectures that will serve as a basis for future research projects.

In what follows we list the main tasks that will be undertaken in this project:

Task 1. Study thoroughly the following papers:

1. A. Facchini, *A tilting module over commutative integral domains*, Comm. Algebra **15(11)** 2235-2250 (1987).
2. R. Göbel and J. Trlifaj, *Cotilting and a hierarchy of almost cotorsion groups*, J. Algebra **224(1)**, 110–122 (2000).
3. S. Bazzoni and L. Salce, *An independence result on cotorsion theories over valuation domains*, J. Algebra **243(1)** 294-320 (2001).
4. J. Trlifaj and S. L. Wallutis, *Tilting modules over small Dedekind domains*, J. Pure Appl. Algebra **172,** 109-117 (2002).
5. L. Salce, *Tilting modules over valuation domains*, Forum Math. **16(4)** 539-552 (2004).
6. S. Bazzoni, *Cotilting and tilting modules over Prüfer domains*, Cortona Conference, 2004.

Task 2. Give at least 12 seminars based on the papers mentioned in Task 1.

Task 3. Attend a summer school/conference in "Commutative Algebra" at which related topics will be discussed with the concerned experts (details p. 16).

Task 4. **(Summer Task)** consists of two parts:
a) Study thoroughly the main results of a book relevant and crucial to "Tilting Theory".
b) Exposing the main results of the book in a "Summer Commutative Algebra Daily Seminar" organized by the senior consultant. This will be achieved through delivering 12-15 seminars during the summer.
The suggested book is: ***Modules over non-Noetherian domains, L. Fuchs & L. Salce, Mathematical Surveys and Monographs, Vol. 84, American Mathematical Society, Providence, RI (2001).***
Remark: The senior consultant, *Prof. Saleh-Eddine Kabbaj*, has indicated that he will, Insha'a Allah, be available at KFUPM during the summer semester 043.

Task 5. Formulate some problems and conjectures based on the study of the papers given above and the discussions in the seminars to be given on the basis of these papers.

Task 6. Write and submit a report on the detailed work done in this project.

5. SCHEDULING

Expected work to be done	**Expected Period**	**Duration**
Task 1 (#1, #2) and Task 2 (4 seminars)	01.02.2005-15.04.2005	$2\frac{1}{2}$ months
Task 1(#3) and Task 2 (2 seminars)	16.04.2005-15.06.2005	2 months
Task 3	Summer 2005	
Task 4 (12-15 seminars)	01.08.2005-31.08.2005	One month in summer
Task 1(#4, #5) and Task 2 (4 seminars)	01.09.2005-30.11.2005	3 months
Task 1(#6) and Task 2 (2 seminars)	01.12.2005-15.02.2006	$2\frac{1}{2}$ months
Tasks 5	16.02.2006-15.04.2007	2 months

6. PERSONNEL REQUIREMENTS

- **Junior researcher** will undertake the six tasks listed above.
- **The consultant**, being an *expert* in "Commutative Algebra", will put the junior researcher into a new path of research by:

  1. Monitoring the work of the junior researcher and the progress of the project.
  2. Monitor delivering of the planned 24-27 seminars by the junior researcher.
  3. Offering guidance and help needed for the junior researcher to fully understand the *papers* & *book* under consideration.
  4. Helping the junior researcher in formulating the new problems and conjectures resulting from this project.

7. MONITORING and EVALUATION

Throughout the research project, the work of the junior researcher will be monitored ***continuously*** by the senior consultant. A brief statement about the evaluation of the junior researcher will be submitted by the senior consultant at the end of the work.

8. UTILIZATION OF RESULTS

The problems and conjectures that we will come up will serve as a basis for future research project.

9. REFERENCES

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10. BUDGET

Project Title: Tilting and Cotilting Modules over Commutative Rings
Junior Researcher: Dr. Jawad Abuihlail (ID # 7030290)
Consultant: Salah-Eddine Kabbaj, (ID # 7020473)

Total Budget	SR 33,567	**Starting Date**	February 1, 2005
Duration	12 months + 1 month in Summer	**Completion Date**	April 15, 2006

#	Budget Item	Allocation (SR)	Remarks
1	Junior Researcher (12×1200)	14,400	
2	Junior Researcher (Compensation for one months in Summer)	09,167	
3	Consultant	--------	**Preferred not to take any financial compensation.**
4.	Summer School/Conference in "Commutative Algebra"	10,000	See below for details
	Grand Total	**33,567**	

The first choice for the summer school/conference the junior researcher intends to attend is:

Summer school in commutative algebra:
Local cohomology and its interactions with algebra, geometry, and analysis
Monday June 20 -- Thursday June 30, 2005 in Snowbird, Utah (USA)

The summer school will include 7 days of lectures, followed by a conference during the last 3 days. During the lecture period, there will be 3 to 4 one hour classes per day, supplemented by problem sessions, computer demos, and discussions. The summer school is intended mainly for graduate students and junior researchers. The homepage of the summer school is: http://www.math.purdue.edu/%7Ewalther/snowbird.html

The junior researcher (Dr. Abuihlail) intends, if possible, to participate at the two parts of the summer school (Lecture series + Conference).

11. RESUME

See the attached CV.