DEANSHIP OF SCIENTIFIC RESEARCH

FAST TRACK RESEARCH PROJECT NO.: …………………………

Title of Proposal:

Semi-Corings and Semi-Comodules

Duration of Project (in months) : 12 months
Starting Date : September1st, 2006
Ending Date : August 31st, 2007
Total Project Cost (SR) : ***28,800 SR***

Submitted by:
Dr. Jawad Abuihlail, Assistant Professor, Mathematical Sciences

Date: Sunday, November 13th, 2005
Revised Version: Wednesday, May 24th, 2006

APPROVALS:

Chairman: ………………………………… **Date:** ………………………
Department of Mathematical Sciences

Chairman,
Research Committee: ……………………… **Date:** ……..………………

Vice Rector for Graduate Studies
and Scientific Research: …………………….. **Date:** ………………………….

عمادة البحث العلمي

مشروع بحث (المسار السريع) رقم :...........................

عنوان البحث :

نصف الحلقات الخلفية و نصف المعايير الخلفية

مدة البحث : 12 شهرا

تاريخ بدء البحث : 1 أيلول، 2006

تاريخ نهاية البحث : 31 آب، 2007

تكلفة المشروع : 28,800 ريال سعودي

مقدم من :

د. جواد يونس أبو اهليل، أستاذ مساعد، العلوم الرياضية

التاريخ: الأحد، 13 تشرين ثاني 2005

النسخة المعدلة: الأربعاء،24 أيار 2006

تصديق

رئيس قسم الرياضيات: ---------------------------- التاريخ: --------------------------------

رئيس لجنة البحث العلمي: ------------------------ التاريخ: ----------------------------------

نائب رئيس الجامعة للدراسات العليا

و البحث العلمي: ---------------------------------- التلريخ: --------------------------------

TABLE OF CONTENTS

1. ABSTRACT

In this project we introduce and investigate the notions of "***Semi-corings and Semi-comodules***" over semi-rings that generalize the notions of "***Corings and Comodules***" over rings, which have been attracting the interest of many algebraists since the begin of the current century. The notions we introduce can also be considered as dual to the well-studied notions of "***Semi-rings and Semi-modules***".

The theory of "Semi-corings and Semi-comodules" will be ***developed from scratch***. In particular we provide non-trivial examples of semi-corings that are not corings and investigate their structure. Particular attention will be paid to the category of semi-comodules for a given semi-coring. Properties of such categories will be studied in details and results we obtained earlier for comodules of corings will be generalized.

As important tools for investigating categories of semi-comodules for semi-corings, we introduce and investigate **two new notions** for semi-modules: **semi-modules *sub-generated by a specific semi-module*** and ***locally projective semi-modules***. These notions (for modules over rings) proved to be very useful in investigating categories of comodules for corings and are expected to play an important role in our project.

We also introduce and investigate the notions of **semi-coalgebras** (respectively **semi-bialgebras, Hopf semi-algebras**) over commutative ground semi-rings, which generalize the classical notions of coalgebras (respectively bialgebras, Hopf algebras) over commutative ground rings.

2. ملخص

في مشروع البحث هذا نقوم بتعريف و دراسة مفاهيم جديدة هي "***نصف الحلقات الخلفية و نصف المعايير الخلفية***" المعرفة على نصف الحلقات، و التي تعمم مفاهيم "***الحلقات الخلفية و المعايير الخلفية***" على الحلقات و هي المفاهيم التي تسترعي اهتمام العديد من الباحثين في علم الجبر منذ مطلع هذا القرن. يمكن اعتبار هذه المفاهيم الجديدة أيضا كمفاهيم مناظرة لمفاهيم "***نصف الحلقات ونصف المعايير***" و التي تمت دراستها بعناية.

سنقوم بتطوير نظرية "*نصف الحلقات الخلفية و نصف المعايير الخلفية*" __**من الأساس**__. بشكل خاص سنقوم بتقديم أمثلة غير بدهية لنصف حلقات خلفية ليست حلقات خلفية و سنقوم بدراسة بنيتها الجبرية. كما سيكون هناك اهتمام خاص بصفات فئة نصف المعايير الخلفية المعرفة على نصف حلقة خلفية معطاة. في هذا المجال سنقوم بتعميم نتائج حصلنا عليها في دراسات سابقة للمعايير الخلفية المعرفة على حلقة خلفية.

كوسائل مهمة في دراسة فئات نصف المعايير الخلفية المعرفة على حلقات خلفية سنقوم بتقديم و دراسة __**مفهومين جديدين**__ لنصف المعايير: ***نصف المعايير نصف المعايير شبه المولدة من نصف معيار معطى، و نصف المعايير المسقطة موضعيا***. هذه المفاهيم (للمعايير المعرفة على الحلقات) أثبتت فاعليتها في دراسة فئات المعايير على الحلقات الخلفية و نتوقع أن يكون لها نفس الفاعلية في مشروع البحث هذا.

سنقوم أيضا بتعريف و دراسة مفاهيم ***نصف الجبريات الخلفية*** (***نصف الجبريات الثنائية، و نصف جبريات هوبف***، على الترتيب) في حالة كون نصف الحلقة الأساس إبدالية. هذه المفاهيم تعمم المفاهيم التقليدية *للجبريات الخلفية* (*الجبريات الثنائية، و جبريات هوبف*، على الترتيب) و المعرفة على حلقات أساس إبدالية.

3. INTRODUCTION

We assume familiarity with the different notions from the theory of "*Corings and Comodules*" as in [BW03] and the theory "*Semi-rings and Semi-modules*" as in [Gol99a].

Our intent is to generalize both notions of rings and corings (modules and comodules) by laying the basis of a theory of ***semi-corings*** (***semi-comodules***). According to our definitions, every ring (module) is a coring (comodule) in a canonical way, which turns to be a semi-coring (semi-comodule) as well. To the best of our knowledge, the notions of *semi-corings* & *semi-comodules* were not considered before which means that we develop the theory from scratch. Our model will be the theory of corings and comodules as developed in the recent monograph by T. Brzezinski and R. Wisbauer [BW03].

Given a semi-ring A and an A-semi-coring C, we find sufficient (and necessary) conditions for the category of right C-semi-comodules to be isomorphic to $\sigma[_{^*C}C]$, the category of ***C-subgenerated*** left ${}^*C$-semi-modules. Here ${}^*C$ is the A-semi-ring of left A-semi-linear maps from C into the ground semi-ring A, with the ***convolution product***. Properties of this category will be investigated. In particular we generalize our results on the category of right comodules for corings in [Abu03] to the category of right semi-comodules for semi-corings. To achieve this we introduce and investigate two new notions for semi-modules: ***subgenerated semi-modules*** and ***locally projective semi-modules*** which proved to be very useful for investigating comodules for corings.

In case the ground semi-ring A is commutative, we introduce and investigate A-semi-coalgebras (respectively A-semi-bialgebras, Hopf A-semi-algebras) and generalize to them results that were obtained earlier for coalgebras (respectively bialgebras, Hopf algebras) over commutative ground rings.

4. LITERATURE REVIEW

First of all we remark that (to the best of our knowledge) ***not a single paper*** is published on the subject of the project and that the notions of *semi-corings* and *semi-comodules* ***have not been even defined yet***. In what follows we collect some background on "*Corings and Comodules*" and "*Semi-rings and Semi-modules*" separately.

Semi-rings and Semi-modules:

Apart from the trivial examples of ***semi-rings*** which are not rings, namely the set of non-negative integers $\mathbb{N}$ and the set of non-negative real numbers R+ with the usual addition and multiplication, the non-trivial examples of semirings first appear in the work of German mathematician *Richard Dedekind* [Ded] in **1894**, in connection with the algebra of ideals of a commutative ring

Later they were studied independently by algebraists, especially by the American mathematician *H. S. Vandiver*, who worked very hard to get them accepted as a fundamental algebraic structure, being basically the “best” structure which includes both rings and bounded distributive lattices [Van34]. He was not successful, however, and – with only a few exceptions – semirings had fallen into disuse and were well on their way to mathematical oblivion until they were “rescued” during the late 1960’s when real and significant applications were found for them. Numerous applications of semi-rings are obtained in *Automata Theory*, *Optimization Theory*, *generalized fuzzy computation*, *Baysian networks and belief propagation*, *algebraic geometry over the optimization algebra*♣. Many of these applications are documented in the

♣This part of the literature review is taken from a recent talk by J. Golan at the INTERNATIONAL CONFERENCE ON ALGEBRA. IN MEMORY OF KOSTIA BEIDAR. National Cheng Kung University. Tainan, Taiwan, March 6-12, 2005.

nice books on the subject by J. Golan [Gol99a], [Gol99b], [Gol03] as well as in [Heb93] and [Kui86].

The theory of ***semi-modules*** over semi-rings was developed by several authors. For the foundations of the theory of semi-modules over semi-rings we refer to the fundamental series of papers by M. Takahashi [Tak79] - [Tak85] in addition to Golan's books [Gol99a], [Gol99b] and [Gol03].

Corings and Comodules:

In **1941**, the first example of ***Hopf algebras*** appeared in algebraic topology in the work of the German mathematician *H. Hopf* [Ann. Math. (2). Vol. 42 (**1941**), pp. 22-52]. However, the first paper to attract the attention of algebraist was on *graded Hopf algebras* by J.W. Milnor and J.C. Moore [Ann. Math., II. Ser. 81, 211-264 (**1965**)].

During the 1960s and 1970s, Hopf algebras were studied intensively from a purely algebraic point of view. The first book in this direction was that of *M. Sweedler* Hopf Algebras, Benjamin, New York (**1969**)]. A subsequent book of this nature, with more flavor of algebraic geometry, is that of *E. Abe* [*Hopf Algebras*, Cambridge Univ. Press (**1980**)]. Hopf algebras appear in many fields of mathematics: number theory (formal groups), algebraic geometry (affine group schemes), Lie algebras (the universal enveloping algebra is a Hopf algebra), graded ring theory (gradings are coactions), Galois Theory, the theory of Azumaya algebras and Brauer groups, etc.

In **1975**, *M. Sweedler* [Trans. Amer. Math. Soc. Vol. 213 (1975), pp. 391-406] introduced the notion of ***corings***. However they did not get much appreciation because of the lack of examples at that time.

Since **1986**, when *V.G. Drinfe'ld* published his milestone "Quantum Groups" [Dri86], the subject received a huge impetus because of the discovery of interesting applications in *quantum mechanics*, *statistical mechanics* and *knot theory*. This resulted in a revival of the algebraic theory of Hopf algebras, since the late 1980s, making it one of the mainstream subjects in mathematics in the 1990's. Many books on **quantum groups**, which are certain *non-commutative* and *non-cocommutative* Hopf algebras, have been published since the late 1980's. In addition, there were two books that concentrated on the purely algebraic aspects of the theory of Hopf algebras: one by *S. Montgomery* [Hopf Algebras and their Actions on Rings, Am. Math. Soc., Providence RI (**1993**)] and another by *S. Dascalescu* et, al. [Hopf algebras. An introduction. Marcel Dekker (**2001**)].

Intensive investigations were conducted by several authors of the so called Doi-Koppinen data and the associated categories of ***Doi-Koppinen modules*** (see e.g. the monograph by S. Caenepeel et. al. [Frobenius and Separable Functors for Generalized Module categories and Nonlinear Equations. Lecture Notes in Mathematics, **Vol.** 1787. *Springer-Verlag, Berlin,* (**2002**)]. Doi-Koppinen modules were generalized in a work in mathematical physics by *T. Brzezinski* and *S. Majid* [Comm. Math. Phys. Vol. 191 (1998), pp. 467–492] to the so called ***entwining structures*** and ***entwined modules***. These attracted (and are still attracting) the attention of many researchers.

In **1999**, *M. Takeuchi* pointed out that *entwining structures,* give rise to new examples of **corings**. This was in fact behind the **revival** of the theory of corings and their comodules in the recent years. With the many new examples discovered, it turned out that corings might have a variety of unexpected and wide-ranging applications, to topics in non-commutative ring theory, category theory, Hopf algebras, differential graded algebras, and non-commutative geometry.

Recent and Current Research (2000 - Now):

Since the revival of their theory at the beginning of the current century, "corings and comodules" are gaining the attention of many algebraists. Many aspects of the theory of corings and comodules are dealt with in the recent monograph by *T. Brzezinski* and *R. Wisbauer* [Corings and Comodules, Cambridge Press (**2003**)], which is the first book to stress the fact that corings (comodules), over arbitrary ground rings, can be thought of as generalization of rings (modules) and not only as dual to them.

Although it is impossible to include all (or even almost all) main trends and aspects in the recent and current research of this *very hot* subject, we include below *samples* of the main papers dealing with aspects that are related to the research interests of the applicant and related to the proposed research project:

* Constructing new classes of corings. In addition to the nice known examples of corings, like ***Sweedler's coring*** associated to an extension of rings and corings arising from entwining structures that led to the revival in the theory of corings, several *new* classes of corings were constructed and extensively studied recently.

One of the most new interesting classes are the so called ***comatrix corings*** (first constructed by first by E. El-Kaoutit and J. Gomez-Torrecillas [EG-T03]). This class of corings has been extensively studied recently and is attracting continuously the attention of researchers in the theory of corings and comodules, e.g. [BG-T03], [G-TV], [EG-T04], [EG-T05b], [G-TV] and [Z-Da].

Recently a new class of corings termed "***base ring extension of a coring by a module***" was constructed and studied in by T. Brzezinski, E. El Kaoutit and J. Gomez-Torrecillas in [BEG-T06].

* <u>Studying the structure of corings</u>. Several papers were devoted to investigating the structure of corings and to study special types of corings, e.g. *semisimple corings*, *coseparable corings*, *semiperfect corings*, etc. Among the papers dealing with these topics are [Brz02], [Brz03a], [Brz03b], [CI05], [EG-TL04] and [G-TL03].

* <u>Establishing a ***Galois Theory*** for corings and comodules</u>. The classical Galois Theory has been generalized to corings and has been investigated by several authors, including the applicant [Abu05a]. Moreover, special types of corings (comodules) called ***Galois corings*** (***Galois comodules***) introduced by T. Brzezinski in [Brz02] ([Brz05]) are intensively investigated. Among the references that investigated these topics, in addition to the three papers above, are [Böh05], [BW05], [Cae04], [Cd-G05], [EG-T03], [EG-TL04], [GT-V], [Kad05], [Wis05a] and [Wis05b].

* <u>Studying Morita Contexts for corings</u>. Several Morita contexts related to corings (having some projectivity properties as modules) were constructed and studied in several papers including one by the applicant, e.g. [Abu05a], [BV], [BKW05], [Cd-G05], [CVW04] and [CVW05].

* Studying Dualities for categories of comodules of corings: *Morita duality* and *Colby-Fuller duality* for categories of comodules for corings were investigated, e.g. [EG-T05a].

* Studying functors related to categories of comodules for corings. Several authors studied adjoint pairs functors related to categories of comodules for corings, e.g. the *induction* and *coinduction functors* between categories of comodules induced by a morphism of corings, e.g. [Abu05b]. Characterizations of functors between categories of comodules for corings to be *Frobenius*, *separable* etc. were investigated in several papers. Among the papers that dealt with these topics are [Brz02], [Brz03], [G-T02], [G-TZ-D] and [ZD-b].

* Equivalences between categories of comodules for corings were studied in several papers e.g. [C-IG-TW03], [Cd-G05], [EG-T03], [EG-T04], [EG-T05a] and [Z-Da]. Moreover, equivalences between categories of comodules and module categories have been studied, e.g. in [Ver].

* Interactions with Non-commutative Geometry. Nice applications and interpretations of theorems in the theory of corings and comodules were found in Non-Commutative geometry, e.g. [BW05].

5. PROJECT OBJECTIVES

In addition to the *general mains objects* of unifying the theories of "semi-rings and semi-modules" and "corings and comodules" and laying the basis for the theory of "semi-corings and semi-comodules", the following *more concrete* objectives should be reached in order to conduct the proposed research project:

- Introducing and investigating the notions of ***semi-corings*** (***semi-comodules***) over arbitrary (not necessarily commutative) semi-rings.

- Constructing non-trivial examples of semi-corings that are not corings and studying their structure.

- Introducing and investigating the notion of ***sub-generated semi-modules*** and the category $\sigma[_T M]$ of *M*-sub-generated semi-modules for some semi-module *M* over a given semi-ring *T*.

- Introducing and investigating the notion of ***locally projective semi-modules.***

- Finding sufficient (and necessary) conditions for the category of right semi-comodules for a semi-coring *C* to be isomorphic to $\sigma[_{^*C}C]$, the category of C-subgenerated left $^*C$ -semi-modules, and investigating the properties of this category.

- Introducing and investigating ***semi-coalgebras*** (resp. ***semi-bialgebras***, ***Hopf semi-algebras***) over commutative semi-rings that generalize coalgebras (resp. bialgebras, Hopf algebras) over commutative rings.

6. PROPOSED RESEARCH

First of all we fix some definitions: with a **semi-ring** we mean an algebraic structure, consisting of a non-empty set *A* on which we have defined two operations, *addition* (usually denoted by "+") and *multiplication* (usually denoted by "·") such that the following conditions hold:

(1) Addition is associative, commutative and has a (unique) neutral element 0_A.

(2) Multiplication is associative and has a (unique) neutral element 1_A.

(3) Multiplication distributes over addition from either side.

(4) $a0_A = 0_A = 0_A a$ for all $a \in A$.

The first *trivial* examples of semi-rings are the set of non-negative integers $\mathbb{N}$ and the set of non-negative real numbers R+ with the usual addition and multiplication.

Given a semi-ring *A*, a ***right A-semi-module*** is a non-empty set *M* on which we have operations of addition and scalar multiplication by elements of *A* (on the right) defined such that:

(1) Addition is associative, commutative and has a (unique) neutral element 0_M.

(2) $(ma)b = m(ab),\ m(a+b) = ma + mb,\ (m+m')a = ma + m'a\ \forall\ a,b \in A\ \&\ m,m' \in M$.

(3) $m1_A = m$ and $m0_A = 0_M = 0_M a$ for all $a \in A\ \&\ m \in M$.

We remark that every Abelian monoid has a *trivial* structure of a right (left) $\mathbb{N}$-semi-module in the obvious way. For two right *A*-semi-modules *M* and *N* we call a map $f : M \to N$ of $\mathbb{N}$-semi-modules *A-**semi-linear***, provided

$$f(0_M) = 0_N \text{ and } f(ma) = f(m)a \text{ for all } m \in M\ \&\ a \in A.$$

Left A-semi-modules and left *A*-semi-linear maps between left *A*-semi-modules are defined similarly. One can also define ***A-semi-bimodules*** and ***A-semi-bilinear*** maps between A-semi-bimodules in the obvious way.

If A is a semi-ring and *W* is a right (*respectively* left) *A*-semi-module, then we have an equivalence relation, called ***Iizuka relation***, defined by

$$w \equiv_{\{0\}} w' \Leftrightarrow \exists w'' \in W \text{ such that } w + w'' = w' + w''.$$

The set of equivalence classes of *W* with respect to this relation is a *cancellative* right (respectively left) *A*-semi-module $e(W) := \{[w] : w \in W\}$ and we have a *surjective* map of right (left) *A*-semi-modules $e_W : W \to e(W)$, $w \mapsto [w]$.

The ***tensor product*** of semi-modules over a semi-ring was defined by Takahashi in [Tak82a]: if *A* is a semi-ring, *M* is a right *A*-semi-module and *N* is a left *A*-semi-module, then $M \otimes_A N$ is an $\mathbb{N}$-semi-module with a map of $\mathbb{N}$-semi-modules $\tau : M \times N \to M \otimes_A N$, $(m,n) \mapsto m \otimes_A n$ satisfying the following universal property:

For every *cancellative* $\mathbb{N}$-semi-module G and every *A-balanced* map $\beta : M \times N \to G$, there exists a unique map of $\mathbb{N}$-semi-modules $\gamma : M \otimes_A N \to G$ such that $\beta = \gamma \circ \tau$, i.e. $\gamma(m \otimes_A n) = \beta(m,n)$ for all $m \in M$, $n \in N$.

Special attention should be paid to the fact that for any right (respectively left) semi-module *W* over a semi-ring *A* we have isomorphisms of right (respectively left) *A*-semi-modules $e(W) \overset{\varsigma_W^r}{\cong} W \otimes_A A$ (respectively $e(W) \overset{\varsigma_W^l}{\cong} A \otimes_A W$).

We define a ***semi-coring*** over a semi-ring *A* as an *A*-semi-bimodule *C* with *A*-semi-bimodule maps $\Delta : C \to C \otimes_A C$, $c \mapsto \sum c_1 \otimes_A c_2$ and $\varepsilon : C \to A$, such that $(id \otimes_A \Delta) \circ \Delta = (\Delta \otimes_A id) \circ \Delta$ and $(id_C \otimes_A \varepsilon) \circ \Delta = \varsigma_C^r \circ e_C$, $(\varepsilon \otimes_A id_C) \circ \Delta = \varsigma_C^l \circ e_C$. For every *A*-semi-ring *C*, there is a semi-ring structure on ${}^{*}C := Hom_{A-}(C,A)$, the set of *left* *A*-semi-linear maps from *C* into *A*, with multiplication given by the *convolution product* $(f *^{l} g)(c) := \sum f(c_1 g(c_2))$ and unity ε.

We define a right semi-comodule over an *A*-semi-coring *C* as a right *A*-semi-module *M* with a right *A*-semi-linear map $\rho : M \to M \otimes_A C$ such that $(id_M \otimes_A \Delta) \circ \rho = (\rho \otimes_A id_C) \circ \rho$ and $(id_M \otimes_A \varepsilon) \circ \rho = \varsigma^r_M \circ e_M$. A *C-semi-colinear* morphism between right *C*-semi-comodules (M, ρ_M) and (N, ρ_N) is defined to be a map of right *A*-semi-modules $f : M \to N$ such that $(f \otimes_A id_C) \circ \rho_M = \rho_N$. The category of right *C*-semi-comodules and right *C*-semi-colinear maps is denoted by SM^C. Similarly, we can define *left C-semi-comodules* and *left C-colinear morphisms* between left *C*-semi-comodules. The *C-semi-bicomodules* and *C-semi-bicolinear morphisms* between them are defined in the obvious way.

<u>Task I. Investigating the notions of "Semi-corings and Semi-comodules":</u> During the 1st stage of the project, we intend to examine and investigate the provided definitions of semi-corings and semi-comodules given above. We construct non-trivial examples of semi-corings that are not corings and study their structure. We introduce and investigate also the notions of right (left) semi-coideals, sub-semi-comodules, simple and semi-simple semi-corings etc. In particular we generalize different basic results on the structure of corings (comodules) over rings to semi-corings (semi-comodules) over semi-rings.

<u>Task II. Introducing and investigating new classes of semi-modules:</u> Given a right semi-module *M* over a semi-ring *T*, we introduce the category $\sigma[{}_T M]$ of *M-**subgenerated** left T-semi-modules*, i.e. left *T*-semi-modules that are sub-semi-modules of *M*-generated left *T*-semi-modules. We investigate to what extent properties of categories of this type for modules over rings, as developed in

[Wis91] and [Wis96], can be generalized to semi-modules over semi-rings. We introduce and investigate also ***locally projective semi-modules*** that generalize locally projective modules as defined in [Zim76]. As in the theory of comodules for coring, local projectivity is expected to play an important role in investigating categories of semi-comodules for semi-corings.

Task III. Investigating the category of Semi-comodules: During the 3rd phase of this project, we plan to investigate the category of semi-comodules for a given semi-coring. In particular we seek conditions for this category to be Grothendieck, respectively of type $\sigma[M]$. Here flatness and local projectivity of the semi-coring under consideration are expected to play an important role. Several results on categories of comodules for corings will be generalized (under suitable conditions) to categories of semi-comodules for semi-corings. Among others *simple*, *semi-simple*, *injective* and *projective* semi-comodules will be introduced and studied extensively.

Task IV. Laying the basis for the theory of Hopf semi-algebras: In the last phase of the project we introduce and investigate the notions of semi-coalgebras (respectively semi-bialgebras, Hopf semi-bialgebras) over commutative semi-rings that generalize the notions of coalgebras (respectively bialgebras, Hopf algebras) over commutative ground rings.

7. PERSONNEL REQUIREMENTS

None

8. SCHEDULING

PHASE I	1st – 3rd month			
PHASE II		4th -6th month		
PHASE III			7th - 9th Month	
PHASE IV				10th – 12th month

Remarks:

(1) The applicant has already spent several months in collecting and reading the references needed to carry out the proposed research project (especially books and papers on *semi-rings* and *semi-modules*). This has started long before proposing the project to shorten the time needed to carry out the project.

(2) The applicant is aware of the many *challenging* "technical difficulties" that will accompany the generalization of results from the theory of "corings and comodules" to "semi-corings and semi-comodules". Among the many expected technical difficulties is dealing with the different notions of *exactness of sequences* of semi-modules and the *subtle* definition of projective semi-modules using surjective maps (which were confused with epimorphisms, see [TW89]). However, careful examining of some of the first expected results by the applicant showed that such difficulties can be overcome. The applicant had a *similar* experience of facing such difficulties as he was generalizing results on coalgebras and Hopf algebras over base fields to the more general case of commutative ground rings and later from the commutative to the arbitrary (non-commutative) case.

(3) Developing the theory of semi-corings and semi-comodules will take indeed more than one year (the period proposed by the applicant). However, the applicant found it more suitable to devote this project to laying the basis of the theory. Depending on the extent to which the investigator succeeds in his tasks, further research projects will be carried out in the future to establish more advanced aspects in the theory of semi-corings and semi-comodules.

9. MONITORING and EVALUATION

The best way to evaluate the project results is to have them published in reputed refereed journals and proceedings of international conferences. We expect to publish **1-2** papers including the expected results of this research project.

10. UTILIZATION OF RESULTS

The project is considered as "basic research". We are *still* not aware of any *direct* applications of semi-corings and semi-comodules as the subject has not been even considered before. However, the results of this project are expected to be very useful in:

(1) Developing a new & wide area of research that was NOT considered before.

(2) Unification of the theories of *"**Semi-rings and Semi-modules**"* and *"**Corings and Comodules**"* which have been developed separately so far. This will bring two big groups of researchers together and will enhance research in both areas.

(3) Investigating new concepts in the theory of "semi-rings and semi-modules", which are interesting on their own, e.g. *subgenerated semi-modules* and *locally projective semi-modules*.

(4) Applications of *semi-corings* are expected in different areas. The expectation is real and justified, due to the fact that semi-rings (which turn out to be *trivial* semi-corings) have many applications, as documented in [Gol99a], [Gol99b], [Gol03], [Heb93] & [Kui86]. Moreover, coalgebras, which are (roughly speaking) semi-corings over commutative ground rings have many nice applications, especially in Computer Science, as clarified in many conferences devoted to such applications, e.g. [FHRR05], [AM04], [Mos02].

11. REFERENCES

[Abe80] Abe, E., *Hopf Algebras*, Cambridge Univ. Press (**1980**).

[Abu03] Abuhlail, J., *Rational modules for corings*, Commun. Algebra. **Vol.** 31 (**2003**), pp. 5793-5840.

[Abu05a] Abuhlail, J., *Morita contexts for corings and equivalences.* Hopf algebras in noncommutative geometry and physics, pp. 1–19, Lecture Notes in Pure and Appl. Math., 239, Dekker, New York (**2005**).

[Abu05b] Abuhlail, J., *A Note on coinduction functors between categories of comodules for corings:* Annali dell'Universita' di Ferrara, sez. VII, Scienze Matematiche: Joint Proceedings of "Ferrara Algebra Workshop" & "Swansea Workshop on Hopf Algebras", June 2004, **Vol.** LI **(2005)**, pp. 151-172.

[AG-TL01] Abuhlail, J., Gómez-Torrecillas, J. and Lobillo, J., *Duality and Rational Modules for Hopf Algebras over Commutative Rings,* Journal of Algebra, **Vol.** 240, Num. 1 (**2001**), pp. 165-184.

[Alt04] Al-Thani, H., *Flat Semimodules,* Int. J. Math. Math. Sci. **Vol.** 2004, (**2004**), pp. 873-880.

[AM04] Adámek, J. and Milius, S., *Proceedings of the Workshop on* "*Coalgebraic Methods in Computer Science*". Papers from the 7th Workshop (CMCS'04) held in Barcelona, March 27-29, 2004. Electronic Notes in Theoretical Computer Science **106**. Elsevier Science B.V., Amsterdam (**2004**).

[BB05] Böhm, G. and Brzeziński, T., *Strong connections and the relative Chern-Galois character for corings,* Int. Math. Res. Not. (**2005), no.** 42, pp. 2579-2625.

[BEG-T06] Brzeziński, T., El Kaoutit, L. and Gómez-Torrecillas, J., *The bicategories of corings*, J. Pure Appl. Algebra. **Vol.** 205 (**2006**), pp. 510–541.

[BG-T03] Brzeziński, T. and Gómez-Torrecillas, J., *On comatrix corings and bimodules.* K-Theory, **Vol.** 29 (**2003**), pp. 101–115.

[BKW05] Brzeziński, T., Kadison, L. and Wisbauer, R., *On coseparable and biseparable corings.* Hopf algebras in noncommutative geometry and physics, pp. 71-87, Lecture Notes in Pure and Appl. Math. **239**, Marcel Dekker (2005).

[Böh05] Böhm, G., *Internal bialgebroids, entwining structures and corings*. Algebraic structures and their representations, pp. 207-226, Contemp. Math., **Vol.** 376, Amer. Math. Soc., Providence, RI (**2005**).

[Brz02] Brzeziński, T., *The structure of corings: induction functors, Maschke type theorem, and Frobenius and Galois-type properties*. Algebr. Represent. Theory, **Vol.** 5 (**2002**), pp. 389–410.

[Brz03a] Brzeziński, T., *The structure of corings with a grouplike element.* Noncommutative geometry and quantum groups (Warsaw, 2001), pp. 21–35, Banach Center Publ., **Vol.** 61, Polish Acad. Sci., Warsaw (**2003**).

[Brz03b] Brzeziński, T., *Towers of corings.* Comm. Algebra, **Vol.** 31 (**2003**), pp. 2015-2026.

[Brz05] Brzeziński, T., *Galois comodules*. J. Algebra **Vol.** 290 (**2005**), pp. 503-537.

[BG-T03] Brzeziński, T. and Gómez-Torrecillas, J., *On comatrix corings and bimodules*, K-Theory, **Vol. 29** (2003), pp. 101- 115.

[BV] Böhm, G. and Vercruysse, J., *Morita theory for coring extensions and cleft bicomodules.* (Preprint: math.RA/0601464).

[BW03] Brzeziński, T. and Wisbauer, R., *Corings and Comodules*, Lond. Math. Soc. Lec. Not. Ser. **309**, Cambridge University Press (**2003**).

[BW05] Brzeziński, T. and Wrightson, A., *Complete coverings and Galois corings*. Int. J. Geom. Methods Mod. Phys. **Vol.** 2 (**2005**), pp. 751–757.

[Cae98[Caenepeel, S., *Brauer groups, Hopf algebras and Galois theory*. K-Monographs in Mathematics **4**. *Kluwer Academic Publishers, Dordrecht* (**1998**).

[Cae04] Caenepeel, S., *Galois corings from the descent theory point of view.* Galois theory, Hopf algebras, and semiabelian categories, pp. 163–186, Fields Inst. Commun. **43**, Amer. Math. Soc., Providence, RI (**2004**).

[Cd-G05] Caenepeel, S. De Groot, E., *Galois corings applied to partial Galois theory*. Proceedings of the International Conference on Mathematics and its Applications (ICMA 2004), pp. 117–134, Kuwait Univ. Dep. Math. Comput. Sci., Kuwait (**2005**).

[CI05] Caenepeel, S. and Iovanov, M., *Comodules over semiperfect corings.* Proceedings of the International Conference on Mathematics and its Applications (ICMA 2004), *135--160,* Kuwait Univ. Dep. Math. Comput. Sci., Kuwait (**2005**).

[C-IG-TW03] Castano Iglesias, F., Gomez-Torrecillas, J. and Wisbauer, R., *Adjoint functors and equivalence of subcategories.* Bull. Sci. Math., **Vol.** 127 (**2003**), pp. 379–395.

[CVW04] Caenepeel, S., Vercruysse, J. and Wang, S., *Morita theory for corings and cleft entwining structures.* J. Algebra, **Vol.** 276 (**2004**), pp. 210–235.

[CVW05] Caenepeel, S., Vercruysse, J. and Wang, S., *Rationality properties for Morita contexts associated to corings.* Hopf algebras in noncommutative geometry and physics, pp. 113–136, Lecture Notes in Pure and Appl. Math. **239**, Dekker, New York (**2005**).

[Ded] Dedekind, R., *Über die Theorie der ganzen algebraiscen Zahlen*. Supplement XI to P. G. Dirichlet, L.: *Vorlesung über Zahlentheorie.* 4 Aufl., Druck und Verlag, Braunschweig (**1894**).

[DNR01] Dascalescu, S., Nastasescu, C. and Raianu, S., *Hopf algebras. An introduction*. Marcel Dekker (**2001**).

[Dri86] Drinfe'ld, V.G., *Quantum Groups*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **Vol.** 155 (**1986**), Differentsialnaya Geometriya, Gruppy Li i Mekh. VIII, **Vol**. 193, pp. 18-49, 193; *translation* in J. Soviet Math. **Vol.** 41 (**1988**), pp. 898-915]

[EG-T03] El Kaoutit, E. and Gómez-Torrecillas, J., *Comatrix corings: Galois corings, descent theory, and a structure theorem for cosemisimple corings.* Math. Z., **Vol.** 244 (**2003**), pp. 887-906.

[EG-T04] El Kaoutit, L. and Gómez-Torrecillas, J., *Infinite comatrix corings*. Int. Math. Res. Not. **(2004**), no. 39, 2017-2037.

[EG-T05a] El Kaoutit, E. and Gómez-Torrecillas, J., *Morita duality for corings over quasi-Frobenius rings*. Hopf algebras in noncommutative geometry and physics, pp. 137-153 (**2005**).

[EG-T05b] El Kaoutit, E. and Gómez-Torrecillas, *Comatrix corings and invertible bimodules*, Annali dell'Universita' di Ferrara, sez. VII, Scienze Matematiche: Joint Proceedings of "Ferrara Algebra Workshop" & "Swansea Workshop on Hopf Algebras", June 2004, **Vol.** LI **(2005)**, pp. 263-280.

[EG-TL04] El Kaoutit, E., Gómez-Torrecillas, J. and F. Lobillo, *Semisimple corings*. Algebra Colloq. **Vol.** 11 (**2004**), pp. 427-442.

[FHRR05] Fiadeiro, J.L., Harman, N., Roggenbach, M. and Rutten, J., *Algebra and coalgebra in computer science*. Proceedings of the 1st International Conference (CALCO 2005) held in Swansea, September 3-6, 2005. Lecture Notes in Computer Science **3629**. *Springer*-Verlag, Berlin (**2005**).

[Gol99a] Golan, J., *Semirings and Their Applications*. Kluwer Academic Publishers, Dordrecht (**1999**).

[Gol99b] Golan, J., *Power Algebras over Semirings. With Applications in Mathematics and Computer Science*. Mathematics and its Applications, **488**. Kluwer Academic Publishers, Dordrecht (**1999**).

[Gol03] Golan, J., *Semirings and Affine Equations over Them. Theory and Applications.* Kluwer Academic Publishers, Dordrecht (**2003**).

[G-T02] Gómez-Torrecillas, J., *Separable functors in corings.* Int. J. Math. Math. Sci., **Vol.** 30 (**2002**), pp. 203–225.

[G-TL03] Gómez-Torrecillas, J. and Louly, A., *Coseparable corings.* Comm. Algebra, **Vol.** 31 (**2003**), pp. 4455-4471.

[G-TV] Gómez-Torrecillas, J., and Vercruysse, J., *Comatrix Corings and Galois Comodules over Firm Rings* (Preprint: math.RA/0509106).

[G-TZ-D] Gómez-Torrecillas, J., and Zarouali-Darkaoui, M., *Frobenius functors for corings.* (Preprint: math.RA/0410626).

[Heb93] Hebisch, U. and Weinert, H.J., *Semirings. Algebraic Theory and applications in computer science*, World Scientific (**1993**).

[Hop41] Hopf, H., *Über die Topologie der Gruppen-Mannigfaltigkeiten und ihre Verallgemeinerungen*, Ann. Math. (2). **Vol.** 42 (**1941**), pp. 22-52.

[Kad05] Kadison, L., *Depth two and the Galois Corings,* Noncommutative Geometry and representation theory in mathematical physics, 149-156, Contemp. Math. **391**, *Amer. Math. Soc., Providence, RI* (**2005**).

[Kat04] Katsov, Y**.,** *On flat semimodules over semirings*. Algebra Universalis, **Vol.** 51 (**2004**), pp. 287-299.

[Kui86] Kuich, W., *Semirings, automata, languages*. Springer-Verlag (**1986**).

[LMS99] Litvinov, G. L. , Maslov, V. P. and Shpiz, G. B., *Tensor products of idempotent semimodules. An algebraic approach.* Math. Notes **Vol.** 65 (**1999**), pp. 479-489.

[MM65] Milnor, J.W. and Moore, J.C., *On the structure of Hopf algebras.* Ann. Math., II. Ser. 81, (**1965**) pp. 211-264.

[Mon93] Montgomery, S., *Hopf Algebras and their Actions on Rings*, Am. Math. Soc., Providence RI (**1993**).

[Mos02] Moss, L.S., *Coalgebraic methods in computer science*. Proceedings of the 5th Workshop held in Grenoble, April 6-7, 2002. Electronic Notes in Theoretical Computer Science **65**, Elsevier Science B.V., Amsterdam (**2002**).

[Sok02a] Sokratova, O., *Projective semimodules.* Algebra Universalis. **Vol.** 4 (**2002**), pp. 389-398.

[Sok02b] Sokratova, O., *On semimodules over commutative, additively idempotent semirings.* Semigroup Forum **Vol.** 64 (**2002**), pp. 1-11.

[Swe69] Sweedler, M., *Hopf Algebras*, Benjamin, New York (**1969**).

[Swe75] Sweedler, M.E., *The predual theorem to the Jacobson-Bourbaki theorem,* Trans. Amer. Math. Soc., **Vol.** 213, pp. 391-406 (**1975**).

[Tak79] Takahashi, M., *A bordism category for the ordinary homology theory*, Math. Sem. Notes Kobe Univ. **Vol.** 7 (**1979**), pp. 547-572.

[Tak80] Takahashi, M., *On the bordism categories. I*. Math. Sem. Notes Kobe Univ. **Vol.** 8 (**1980**), pp. 527-546.

[Tak81] Takahashi, M., *On the bordism categories. II*. *Elementary properties of semimodules.* Math. Sem. Notes Kobe Univ. **Vol.** 9 (**1981**), pp. 495-530.

[Tak82a] Takahashi, M., *On the bordism categories. III. Functors Hom and ⊗ for semimodules.* Math. Sem. Notes Kobe Univ. **Vol.** 10 (**1982**), pp. 551-562.
[Tak82b] Takahashi, M., *Extensions of semimodules. I.* Math. Sem. Notes Kobe Univ. **Vol.** 10 (**1982**), pp. 563-592.

[Tak83] Takahashi, M., *Extensions of semimodules. II.* Math. Sem. Notes Kobe Univ. **Vol.** 11 (**1983**), pp. 83-118.

[Tak84a] Takahashi, M., *On semimodules. I.* Kobe J. Math. **Vol.** 1 (**1984**), pp. 67-97.

[Tak84b] Takahashi, M., *On semimodules. II. System of generators.* Kobe J. Math. **Vol.** 1 (**1984**), pp. 177-190.

[Tak85] Takahashi, M., *On semimodules. III. Cyclic semimodules.* Kobe J. Math. **Vol.** 2 (**1985**), pp. 131-141.

[TW89] Takahashi, M. and Wang, H.X.; *On epimorphisms of semimodules*. Kobe J. Math. **Vol.** 6 (**1989**), 297-298.

[Van34] Vandiver, H.S., *Note on a simple type of algebra in which cancellation law of addition does not hold*, Bull. Amer. Math. Soc. **Vol.** 40 (**1934**), pp. 914-920.

[Ver] Vercruysse, J., *Equivalences between categories of modules and categories of comodules.* (Preprint: math.RA/0604423).

[Wis91] Wisbauer, R., *Foundations of Module and Ring Theory, a Handbook for Study and Research*, Gordon and Breach Science Publishers (**1991**).

[Wis96] Wisbauer, R., *Modules and Algebras: Bimodule Structure and Group Action on Algebras*, Addison Wesely Longman Limited (**1996**).

[Wis01] Wisbauer, R., *Weak corings***.** J. Algebra**, Vol.** 245 (**2001**), pp. 123-160.

[Wis02] Wisbauer, R., *On the category of comodules over corings*. Mathematics & mathematics education (Bethlehem, 2000), pp. 325-336, World Sci. Publishing, River Edge, NJ (**2002**).

[Wis05a] Wisbauer, R., *On Galois corings*. Hopf algebras in noncommutative geometry and physics, pp. 309–320, Lecture Notes in Pure and Appl. Math., 239, Dekker, New York (**2005**).

[Wis95b] Wisbauer, R., *From Galois field extensions to Galois comodules*. Advances in ring theory, pp. 263-281, World Sci. Publ., Hackensack, NJ (**2005**).

[Z-Da] Zarouali-Darkaoui, M., *Comatrix Coring generalized and Equivalences of Categories of Comodules.* (Preprint: math.RA/0505534).

[Z-Db] Zarouali-Darkaoui, M., *Adjoint and Frobenius Pairs of Functors for Corings*. (Preprint: math.RA/0505478).

[Zim76] Zimmermann-Huisgen, B., *Pure submodules of direct products of free modules*, Math. Ann. **224**, 233-245 (1976).

12. BUDGET

A. Principal Investigator: Dr. Jawad Abuihlail (ID # 7030290) Project Duration: 12 months Compensation @ SR1,200/- per month	14,400
B. Conferences/Scientific Visits:	10,000
C. Graduate Student	02,400
D. Secretary	01,000
E. Equipment & Stationary:	01,000
TOTAL	**28,800**

Note: The increase in the budget (i.e. adding items C, D and increasing the budget for item B is upon suggestion of the University Research Committee. Attached please find the letter emailed to the applicant on May 11th, 2006).

13. RESUME

See the attached CV