King Fahd University of Petroleum & Minerals
Deanship of Scientific Research

RESEARCH PROJECT NO: ....

Research Project Entitled

"The Structure of Tilting Modules over Commutative Rings"

Duration of Project:	24 months
Proposed Starting Date:	01.09.2007
Ending Date:	31.08.2009
Total Project Cost:	61,000 SR

Submitted (on 15.11.2006) by:

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Contents

1. Abstract

Tilting modules are considered as generalization of progenerators (which characterize Morita equivalences between categories of modules) and are attracting the attention of many researchers in different aspects of mathematics, including mainly "*Representation Theory*" of (finite dimensional, Artin) algebras, "*Categories of Modules*" and "*Commutative Algebra*".

The structure of tilting modules over special classes of commutative rings and domains was investigated by several authors. In particular, tilting modules over Prüfer domains are completely characterized. Moreover, a simple and nice description of tilting modules over Dedekind and valuation domains is obtained. However, a complete description of the structure of tilting modules over (non-Prüfer) domains and more general classes of commutative rings is still not known.

In this project, we aim to investigate and hopefully give a complete description of tiling modules (of arbitrary finite projective dimension) over special classes of non-Prüfer rings (e.g. Matlis domains, Krull domains and their generalizations, Iwangsawa-Gorenstein rings, etc.). For $n \geq 2$, we we investigate (classical) n-tilting modules over n-Prüfer domains and study integral domains for which every (finitely generated) non-zero ideal is (classical) n-tilting.

2000 Mathematics Subject Classification:

13C05: Theory of modules and ideals - Structure, classification theorems
13D07: Homological functors on modules
13F05: Dedekind, Prüfer and Krull rings and their generalizations
13F30: Valuation rings

16D50: Injective modules, self-injective rings
16D90: Module categories
16E10: Homological dimension
16E65: Homological conditions on rings

Keywords: Tilting Modules, $*^n$-modules, Divisible Modules, Homological Dimensions, Iwangsawa-Gorenstein Rings, n-Prüfer Domains, n-coherent rings, Dedekind Domains, Valuation Domains, Krull domains, Matlis domain.

2. Introduction

In a rather restrictive way, *finitely generated tilting modules of projective dimension at most* 1 were introduced by S. Brenner and M. Butler [BB:1980] and then generalized and developed by D. Happel and C. Ringel [HR:1982] (for more on this see [Ass:1990] and the recent monographs [ASS:2006] and [GT:2006]). *Tilting Theory* can be considered, in some sense, as generalization of *Morita Theory* on equivalences between categories of modules over a pair of rings R and S : while a progenerator $_RP$ induces an equivalence of module categories $_R\mathbb{M} \approx {}_{\mathrm{End}(_RP)^{op}}\mathbb{M}$, every tilting module $_RT$ satisfying suitable *finiteness conditions* induces an equivalence between suitable full subcategories of $_R\mathbb{M}$ and $_{\mathrm{End}(_RT)^{op}}\mathbb{M}$ (e.g. the **Tilting Theorem** of Brenner and Butler [BB:1980], see also [HR:1982], [Bon:1981], and its generalizations, e.g. Y. Miyashita [Miy:1986]).

Although the first examples of tilting modules were finitely generated, many interesting examples of *infinitely generated* modules having the tilting property were discovered, e.g. the Fuchs divisible module ∂ over an arbitrary integral domain (as shown first by A. Facchini [Fac:1987], [Fac:1988]), the Bass tilting module $B := \bigoplus\limits_{\mathrm{ht}(\mathfrak{q})=0} \mathrm{E}(R/\mathfrak{q}) \oplus \bigoplus\limits_{\mathrm{ht}(\mathfrak{p})=1} \mathrm{E}(R/\mathfrak{p})$ over a commutative 1-Gorenstein ring R (e.g. L. Angelerei-Hügel et. al. [AHT:2006]) and the Abelian group $\mathbb{Q} \oplus \mathbb{Q}/\mathbb{Z}$ is a tilting $\mathbb{Z}$-module (e.g. R. Göbel and J. Trlifaj [GT:2000]). Moreover, finitely generated tilting modules of arbitrary projective dimension were introduced by Y. Miyashita [Miy:1986].

The following definition of (*generalized*) *tilting modules* of arbitrary finite projective dimension over a ring R is due to L. Angeleri-Hügel and F. Coelho [AC:2001]:

Definition 1. An R-module T is called a **tilting module**, provided:

1. proj.dim.$(_RT) < \infty$;
2. $\mathrm{Ext}^i_R(T, T^{(\Lambda)}) = 0$ for every index set Λ and all $i \geq 1$;
3. There exist $T_0, ..., T_k \in \mathrm{Add}(_RT)$ (the class of direct summands of arbitrary direct sums of $_RT$) fitting in an exact sequence of R-modules

$$0 \to R \to T_0 \to T_1 \to ... \to T_k \to 0. \quad (1)$$

Definition 2. A tilting R-module T with proj.dim.$({}_RT) \leq n$ for some $n \in \mathbb{N}$ will be called n**-tilting**. A finitely generated (n)-tilting module is called **classical** (n)**-tilting.**

As clarified by Bazzoni [Baz:2004(b)], an R-module T is n-tilting module $(1 \leq n < \infty)$ if and only if $\mathrm{Pres}_n(T) = T^{\perp_\infty}$, where $\mathrm{Pres}_n(T)$ is the class of T-n-presented R-modules and $T^{\perp_\infty} := \bigcap_{i=1}^{\infty} \mathrm{Ker}(\mathrm{Ext}^i_R(T, -))$.

Although there is a wide literature existing on tilting modules since their appearance more than 25 years ago, *their structure is still not well understood in most cases.* This surprising fact becomes more clear, if one considers infinitely generated tilting modules (e.g. [Sal:2004]).

Restricting our attention to tilting modules over commutative rings, a *complete* characterization of tilting modules and tilting classes was obtained only in the recent years and only for special classes of integral domains, e.g. Dedekind domains by J. Trlifaj et. al. (e.g. [TW:2002], [TW:2003], [BET:2005]) and Prüfer domains by L. Salce [Sal:2005] (for valuation domains, [GT:2006, Theorem 6.2.21.] combines ideas of [Sal:2005] and [Sal:2004] to remove the technical and set theoretic assumptions in the later article).

The goal of this project is to attack the following main *open problem:*

Investigate and characterize the tilting modules and tilting classes over special classes of commutative (non Prüfer) rings.

In particular, we aim to investigate, and hopefully characterize, tilting modules over Krull domains and their generalizations, Matlis domains, n-Gorenstein rings (among other classes of commutative rings). As S. Bazzoni showed recently [Baz] that all tilting modules over Prüfer domains are 1-tilting, we study n-tilting module over n-Prüfer domains for $n \geq 2$.

Motivated by the observation that 1-Prüfer domains can be characterized as those integral domains for which every finitely generated non-zero ideal is classical 1-tilting, we study integral domains for which every non-zero ideal is (classical) n-tilting, where $n \geq 2$.

Although cotilting modules emerge in many cases as dual to tilting modules, we will restrict ourselves to studying tilting modules over commutative rings and will consider the dual notion only when it is necessary to do so.

3. Literature Review

The interest in tilting modules stems mainly from the fact that they allow generalization of the classical Morita equivalences as well as being a main tool in the study of the representation theory of (finite dimensional, Artin) algebras.

In what follows we present a *brief* literature review of *Tilting Modules* with emphasis on the case where the ground ring is commutative. We remark that we restate some of the results in the literature according to the definitions we adopt in the introduction (*and sometimes in view of later results*).

- *Tilting Theory* originated with the study of reflection functors by Bernstein, Gelfand and Ponomarev [BGP:1973].

- The first instance of a *tilting module* goes back to the work by Auslander, Platzeck and Reiten on a module-theoretic interpretation of Coxeter functors [APR:1979]. These modules are refereed to as the *APR-tilting* modules and were constructed using a projective resolution of a suitable simple module.

- The construction of the *APR-tilting modules* was then extended by Brenner and Butler [BB:1980], who introduced, in a rather restrictive way, tilting modules of finite dimensional algebras over base fields. In fact they were the first to begin an axiomatic approach to the study of tilting modules.

- The theory of *tilting modules* was generalized and extensively developed by Happel and Ringel [HR:1982], who gave the generally accepted set of axioms of a tilting modules over (finite dimensional, Artin) algebras.

- Direct proofs for the main results on *tilted algebras* obtained by Happel and Ringel were provided by Bongartz [Bon:1981], who gave also various applications of the theory, in particular to *representation-finite algebras.*

- Miyashita [Miy:1986] considered finitely generated tilting modules of *arbitrary finite projective* dimension and studied the equivalences of categories induced by them. Later, Miyashita's construction of finitely generated tilting modules of finite projective dimension was (properly) generalized by Fujita [Fuj:1992].

- An *infinitely generated* tilting module of projective dimension 1 over an arbitrary commutative integral domain R is the so called **Fuchs divisible module** ∂ due to Fuchs [Fuc:1984]. This module was investigated intensively by Facchini in [Fac:1987] and [Fac:1988], who showed that ∂ has the tilting property.

- $*$-modules were introduced by Menini and Orsatti [MO:1989] as those R-modules P for which the subcategory $\mathrm{Gen}({}_RP)$ of P-generated R-modules is equivalent to the subcategory $\mathrm{Cogen}({}_SP^*)$ of P^*-cogenerated S-modules, where $S := \mathrm{End}({}_RP)^{op}$, ${}_RQ$ is an injective cogenerator and $P^* := \mathrm{Hom}_R(P, Q)$. These modules were shown by Trlifaj [Trl:1994] to be finitely generated.

- In [Zan:1990], Zanardo characterized $*$-modules over a valuation ring R as those that are isomorphic to $(R/I)^n$ for some suitable $n > 0$ and a suitable ideal $I \lhd R$.

- Colpi and Menini proved in [CM:1993] that a module ${}_RT$ is a classical 1-tilting if and only if ${}_RT$ is a $*$-module that generates all injective R-modules.

- In case R is a commutative ring, Colpi and Menini showed in [CM:1993] that an R-module T is a $*$-module (a classical 1-tilting module) if and only if ${}_RT$ is quasi-progenerator (progenerator).

- In [Trl:1992], Trlifaj showed that classical 1-tilting modules over commutative rings, local rings and Von Neumann regular rings, are progenerators.

- The notion of *infinitely generated* tilting modules over arbitrary rings was introduced by Colpi and Trlifaj in [CT:1995] for the one dimensional case. Such modules were characterized as R-modules T for which $\mathrm{Gen}({}_RT) = \{{}_RM \mid \mathrm{Ext}^1_R(T, M) = 0\}$.

- *Quasi-tilting modules* were presented by Colpi et. al. [CDT:1997] as those finitely generated modules ${}_RT$, for which $\mathrm{Gen}({}_RT) = \{M \in$

$\sigma[{}_RT] \mid \mathrm{Ext}^1_R(T,M) = 0\}$, where $\sigma[{}_RT] \subseteq {}_R\mathbb{M}$ is the subcategory of T-*subgenerated* R-modules (e.g. [Wis:1991]). If R is commutative, it was shown that an R-modules is classical 1-tilting if and only if ${}_RT$ is faithful and quasi-tilting.

- Wisbauer [Wis:1998] considered arbitrary, possibly infinitely generated quasi-tilting modules and called them *self-tilting modules* (as such a self-tilting module ${}_RT$ has the tilting properties in $\sigma[{}_RT]$ the subcategory of all R-modules subgenerated by ${}_RT$). Moreover, their relations with 1-tilting modules and other classes of modules are clarified. The $*$-modules (in the sense of Menini and Orsatti [MO:1989]) were shown to coincide with the self-small self-tilting modules.

- Angeleri-Hügel and Coelho extended in [AC:2001] both the notions of infinitely generated 1-tilting modules and finitely generated n-tilting modules by introducing the notion of *infinitely generated n-tilting modules.*

- Göbel and Trlifaj gave in [GT:2000] a complete description of (partial) tilting Abelian groups assuming Gödel's Axiom of Constructability, $V = L$.

- In [TW:2002] (and [TW:2003]), Trlifaj and Wallutis extended in a natural way the characterizations of tilting $\mathbb{Z}$-modules to tilting modules over *small* Dedekind domains assuming Gödel's Axiom of Constructibility ($V = L$). Recently, Bazzoni et. al. obtained in [BET:2005] the same characterizations of tilting modules over *arbitrary* tilting modules without assuming $V = L$.

- In [Sal:2004], Salce investigated the structure of tilting modules over valuation domains. Assuming T is a module of countable type over a valuation domain R, or that $\left|\widehat{R}\right| \leq 2^{\aleph_0}$ and $V = L$, sufficient and necessarily conditions are given for ${}_RT$ to be tilting in case ${}_RT$ is torsion-free or p.d.$(R_{T^\#}) \leq 1$. Here $\widehat{R}$ is the pure-injective hull of R and $T^\# := \{r \in R \mid rT \subsetneqq T\}$.

- In [Baz], S. Bazzoni considered tilting modules over Prüfer domains and showed that they are of projective dimension at most 1, i.e. 1-tilting.

- The structure of tilting modules over Prüfer domains was investigated by Salce [Sal:2005]. In particular, he showed that the tilting torsion classes over a Prüfer domain R correspond bijectively to *finitely generated localizing systems* (*Gabriel filters*) of R-ideals. For such a system $\mathfrak{F}$, a *generalized Fuchs divisible module* $\partial_{\mathfrak{F}}$ is constructed which turns out to be tilting and to generate the corresponding tilting torsion class.

- In [AHT:2005], Angeleri Hügel et. al. studied the relation between localizations and tilting modules. In particular, they showed that if R is a ring and $\mathcal{O} \subseteq R$ is a left Ore set of *regular elements* of R, then $\text{p.d.}(\mathcal{O}^{-1}R) \leq 1$ if and only if $\mathcal{O}^{-1}R \oplus \mathcal{O}^{-1}R/R$ is a tilting R-module. Moreover, they constructed a divisible module $\partial_{\mathcal{O}}$ that extends the Fuchs-Salce divisible module to the case of non-commutative rings.

- The relation between classical tilting modules and equivalences between subcategories of module categories was investigated in details in the recent monograph [CF:2004].

- So far, all known tilting modules were known to be of finite type. Tilting modules over special classes of ground rings were shown to be of finite type by several authors. For example, Bazzoni showed in [Baz] that every tilting module over a Prüfer domain are of finite type. Making use of the crucial reduction to the *countable case* by Šťovíček and Trlifaj [ST], it was shown recently by Bazzoni and Šťovíček [BS] that over any ground ring "*all tilting modules are of finite type*".

- In a number of recent papers (e.g. [HHTW-2003], [Wei:2005]), J. Wei et. al. generalized the notions of $*$-modules to those of $*^n$-modules and showed that an R-module ${}_RT$ is a classical n-tilting module if and only if ${}_RT$ is a (self-small) $*^n$-module and all injective R-modules admit an Add-Tpresentation of length n.

- In [Wei:2005b] J. Wei generalized the notions of self-small $*^n$-modules (classical n-tilting modules of finite projective dimension $\leq n$) to self-small $*^\infty$-modules (∞-tilting modules of possibly infinite projective dimension). He showed in particular, that classical n-tilting modules in the sense of Miyashita [Miy:1986] are precisely the ∞-tilting modules of finite projective dimension $\leq n$ and that Miyashita's generalization of the Brenner-Butler's Tilting Theorem holds for ∞-tilting modules.

4. Project Objectives and Proposed Research

Our **main objectives** are:

1. to develop a "*Multiplicative Ideal Theory*"-approach to study the structure of tilting modules over commutative rings.
2. to "characterize the structure of tilting modules over different classes of (non Prüfer) commutative rings".

As classical 1-tilting modules over commutative rings are faithful $*$-modules (faithful self-small self-tilting modules), hence progenerators we make the following

Observation: An integral domain R is a Prüfer domain if and only if every non-zero finitely generated ideal is classical 1-tilting.

Task I. Motivated by the observation above, we investigate and hopefully characterize the integral domains R such that every non-zero (finitely generated) ideal $0 \neq I \lhd R$ is n-tilting (where $n \geq 2$).

Task II. We investigate the structure of self-tilting modules over special classes of commutative rings (e.g. Prüfer domains, Dedekind domains, valuation domains).

Task III. We investigate the possible extension of the structure of 1-tilting modules over Dedekind domains to commutative 1-Gorenstein rings. Moreover, we study tilting modules over n-Gorenstein rings for $n \geq 2$.

Task IV. We investigate the structure of tilting modules over special classes of *non-Prüfer* commutative rings (e.g. Matlis domains, Krull domains and their generalizations). Moreover, for $n \geq 2$ we study the structure of n-tilting modules over n-Prüfer domains (in the sense of Costa [Cos:1994]). We notice here that all n-tilting modules over 1-Prüfer domains are 1-tilting by [Baz], hence completely characterized as in [Sal:2005].

Remark: The principal investigator has already carried out an intensive survey and literature review of the subject (this is why this was not included in the tasks above).

5. Time Schedule

Task I	1st -8th month			
Task II:		9th - 12th month		
Task III:			13th - 15th month	
Task IV:				16th - 24th month

6. Personnel Requirements

1 Secretary (24 months)

7. Monitoring and Evaluation

The best way to evaluate the results of project is to have them published in reputed refereed journals and proceedings of international conferences.

8. Utilizing the Results

The results of the project are expected to be very useful in developing the Tilting Theory over commutative rings. In particular, such results will develop and establish a multiplicative ideal theory approach to tilting theory. Moreover, they will serve as a bridge between the two theories.

9. Budget

Principal Investigator (1st Year)		14,400 SR
Principal Investigator (2nd Year)		14,400 SR
Graduate Student (1st Year)		09,600 SR
Graduate Student (2nd Year)		09,600 SR
Consultant (1st Year)		
Consultant (2nd Year)		
Conferences/Scientific Visits		10,000 SR
Books		01,000 SR
Stationary		02,000 SR
Total		**61,000 SR**

10. Resumes

(Separate)

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