DEANSHIP OF SCIENTIFIC RESEARCH

RESEARCH PROJECT NO: IN000296

Title of Proposal:

Primeness and Coprimeness Conditions for Comodules and Corings

Duration of Project (in months) : **27 months**
Proposed Starting Date : **December 15th, 2005**
Ending Date : **March 14th, 2008**
Total Project Cost (SR) : **48,900 SR**

Submitted by:

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Original Version: Tuesday, November 23rd, 2004

Revised Version: Saturday, May 7th, 2005

APPROVALS:

Chairman: ……………………………………… **Date:** ………………………
Department of Mathematical Sciences

Chairman,
Research Committee: ……………………… **Date:** ……..………………

Vice Rector for Graduate Studies
and Scientific Research: …………………….. **Date: ………………………….**

عمادة البحث العلمي

مشروع بحث رقم : IN000296

عنوان البحث : الصفات الأولية و الأولية الخلفية للمعايير و الحلقات الخلفية

مدة البحث	:	27 شهرا
تاريخ بدء البحث	:	15 كانون أول 2005
تاريخ نهاية البحث	:	15 آذار2008
تكلفة المشروع	:	48,900 ريال سعودي

مقدم من :

د. جواد يونس أبو اهليل، أستاذ مساعد، العلوم الرياضية

النسخة الأولى: الثلاثاء، 23 تشرين ثاني 2004

النسخة المعدلة: السبت، 7 أيار 2005

تصديق

رئيس قسم الرياضيات: ---------------------------- التاريخ: --------------------------------

رئيس لجنة البحث العلمي: ------------------------ التاريخ: ---------------------------------

نائب رئيس الجامعة للدراسات العليا

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1. ABSTRACT

In 2001, R. Nekooei and L. Torkzadeh [NT01] introduced the concept of *coprime coalgebras* over base fields as a generalization of simple coalgebras. On the set of *coprime subcoalgebras* of a given coalgebra, they defined a topology whose properties reflect (and are reflected by) the structure of the coalgebra under consideration. For reasons to be clarified in the sequel, we call these coalgebras *pre-coprime.*

Through this research project, we generalize that concept pf pre-coprime coalgebras to *pre-coprime comodules* for corings over (not necessarily commutative) ground rings by replacing the wedge product of subcoalgebras in the original definition of (pre-)coprime coalgebras with the *internal coproduct* of fully invariant subcomodules of the comodule under consideration. Moreover we investigate the properties and the structure of E-*prime* (E-*semiprime*) comodules, i.e. comodules for which the ring of colinear endomorphisms is prime (semiprime). We investigate also the relations between these new coprimeness conditions of a comodule of a given coring with other (co)primeness conditions of the comodule in the literature (considered as a bimodule over the dual ring of its ground coring and over its ring of comodule endomorphisms), as well as their relations with simple, semisimple and irreducible comodules.

In this project, we plan mainly to undertake an extensive and intensive study of these [new] notions with the goal of obtaining a

complete characterization of comodules having these properties. We plan also to construct a topology on the set of pre-coprime subcomodules of a given comodule and study the interplay between the topological properties of that topological space and the structure of the comodule under investigation.

The results we get for comodules will be applied then to develop a theory of *pre-coprime* corings that not only dualizes but also generalizes the theory of prime rings (recall that every ring is a coring in a natural way).

With the results we get on pre-coprime corings and pre-coprime comodules we will try to shed some light on the answer of the following crucial problem:

How to define colocalizations for corings and comodules that dualize and generalize the classical localizations of rings and modules?

We hope to be able to give (at least partial) answer to this open question.

2. ملخص

في العام 2001 قام كل من نيكوي و تركزاده (إيران) في بحث مشترك بتعريف ودراسة مفهوم الجبريات الخلفية الأولية الخلفية (و التي سنسميها الجبريات الخلفية قبل الأولية الخلفية، لأسباب سنذكرها لاحقا) كتعميم لمفهوم الجبريات الخلفية البسيطة المعرفة على حقل تبديلي. الهدف الرئيس من البحث كان تعريف فراغ توبولوجي على مجموعة الجبريات الخلفية الأولية الخلفية الجزئية من جبرية خلفية معطاة، تعكس خصائصه التوبولوجية الخصائص الجبرية للجبرية الخلفية قيد الدراسة.

من خلال مشروع البحث هذا نهدف الى تمديد هذا المفهوم الجديد الى مفهوم المعايير الخلفية قبل الأولية الخلفية للحلقات الخلفية المعرفة على حلقات أساسية (ليست تبديلية بالضرورة) باستخدام مفهوم الضرب الخلفي الداخلي للمعايير الجزئية الثابتة. كما نقوم بتعريف و دراسة مفاهيم جديدة مثل المعايير الخلفية نصف الخلفية الأولية، المعايير الخلفية الأولية الخلفية التامة، المعايير الخلفية نصف الأولية الخلفية التامة، المعايير الخلفية البدائية و المعايير الخلفية نصف البدائية، و ندرس علاقة هذه المفاهيم ببعضها البعض و علاقتها بالمعايير الخلفية البسيطة و المعايير الأولية نصف البسيطة. نهدف بعد ذلك الى دراسة علاقة هذه الخواص الجديدة بغيرها من الخصائص الأولية للمعايير الخلفية و التي يمكن اعتبارها كمعايير مزدوجة على الحلقة النظيرة للحلقة الخلفية و كذلك على الحلقة "ي" المكونة من جميع الدوال خلف الخطية من المعيار الخلفي إلى نفسه

نجري في مشروع البحث هذا دراسة مكثفة و شاملة لتركيبة هذه المعايير الخلفية التي تتحقق فيها هذه الصفات الجديدة بهدف الحصول على توصيف كامل لها. بعد ذلك نقوم بتعريف فراغ توبولوجي على مجموعة المعايير الخلفية قبل الأولية الخلفية الجزئية من معيار خلفي معطى تعكس خصائصه التوبولوجية الخصائص الجبرية للمعيار الخلفي قيد الدراسة.

بعد دراسة المعايير الخلفية الأولية الخلفية و غيرها من الصفات للمعايير الخلفية سنقوم بتطبيق النتائج التي نحصل عليها على الحلقات الخلفية (و التي يمكن اعتبارها كمعايير خلفية بشكل طبيعي) بهدف الحصول على نظرية تناظر و تعمم نظرية الحلقات الأولية.

من خلال هذا البحث سنحاول أن نلقي بعض الضوء على امكانيات الاجابة على التساؤل التالي المهم:

كيف يمكن تعريف الموضعة الخلفية بالنسبة للحلقات و المعايير الخلفية بشكل يناظر و يمدد عملية الموضعة الكلاسيكية للحلقات و المعايير؟

في المرحلة النهائية من هذا البحث سنحاول الحصول على إجابة (ولو جزئية) لهذا التساؤل المفتوح.

3. INTRODUCTION

We assume familiarity with the different notions from the theory of "Corings and Comodules" as in [BW03]. For any undefined notions from the theory of "Rings and Modules" we refer to [Wis91].

Coprime coalgebras over commutative base fields were introduced by R. Nekooei and L. Torkzadeh [NT01] as a generalization of simple coalgebras: simple coalgebras are coprime; and the converse holds for finite dimensional coalgebras. Although the original definition of coprime coalgebras over base fields using the so called *wedge product* of subcoalgebras is not applicable for corings over arbitrary ground rings, we observe that the wedge product of subcoalgebras over base fields is nothing but the *internal coproduct* of these subcoalgebras (in the sense of [RR-MW]). Since there are several notions of coprime modules in the literature (e.g. [Ann], [BJKN80], RR-MW] and since coprime modules in the sense of [RR-MW] are exactly those for which some specific preradical is coprime, we call them pre-coprime modules and call coprime coalgebras in the sense of [NT01] *pre-coprime coalgebras*.

Let C be a coring over a ground ring A, ${}_{A}C$ flat and M be a right C-comodule. The *internal coproduct* of fully invariant C-subcomodules $L', L \subseteq M$ is the fully invariant C-subcomodule $(L' :_{M}^{C} L) := \bigcap\{f^{-1}(L) : f \in \mathrm{End}^{C}(M), f(L') = 0\}$. A fully invariant C-subcomodule $K \subseteq M$ will be called pre-coprime in M (pre-cosemiprime in M), iff for any two (equal) fully-invariant C-subcomodules $L', L \subseteq M$, the inclusion $K \subseteq (L' :_{M}^{C} L)$ implies $K \subseteq L'$ or $K \subseteq L$. The C-comodule M will be called *pre-coprime* (*pre-cosemiprime*), iff M is M-coprime (M-cosemiprime).

As a coalgebras C over base fields is pre-coprime if and only if the dual algebra $C^* \cong \mathrm{End}^C(C)^{op}$ is prime, we investigate in general right comodules M for which the ring of C-colinear endomorphisms $E_M^C := \mathrm{End}^C(M)^{op}$ is prime (respectively semiprime, domain, reduced) and call these comodules *E-coprime* (respectively *E-cosemiprime*, *completely E-coprime*, *completely E-cosemiprime*).

Our *pre-coprime* and *E-prime* comodules generalize simple comodules in the following sense: every simple comodule is pre-coprime; and the converse holds for finitely generated comodules of a left locally projective coring over a right Artinian base ring (e.g. finite dimensional comodules of coalgebras over base fields).

One of the goals of this project is to study these new coprimeness notions for comodules and get complete characterizations for comodules having them. We also consider different (co)primeness conditions from the literature for a right C-comodule M, considered as a left ${}^*C$-module and as a right E_M^C-module, and study their relations with the new coprimeness conditions. We intend also to find out special classes of comodules for which the different primeness and coprimeness conditions coincide.

One of the means we intend to apply in studying the structure of a given comodule is to construct a topology on the set of pre-coprime subcomodules of the comodule under consideration and study the interplay between the topological properties of the topological space we obtain and the algebraic structure of the comodule under consideration. Staring with a pre-coprime or irreducible comodule, that topological space is expected to have nice properties that we intend to investigate.

The results we get for comodules will be applied then to develop a theory of *pre-coprime corings* (recall that any coring can be considered as a bicomodule over itself in a natural way). A coring C will be called *right pre-coprime* (resp. *right pre-cosemiprime*) provided C is so as a right C-comodule. The left notions are defined similarly. We also characterize corings for which the dual rings ${}^{*}C \cong {}^{C}End(C)$ and $C^{*} \cong End^{C}(C)^{op}$ are prime (respectively semiprime, domain, reduced).

In particular, a ring T, considered as a coring via the natural isomorphism $T \cong T \otimes_{T} T$, is right pre-coprime (pre-cosemiprime) as a coring if and only if $T \cong End_{-T}(T) \cong End^{T}(T)$ is prime (semiprime) as a ring. So, the notions and results we get for corings generalize the corresponding ones for rings.

Till now, there is a lack of a good concept of localization for coalgebras (corings). This hampered, for example, the development of cohomology theories for coalgebras. One of the main tasks in this research project will be a *serious attempt,* depending on the results we get for pre-coprime corings, to shed some light on this problem and try to lay the basis for a colocalization theory for corings and comodules dual to the classical theory of localization for rings and modules.

<u>4. LITERATURE REVIEW</u>

First of all we remark that (to the best of our knowledge) *all* published papers in the literature that dealt with *coprime coalgebras* or localization in the theory of Hopf algebras were in the case of commutative base fields.

In what follows we collect background and main contributions mainly from all papers on "coprime coalgebras" that we are aware of:

In [NT01] the authors introduced the notion of coprime (sub)coalgebras. A non-zero subcoalgebra P of a given coalgebra (C,Δ,ε) (over a base field) is said to be <u>*coprime in* C</u> if and only if $P \subseteq X \wedge Y \Rightarrow P \subseteq X \text{ or } P \subseteq Y$ for any subcoalgebras X' and X of C, where the *wedge product* of X' and X is defined as $X' \wedge X := \mathrm{Ker}\,(C \xrightarrow{\Delta} C \otimes C \xrightarrow{\pi_{X'} \otimes \pi_X} C/X' \otimes C/X)$. The coalgebra C is said to be coprime iff C is coprime in C. In Proposition 1.2, it is proved that a subcoalgebra $P \subseteq C$ is coprime in C if and only if $P^{\perp} := \{f \in C^* \mid f(P) = 0\}$ is a prime ideal, where C^* is the dual algebra with the *convolution product*. In particular C is a coprime coalgebra if and only if $0_{C^*} = C^{\perp}$ is a prime ideal, equivalently C^* is a prime algebra.

For a coalgebra C, let X be the set of all coprime subcoalgebras of C. The authors prove then that every simple coalgebra is coprime, and that finite dimensional coprime coalgebras are necessarily simple. After proving that every cocommutative coprime coalgebra has a unique simple subcoalgebra, they conjecture that every infinite dimensional (cocommutative) coalgebra with a unique simple subcoalgebra is necessarily coprime. For any subcoalgebra $E \subseteq C$, let $V(E)$ be the set of coprime subcoalgebras of C contained in E and

$\tau = \{\text{X-V(E)} : \text{E} \subseteq \text{C subcoalgebra}\}$. The authors show that (X, τ) is a topological space, whose properties reflect and are reflected by the structure of the coalgebra C. So, they show for example that (X, τ) is connected if C is irreducible (Proposition 2.3), and is compact (Lindelof) if C has a finite (countable) number of simple subcoalgebras (Proposition 2.4); if (X, τ) is Hausdorff then every coprime subcoalgebra of C is simple (Theorem 2.2), etc.

In [JMR], P. Jara et. al. study coprime subcoalgebras of *path coalgebras* over base fields. After defining the path coalgebra C associated to a graph, they show in particular that the path coalgebra defined by a graph (V,E) is coprime if and only if the graph is strongly connected (Theorem 3.3). The problem of characterizing coprime subcoalgebras of path coalgebras is reduced then to the case of path coalgebras with at most two vertices (Theorem 6.2).

In [XLZ92], the authors use the structure of a given coalgebra C over a base filed to describe some properties of the dual algebra C^*. In particular they gave sufficient and necessary conditions for the dual algebra of coalgebra to be prime (Theorem 3), domain or reduced (Corollary, page 509). Moreover they prove that if for an infinite dimensional coalgebra C, there exists some $f \in C^*$ with $\dim(f \cdot C) < \dim(C)$ then C^* is not prime (Theorem 2).

In her Ph.D. thesis, V. Rodrigues (a student of M. Ferrero, Universidade Federal do Rio Grande do Sul, Brazil) studied *prime* (*semiprime*) *comodule*s and *prime* (*semiprime*) *coalgebras* over base fields (the main results are included in [FR]): A right comodule M of a given coalgebra C over a base filed is said to be *prime* provided M is a prime rational left C^*-module, and a coalgebra C was said to be *prime* provided the right C-comodule $C \otimes C^{op}$ is prime, i.e. $C \otimes C^{op}$ is a prime rational left $(C \otimes C^{op})^*$-module. Observing that for any comodule of a coalgebra C over a base field, the algebra $C^*/\mathrm{ann}_{(C^*}M)$ is left Artinian, prime (semiprime) comodules were characterized as those that are direct sums of simple (prime) subcomodules. Investigating prime coalgebras carefully it turned out that these are just the simple coalgebras (i.e. finite dimensional coprime coalgebras in the sense of [NT01]).

A study of primeness conditions of comodules of coalgebras over their rings of collinear endomorphisms is carried currently by Inda Wijayanti (a current Ph.D. student of Prof. Wisbauer, Heinrich-Heine Universitaet, Duesseldorf-Germany) [Wij]. The applicant is in continuous contact with Mrs. Wijayanti on the new developments in her research.

A well-established theory of "colocalization" for coalgebras is still far from being achieved. Given a coalgebra C (over a base filed), it's not clear how to choose a good "*localizing set*" S to define a new coalgebra $C_{[S]}$. One approach to handle this problem was suggested by M. Takeuchi [Tak85], namely to work in the category of topological coalgebras. In [FS98], M. Farinati and A. Solotar followed this approach and considered the localizing sets as multiplicatively closed subsets of the center of the *topological dual algebra*. Such localizations were used then to study cohomology theories for topological coalgebras.

In [NT96] C. Nastasescu and B. Torrecillas applied results on colocalizations for general Grothendieck categories to study colocalizing full subcategories of the category of right C-comodules of a given coalgebra C over a base field.

5. PROJECT OBJECTIVES

In order to conduct the research under consideration we have to:

- Develop a theory of pre-coprime comodules.
- Study different possible notions of pre-coprime comodules, clarify the relations between them and give conditions for these various coprimeness notions to coincide.
- Construct a topology on the set of pre-coprime subcomodules of a given comodule, study its properties and clarify the interplay between the topological properties of this space and the algebraic structure of the comodule under consideration.
- Apply the results we get for comodules to corings over arbitrary base rings to develop a theory of pre-coprime corings that dualizes and generalizes the theory of prime rings.
- Shed some light on the problem of colocalization for corings and comodules in the hope to share in laying the basis of a theory of colocalization for corings analogous to the classical localization theory of rings and modules.

6. PROPOSED RESEARCH

First of all we fix some notation:

Throughout, C denotes a non-zero coring over a (not necessarily commutative) ring A, ${}^{*}\mathrm{C}:=\mathrm{Hom}_{\mathrm{A}-}(\mathrm{C},\mathrm{A})$ denotes the ring of left A-linear maps from C into A with multiplication $(f *^{l} g)(c) := \sum f(c_1 g(c_2))$ and M^{C} denotes the category of right C-comodules. To avoid any technical difficulties we assume C to be flat as a left A-module, so that M^{C} becomes a Grothendieck category. With M we denote a non-zero right C-comodule and with $\mathrm{E}_{\mathrm{M}}^{\mathrm{C}}:=(\mathrm{End}(\mathrm{M})^{\mathrm{C}})^{op}$ its ring of C-comodule endomorphisms (with the opposite composition). We consider M as a $({}^{*}\mathrm{C},\mathrm{E})$-bimodule in the usual way and call a C-subcomodule $\mathrm{K} \subseteq \mathrm{M}$ *fully invariant* provided K is also a right E-submodule.

A fully invariant C-subcomodule $\mathrm{K} \subseteq \mathrm{M}$ will be called E-coprime (resp. E-cosemiprime, completely E-coprime, completely E-cosemiprime) in M, iff the ideal $\mathrm{Ann}_{\mathrm{E}}(\mathrm{K}) := \{\mathrm{f} \in \mathrm{E}_{\mathrm{M}}^{\mathrm{C}} \mid \mathrm{f}(\mathrm{K}) = 0\}$ is prime (resp. semiprime, completely prime, completely semiprime). In particular we call M an E-*coprime* (resp. E-*cosemiprime, completely* E-*coprime, completely* E-*cosemiprime*) comodule, iff the ring $\mathrm{E}_{\mathrm{M}}^{\mathrm{C}}$ is prime (resp. semiprime, domain, reduced). The corresponding notions for left C-comodules are defined analogously.

The coring C will be called *right* E-*coprime* (resp. *right* E-*cosemiprime*, *completely right* E-*coprime*, *completely right* E-*cosemiprime*) provided C is so as a right C-comodule. We all C *left* E-*coprime* (resp. *left* E-*cosemiprime*, *completely left* E-*coprime*, *completely left* E-*cosemiprime*) provided it's so as a left C-comodule. The coring C will be called E-*coprime* (resp. E-*cosemiprime*, *completely* E-*coprime*, *completely* E-*cosemiprime*), iff it's so as a right and as a left C-comodule.

In what follows we give a brief description of the tasks we plan to undertake during this research project.

Task I. Developing a Theory of Coprime Comodules: During the 1st phase of this project, we plan to undertake an extensive and intensive study of these *new* coprimeness conditions for comodules. This will include for example:

1. Clarifying the relations between E-coprime (E-cosemiprime) & simple (semisimple) comodules. This will include in particular constructing non-trivial examples which show that these are different concepts in general as well as studying the cases where they coincide.
2. Giving sufficient and necessary conditions for a comodule to be (completely) E-coprime, E-cosemiprime and obtaining a complete characterization of these comodules.
3. Considering the right C-comodule comodule M as a $({}^{*}C, E_{M}^{C})$-bimodule in the usual way, we clarify the relation between the E-coprimeness (E-cosemiprimeness) of M as a right C-comodule and the different primeness (semiprimeness) properties of M as left ${}^{*}C$-module and as a right E_{M}^{C}-module.

In addition, we consider *pre-coprime comodules* inspired by the notion of internal coproduct of fully invariant submodules (e.g. [BJKN80], [RR-MW]). Given a right C-comodule M, we define the internal coproduct of two fully invariant C-subcomodules $L, L' \subseteq M$ as $(L' :_{M}^{C} L) := \{f^{-1}(L) : f \in \mathrm{End}^{C}(M), f(L') = 0\}$. A

C-subcomodule $K \subseteq M$ will be called *pre-coprime* (*pre-cosemiprime*) *in M*, iff for any tow (equal) fully invariant C-subcomodules $L, L' \subseteq M$, the inclusion $K \subseteq (L :^{C}_{M} L')$ implies $K \subseteq L$ or $K \subseteq L'$. In particular we call M a *pre-coprime comodule* (*pre-cosemiprime comodule*) provided M is pre-coprime (pre-cosemiprime) in M.

This notion of pre-coprime (pre-cosemiprime) comodules will be studied in details. In particular:

1. We study the relation between pre-coprime (pre-cosemiprime) and E-coprime (E-cosemiprime) comodules and give sufficient conditions for both concepts to coincide.
2. We study the properties and structure of pre-coprime (pre-cosemiprime) comodules. As pre-coprime (pre-cosemiprime) comodules are closely related to E-prime (E-semiprime) comodules, the results we get for them are expected to be analogous to those we expect to get for E-coprime (E-cosemiprime) comodules.

Task II. Study the pre-coprime topology: During the 2nd phase of this project, we plan to construct a topology (X, τ) on the set X of pre-coprime subcomodules of a given right C-comodule M (analogous to the one defined in [NT01] on the set of (pre)-coprime subcoalgebras of a given coalgebra over a base field). In this phase we intend mainly to clarify the interplay between the topological properties of (X, τ) and the comodule structure of M. In particular we aim to answer several questions like:

(1) When is (X, τ) Hausdorff, connected, compact or Lindelof?

(2) What can we say about (X, τ) in case M is pre-coprime or irreducible?

Task III. Study of coprime corings: In this phase we apply the results we get in the 1st and the 2nd phases to corings. Corings can be considered not only as dual structures to rings but also as generalizations of rings. So in the 3rd phase of this research project we seek possible generalizations of results on prime rings to pre-coprime corings.

Task IV. Lay the basis for colocalization theory: In this phase we use the results we obtain for pre-coprime comodules and pre-coprime corings in the first three phases to shed some light on the problem of colocalization for corings and comodules. Among the first and main questions we have to answer in this stage is: *In localization of corings and comodules, what can play the role of prime (maximal) ideals in the classical localization theory of rings and modules?*

7. SCHEDULING

PHASE I	1st - 8th month			
PHASE II		9th - 14th month		
PHASE III			17th - 20th month	
PHASE IV				21st - **27**th month

Remarks:

1. The scheduling given above is approximate.
2. Parts of the work in the 3rd phase will be done parallel to the work done in the first two phases (this is why the scheduled time for that phase is restricted to 4 months).
3. The 4th task is expected to be very difficult; however we expect that the results we get during the first three phases of the research project will provide us by the necessary means to undertake the fourth task.

8. PERSONNEL REQUIREMENTS

(None)

9. MONITORING and EVALUATION

The best way to evaluate the project results is to have them published in reputed refereed journals and proceedings of international conferences. I expect to publish (at least) **three** papers including the expected results of this research project.

10. UTILIZATION OF RESULTS

The results of this project are expected to very useful in developing a theory of colocalization for corings and comodules that not only dualizes but also generalizes the theory of localization for rings and modules.

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12. BUDGET

A. Principal Investigator:
Dr. Jawad Abuihlail (ID # 7030290)
Project Duration: 30 months
Compensation @ SR1,200/- per month 32,400

B. Conference/Scientific Visits: 10,000

C. Equipment & Stationary: 06,5000

Printer	1500
Scanner	0500
Software	3000
Stationary & Miscellaneous (Including Software)	1500

TOTAL **48,900 SR**

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13. RESUME

See the attached CV