King Fahd University of Petroleum & Minerals
Deanship of Scientific Research

INTERNAL KFUPM FUNDED/RESEARCH PROJECT NO:
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Research Project Entitled

"Semibialgebras and Hopf Semialgebras"

Duration of Project:	**24 months**
Proposed Starting Date:	01.03.2008
Ending Date:	28.02.2010
Total Project Cost:	63,000 SR

Submitted (on 2.11.2007) by:

Principal Investigator: **Dr. Jawad Abuihlail** (7030290):
Associate Professor, Department of Mathematics and Statistics

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Approvals:

Chairman: .... Date: ....
Department of Mathematics and Statistics

Chairman: .... Date: ....
Research Committee

Vice Rector: .... Date: ....
for Graduate Studies and Scientific Research

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Name of the Principal Investigator: **Dr. Jawad Abuihlail** (7030290)

Date: 2.11.2007

Contents

1. Abstract

In this project we develop - *from scratch* - the theory of "*semibialgebras*" and "*Hopf Semialgebras*" over commutative semirings (semifields), which generalizes and extends the theory of bialgebras and Hopf Algebras over commutative rings (fields). In particular, we provide non-trivial examples of Hopf semialgebras (semibialgebras) that are not Hopf algebras (not bialgebras) and study their structures. For a given semibialgebra H, and analogous to the classical case, we introduce and study H-semi(co)module semi(co)algebras, Doi-Koppinen structures (H, A, C) and the associated categories of Doi-Koppinen semimodules $\mathbb{SM}(H)_A^C$. Assuming suitable conditions, we consider *dual semicoalgebras* (reps. *dual semibialgebras*, *dual Hopf semialgebras*) of semialgebras (resp. semibialgebras, Hopf semialgebras) over special classes of commutative semirings.

2000 Mathematics Subject Classification:

16: Associative Algebras
- **16W**: Rings and algebras with additional structure
  - **16W30**: Coalgebras, bialgebras, Hopf algebras; rings, modules, etc. on which these act
- **16S**: Rings and algebras arising under various constructions
- **16Y**: Generalizations
  - **16Y60**: Semirings

Keywords: Semifields; Semirings; Semimodules; Semialgebras; Semicoalgebras; Semibialgebras; Hopf Semialgebras; Doi-Koppinen Structures; Doi-Koppinen Semimodules; Semimodule Semialgebras; Semimodule Semicoalgebras; Semicomodule Semialgebras; Semicomodule Semicoalgebras; Dual Semicoalgebras; Dual Semibialgebras; Dual Hopf Semialgebras

2. Introduction

The theory of *Hopf Algebras* is one of the most interesting theories in mathematics that is attracting continuously the attention of an increasing number of researchers in several areas of Mathematics and related disciplines (e.g. Physics and Computer Science). On the other hand, *semirings* and the associated categories of *semimodules* are well studied and have been shown to have a wide spectrum of applications (e.g. Automata Theory, Optimization Theory, Generalized Fuzzy Computation, Baysian Networks and Belief Propagation).

Our main goal in this project is to generalize the notions of Hopf algebras (bialgebras) to Hopf semialgebras (semibialgebras). According to our definition, every commutative semiring R is a *trivial* Hopf semialgebra and the associated category of R-semimodules is isomorphic to the category of R-semicomodules. So the theory of Hopf semialgebras (as we develop) generalizes the theory of semimodules. This approach was applied in [Abu] too, where any arbitrary semiring A was considered as a trivial *semicoring* and the associated category of right (left) A-semimodules is isomorphic to the category of right (left) A-semicomodules.

In this project we develop - from scratch - the theory of semibialgebras and Hopf semialgebras over commutative semirings (semifields) and generalize to them several results known for bialgebras and Hopf algebras over commutative rings (fields). The results we are targeting in this product are generalizations of the corresponding results for bialgebras and Hopf algebras in [BW2003, Chapter II] ([Swe1969, Chapters III - VI]) as well as in our previous papers [AG-TW2000] and [Abu2005]. Given a semibialgebra H, we investigate the Hopf H-semimodules and prove the *Fundamental Theorem of Hopf Semialgebras*. We also introduce Doi-Koppinen structures (H, A, C) and the associated categories of Doi-Koppinen semimodules $\mathbb{SM}_A^C(H)$. Given a commutative semiring R and an R-semialgebra (resp. R-semibialgebra, Hopf R-semialgebra) A, we consider possible structures of R-semicoalgebras (resp. R-semibialgebras, Hopf R-semialgebras) over the finite dual $A^\circ \subseteq A^*$ of continuous semilinear maps from A (considered with the cofinite topology) to the ground semiring R (considered with the discrete topology).

3. Literature Review

In [Abu] we introduced the notions of *semibialgebras* and *Hopf semialgebras* over commutative semirings (semifields). To the best of our knowledge, we are the first to introduce and investigate these concepts and there is no single paper on the subject of the current project in the published literature.

In what follows we present some *highlights* from the literature on topics related to the current research project:

Semi-rings and Semi-modules:

Semiring (semimodules) can be *roughly* defined as rings (modules) without subtraction. Apart from the trivial examples of *semirings* which are not rings, namely the set $\mathbb{N}_0 := \{0, 1, 2, 3, ...\}$ of non-negative integers and the set $\mathbb{R}^+ := [0, \infty)$ of non-negative real numbers with the usual addition and multiplication, the first non-trivial example appeared first in the work of German mathematician Richard Dedekind [Ded1894], in connection with the algebra of ideals of a commutative ring

Later they were studied independently, especially by the American mathematician H.S. Vandiver, who worked very hard to get them accepted as a fundamental algebraic structure, being basically the "*best*" structure which includes both rings and bounded distributive lattices [Van1934]. He was not successful, however, and – with only a few exceptions – semirings had fallen into disuse and were well on their way to mathematical oblivion until they were "rescued" during the late 1960's when real and significant applications were found for them. Numerous applications of semirings are obtained in Automata Theory, Optimization Theory, Generalized Fuzzy Computation, Baysian Networks and Belief Propagation, Algebraic Geometry over the optimization algebra. Many of these applications are documented in a series of books on the subject by J. Golan [Gol1999a], [Gol1999b], [Gol2003] as well as in [Heb1993] and [Kui1986]. The theory of semi-modules over semi-rings was developed by several authors. We refer mainly to the fundamental series of papers by M. Takahashi [Tak1979] - [Tak1985] in addition to Golan's books mentioned above.

Bialgebras and Hopf Algebras:

In 1941, the first example of Hopf algebras appeared in algebraic topology in the work of the German mathematician H. Hopf [Hop1941]. However, the first paper to attract the attention of algebraist was on graded Hopf algebras by J.W. Milnor and J.C. Moore [MM1965]. During the 1960s and 1970s, Hopf algebras were studied intensively from a purely algebraic point of view. The first book in this direction was [Swe1969] by M. Sweedler. A subsequent book of this nature, with more flavor of algebraic geometry, is that of E. Abe [Abe1980]. Hopf algebras appear in many fields of mathematics: number theory (formal groups), algebraic geometry (affine group schemes), Lie algebras (the universal enveloping algebra is a Hopf algebra), graded ring theory (gradings are coactions), Galois Theory, the theory of Azumaya algebras and Brauer groups, etc.

In 1988 V.G. Drinfel'd published his famous paper "*Quantum Groups*" [Dri1988]. Since then, the subject received a huge impetus because of the discovery of interesting applications in Quantum Mechanics, Statistical Mechanics and Knot Theory. This resulted in a revival of the algebraic theory of Hopf algebras, since the late 1980s, making it one of the mainstream subjects in mathematics in the 1990's. Many books on quantum groups, which are certain *non-commutative and non-cocommutative Hopf algebras*, have been published since the late 1980's (e.g. [Maj1995], [Kas1995]). In addition, there were two books that concentrated on the purely algebraic aspects of the theory of Hopf algebras: one by S. Montgomery [Mon1993] and another by S. Dascalescu et. al. [DNR2001].

Corings and Comodules:

Intensive investigations of the so called Doi-Koppinen structures and the associated categories of Doi-Koppinen modules were conducted by several authors (see the monograph by S. Caenepeel et. al. [CMZ2002]. Doi-Koppinen modules were generalized by T. Brzeziński and S. Majid [BM1998] to the so called entwining structures and entwined modules. These attracted (and are still attracting) the attention of many researchers. In 1999, M. Takeuchi pointed out that *entwining structures*, give rise to new examples of *corings* (defined first by M. Sweedler [Swe1975]). This was in fact behind the revival of the theory of corings and their comodules in the recent years.

With the many new examples discovered, it turned out that corings might have a variety of unexpected and wide-ranging applications, to topics in Non-commutative Ring Theory, Category Theory, Hopf Algebras, Differential Graded Algebras, and Non-commutative Geometry. For more on this, one may refer to the monograph [BW2003] by T. Brzeziński and R. Wisbauer in addition to that of S. Caenepeel et. al. [CMZ2002] as well as to several articles on the subject by G. Böhm, T. Brzeziński, S. Caenepeel, L. Kadison, E. El Kaoutit, J. Gómez-Torrecillas, J. Vercruysse and R. Wisbauer in addition to J. Abuhlail (e.g. [Abu2003], [Brz2002], [BB2005], [BKW2005], [Boh2005], [Brz2002], [BV2007], [Cae2004], [CdGV2006], [CdGV2007], [EG-T2003], [EG-T2004a], [EG-T2004b], [EG-T2005a], [EG-T2005b], [Kad2005]).

Semicorings and Semicomodules:

In [Abu], we developed - from scratch - the theory of "*semicorings and semicomodules*" over arbitrary (not necessarily commutative) semirings. In particular we construct non-trivial examples of semi-corings that are not corings and studied their structure; we introduced and investigated the notions of cosemisimple semicorings and generalized different basic results on the structure of corings (comodules) over rings to semicorings (semicomodules) over semirings. We investigated also the category of semicomodules for a given *locally projective* semicoring as we did previously with the category of comodules of a locally projective coring in [Abu2003].

Semibialgebras and *Hopf semi-algebras* over commutative ground semirings (semifields) were defined first by us in [Abu]; however, we did not have the chance to investigate them deeply as we intend to do in this project.

4. Objectives and Proposed Research

With a *commutative semiring* we mean an algebraic structure $(R, +_R, \cdot_R)$, where R is a non-empty set and

$$+_R : R \times R \to R \ \& \ \cdot_R : R \times R \to R,$$

are binary operations such that:

- $(R, +_R, 0_R)$ is an Abelian monoid with neutral element 0_R;
- $(R, \cdot_R, 1_R)$ is a commutative monoid with neutral element 1_R.
- $a \cdot (b + c) = a \cdot b + a \cdot c$ for all $a, b, c \in R$.

Example of commutative semi-rings are the set $\mathbb{N}_0 = \{0, 1, 2, 3, ...\}$ of non-negative integers and the set of $R^+ := [0, \infty)$ of non-negative real numbers with the usual addition and multiplication, and the set of ideals of a commutative ring.

Let R be a semiring. An *R-semimodule* is a triple $(M, +, \rightharpoonup)$, where M is a non-empty set and

$$+_M : M \times M \to M \ \& \ \rightharpoonup_M : R \times M \to M,$$

are maps defined such that for all $r_1, r_2, r \in R$ and $m_1, m_2, m \in M$:

- $(M, +_M)$ is an Abelian monoid with neutral element 0_M;
- $r(m_1 + m_2) = rm_1 + rm_2$, $(r_1 + r_2)m = r_1m + r_2m$; $(r_1r_2)m = r_1(r_2m)$;
- $1_Rm = m$ and $0_Rm = 0_M = r0_M$.

In [Abu], we began the investigation of semicorings and semicomodules. The last task of that project was "*laying the basis for the theory of Hopf semialgebras*". In the last phase of the project "*we introduced and investigated the notions of semicoalgebras* (*respectively semibialgebras, Hopf semibialgebras*) *over commutative semirings that generalize the notions of coalgebras* (*respectively bialgebras, Hopf algebras*) *over commutative ground rings*".

However, that was just the first step!!. In this project, we take further steps towards our main goal:

Developing the theory of semibialgebras & Hopf semialgebras.

To achieve this we begin a deep study and investigation of *semibialgebras* and *Hopf semialgebras* over commutative semirings (semifields) as introduced earlier in [Abu], and generalize to them know results for bialgebras and Hopf algebras over commutative rings, e.g. Chapter 2 of [BW2003] (Chapters 3-6 of [Swe1969]).

The main task will be the following (where R is a commutative semiring):

- **Task I (Semibialgebras)**: We investigate the notion of *R-semibialgebras*, and construct non-trivial examples of semibialgebras that are not bialgebras. Given an R-semibialgebra B, we investigate the invariants and coinvariants of B-comodules. We also introduce and investigate the category of Hopf B-modules $\mathbb{M}_B^B$. Integrals in and on B will be also introduced and investigated.

- **Task II (Hopf Semialgebras)**: We investigate the notion of *Hopf R-semialgebras*, and construct non-trivial examples of Hopf R-semialgebras that are not Hopf algebras. We also prove the *Fundamental Theory of Hopf Semialgebras* (analogous to the Fundamental Theory of Hopf Algebras). We investigate *trace ideals* and (*total*) *integrals*, Hopf semialgebras with bijective antipodes and *cosemisimple Hopf semialgebras*. *Semiperfect Hopf semialgebras* over special types of semirings and *separable Hopf semialgebras* (over semifields) will also be investigated.

- **Task III (Category of Doi-Koppinen Semimodules)**: Let H be an R-semibialgebra. We introduce and study H-semimodule semialgebras (resp. H-semimodule semicoalgebras, H-semicomodule semialgebras, H-semicomodule semicoalgebras). We also introduce and investigate the notions of *Doi-Koppinen semistructures* (H, A, C) and the associated categories $\mathbb{SM}_A^C(H)$ of *Doi-Koppinen semimodules*.

- **Task IV (Duality)**: Given an R-semialgebra (resp. a semibialgebra, a Hopf semialgebra) A, we consider possible structures of R-semicoalgebras (resp. R-semibialgebras, Hopf R-semialgebras) on the *finite dual*

$$A^\circ := \varinjlim\{(A/I)^* \mid I \lhd A,\ A/I \text{ is finitely generated } R\text{-semimodule}\}.$$

Other duals of (suitable (A, A)-subsemibimodules of $A^* := \mathrm{Hom}_R(A, R)$) will be considered too. In case the semiring R is not a semifield, suitable assumption will be assumed as was done in [AG-TW2000] and [Abu2005].

5. Rough Time Schedule

Task I	1st - 6th month			
Task II:		7th -14th month		
Task III:			15th - 18th month	
Task VI:				19th - 24th month

6. Personnel Requirements

1 Secretary (24 months)

7. Monitoring and Evaluation

The best way to evaluate the results of the project is to have them published in reputed refereed journals and/or proceedings of international conferences. I highly expect to publish the main results of the project in (at least) **two papers.**

8. Utilizing the Results

The results of the project are expected to be very useful in:

- Opening a **NEW** and **WIDE** area of research (we develop the theory from scratch).

- *Quantum groups* (which are suitable non-commutative non-cocommutative Hopf algebras) showed to have important applications in several areas of Mathematics, Statistical Mechanics and Physics. The current project is hoped to lead us to a generalized concept of *Quantum Semigroups* (which we could define roughly as suitable non-commutative non-cocommutative Hopf semialgebras). We should warn here that this definition is still to be examined carefully and is not final.

- Semirings have many applications and so do bialgebras and Hopf algebras. We think that *semibialgebras* and *Hopf semialgebras* will have a wider spectrum of applications in different areas of Mathematics, Physics and Computer Science.

- Applications of semibialgebras and Hopf semialgebras are expected in different areas, since commutative semi-rings (which are trivially Hopf semialgebras) have many applications, as documented in [Gol1999a], [Gol1999b], [Gol2003], [Heb1993] & [Kui1986]. Moreover, coalgebras over commutative ground rings have many nice applications, especially in Computer Science, as clarified in proceedings of conferences devoted to such applications, e.g. [FHRR2005], [AM2004] and [Mos2002].

9. Budget

Principal Investigator (24 months)		28,800 SR
Graduate Students (23 months)		19,200 SR
Conferences/Scientific Visits		10,000 SR
Secretary		01,000 SR
Books		02,000 SR
Stationary		02,000 SR
Total		**63,600 SR**

10. Resume

(See the attached CV)

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