On the Symmetry Structures of the Minkowski Metric and a Weyl Re-Scaled Metric

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Abstract Symmetries of spacetime manifolds which are given by Killing vectors are compared with the symmetries of a Lagrangian constructed from a Weyl re-scaled metric used in discussing disorder operators in Gauge theories. We find the point generators of the one parameter Lie groups of transformations that leave invariant the action integral corresponding to the Lagrangian (Noether symmetries). It is shown that the Noether symmetries obtained by considering the Lagrangian provide additional symmetries which are not provided by the Killing vectors. New conservation law/s are determined.

1 Introduction

The general theory of relativity, which is a filed theory of gravitation, is described by the Einstein field equations (EFE). These equations are expressed in terms of Lorentzian metric g_{ab} and are highly nonlinear. Because of this nonlinearity it is quite difficult to find their exact solutions. On the other hand if Lorentzian metric is chosen to define the Einstein tensor, then any arbitrary g_{ab} shall form a solution of the EFE. However, such arbitrary solutions can not be of any interest as they do not represent a physically plausible situation. Thus to find a physically interesting solution of the EFE some restrictions (e.g. spherical or axial symmetry etc.) are imposed on the spacetime metric they depend upon. These restrictions are generally known as Killing vectors or isometries. A KVs [1] is the one along which Lie

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