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Symmetry classifications and reductions of some classes of (2 + 1)-nonlinear heat equation

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Abstract

The (2 + 1)-nonlinear heat equation $u_t - f(u)(u_{xx} + u_{yy}) = 0$ is considered. A symmetry classification of the equation using Lie group method is presented and reduction to the first- or second-order ordinary differential equations is provided.

Keywords: Two-dimensional nonlinear heat equation; Symmetry classification

1. Introduction

The one-dimensional heat equation is extensively studied from the point of view of its Lie point symmetries. A detailed symmetry analysis of this equation can be found in Cantwell [1], Ibragimov [2] and Bluman and Kumei [3]. Since thermal diffusivity of some materials may be a function of temperature, it introduces nonlinearities in the heat equation that models such phenomenon. This shows that whereas nonlinear heat equation models real world problems the best, it may be difficult to tackle such problems by usual methods. In an attempt to study nonlinear effects Saied and Hussain [4] gave some new similarity solutions of the (1 + 1)-nonlinear heat equation. Later Clarkson and Mansfield [5] studied classical and nonclassical symmetries of the (1 + 1)-heat equation and gave new reductions for the linear heat equation and a catalogue of closed-form solutions for a special choice of the function $f(u)$ that appears in their model. In higher dimensions Servo [6] gave some conditional symmetries for a nonlinear heat equation while Goard et al. [7] studied the nonlinear heat equation in the degenerate case. Nonlinear heat equations in one or higher dimensions are also studied in literature by using both symmetry as well as other methods [8,9]. An account of some interesting cases is given by Polyanin [10].

As pointed out above, the thermal diffusivity of materials such as gases is not a constant, but depends upon the temperature of the body. Physically it is quite an interesting situation and can be modelled by (2 + 1)-nonlinear heat equation

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