# Collineations of the Ricci tensor

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Ricci collineations for the Ricci tensor which is constructed from a general spherically symmetric and static metric are classified for all possibilities of $R_{ab}(r)$ (such that $R_{ab} \neq 0$ for a=b). It turns out that the only collineations admitted by this tensor can be ten, six, or four and there does not appear any case in between.

## I. INTRODUCTION

The general theory of relativity, which is a field theory of gravitation, is described by the Einstein field equations. These equations, whose fundamental constituent is the space–time metric $g_{ab}$, are highly nonlinear partial differential equations (pde's) and, therefore it is very difficult to obtain their exact solutions. They become still more difficult to solve if the space–time metric depends on all coordinates.[1] This problem, however, can be simplified to some extent if some geometric symmetry properties are assumed to be possessed by the metric tensor. These geometric symmetry properties are described by killing vector fields (the number of independent killing vector fields fixes the symmetry structure of different space–times) and lead to conservation laws in the form of first integrals (i.e., constants of motion) of a dynamical system.[2,3] There exist, by now, a reasonably large number of solutions of the Einstein field equations possessing different symmetry structures.[4] These solutions have been further classified according to their properties and groups of motions admitted by them.[5]

After the space–time metric, the curvature and the Ricci tensors are other important candidates which play a significant role in understanding the geometric structure of space–times in relativity.[6] In reviewing the topic of new principles of classifying geometries, the possible value of general symmetry considerations (e.g., Ricci collineations, curvature collineations) is stressed as being a basis for invariant classification.[7] Katzin, Lavine, and Davis[8] were the pioneers in carrying out a detailed study of curvature and Ricci collineations in the context of the related particle and field conservation laws that may be admitted in the standard form of general relativity.[9] Some more rigorous work was done on the important role of Ricci collineations admitted by certain relativistic matter fields and the related conservation laws that may be admitted in the corresponding Riemannian space–times.[10] In this work it was emphasized that Ricci collineations provide an invariant classification scheme for certain types of matter fields.

Following a remark[11] and keeping in view the importance of Ricci collineations in classifying certain types of matter fields, we again readdress this problem from the point of view of obtaining a complete classification of Ricci collineations for the Ricci tensor constructed from a spherically symmetric and static metric. To be able to classify these collineations we employ, in this article, a modified procedure used to classify general space–times,[12] which was developed by us previously to classify spherically symmetric and static space–times.[13] By an exhaustive application of this procedure it turns out that a Ricci tensor, with nonzero diagonal components, constructed from a spherically symmetric and static metric tensor can possess ten, six, or four collineations only and there does not appear any case in between. The relationship between the Ricci collineations and the isometries will be discussed in detail in a forthcoming article.[14]

The plan of this article is as follows. In the next section we briefly outline our method to classify Ricci collineations by writing them down in a form in which they become known functions of $\vartheta$ and $\phi$ and unknown in those of $t$ and $r$ coordinates. We then substitute these


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