Hashing: Collision Resolution Schemes

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Collision Resolution Techniques

- There are two broad ways of collision resolution:

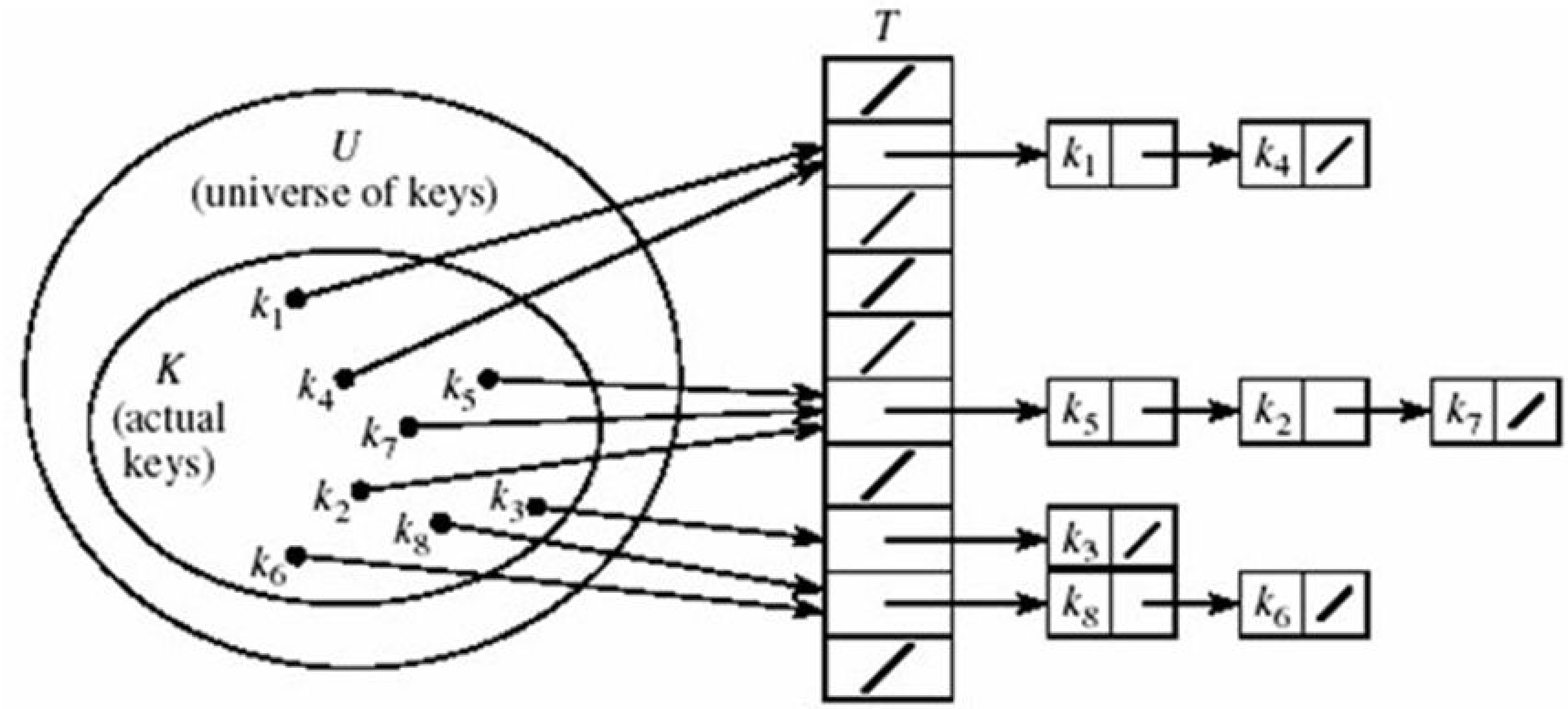
1. Separate Chaining: An array of linked list implementation.

2. Open Addressing: Array-based implementation.

   (i) Linear probing (linear search)
   (ii) Quadratic probing (nonlinear search)
   (iii) Double hashing (uses two hash functions)

Separate Chaining

- The hash table is implemented as an array of linked lists.
- Inserting an item, **`r`**, that hashes at index **`i`** is simply insertion into the linked list at position **`i`**.
- Synonyms are chained in the same linked list.



Separate Chaining (cont'd)

- Retrieval of an item, **r**, with hash address, **i**, is simply retrieval from the linked list at position **i**.
- Deletion of an item, **r**, with hash address, **i**, is simply deleting **r** from the linked list at position **i**.
- **Example:** Load the keys **23, 13, 21, 14, 7, 8, and 15** , in this order, in a hash table of size **7** using separate chaining with the hash function: **h(key) = key % 7**

h(23) = 23 % 7 = 2

h(13) = 13 % 7 = 6

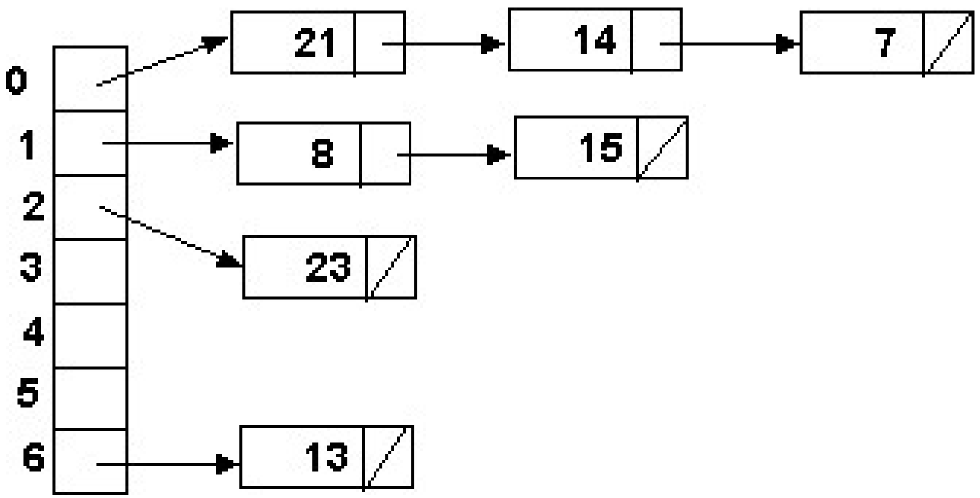
h(21) = 21 % 7 = 0

h(14) = 14 % 7 = 0 collision

h(7) = 7 % 7 = 0 collision

h(8) = 8 % 7 = 1

h(15) = 15 % 7 = 1 collision



Separate Chaining with String Keys

- Recall that search keys can be numbers, strings or some other object.
- A hash function for a string s = c0c1c2…cn-1 can be defined as:

$$hash = (c_0 + c_1 + c_2 + \ldots + c_{n-1}) \% tableSize$$

this can be implemented as:

```
public static int hash(String key, int tableSize){
   int hashValue = 0;
   for (int i = 0; i < key.length(); i++){
       hashValue += key.charAt(i);
   }
   return hashValue % tableSize;
}
```

- Example: The following class describes commodity items:

```
class CommodityItem {
    String name;     // commodity name
    int quantity;    // commodity quantity needed
    double price;    // commodity price
}
```

Separate Chaining with String Keys (cont'd)

- Use the hash function **hash** to load the following commodity items into a hash table of size **13** using separate chaining:

```
onion       1     10.0
tomato      1     8.50
cabbage     3     3.50
carrot      1     5.50
okra        1     6.50
mellon      2     10.0
potato      2     7.50
Banana      3      4.00
olive       2     15.0
salt        2     2.50
cucumber    3     4.50
mushroom    3     5.50
orange      2     3.00
```

- Solution:

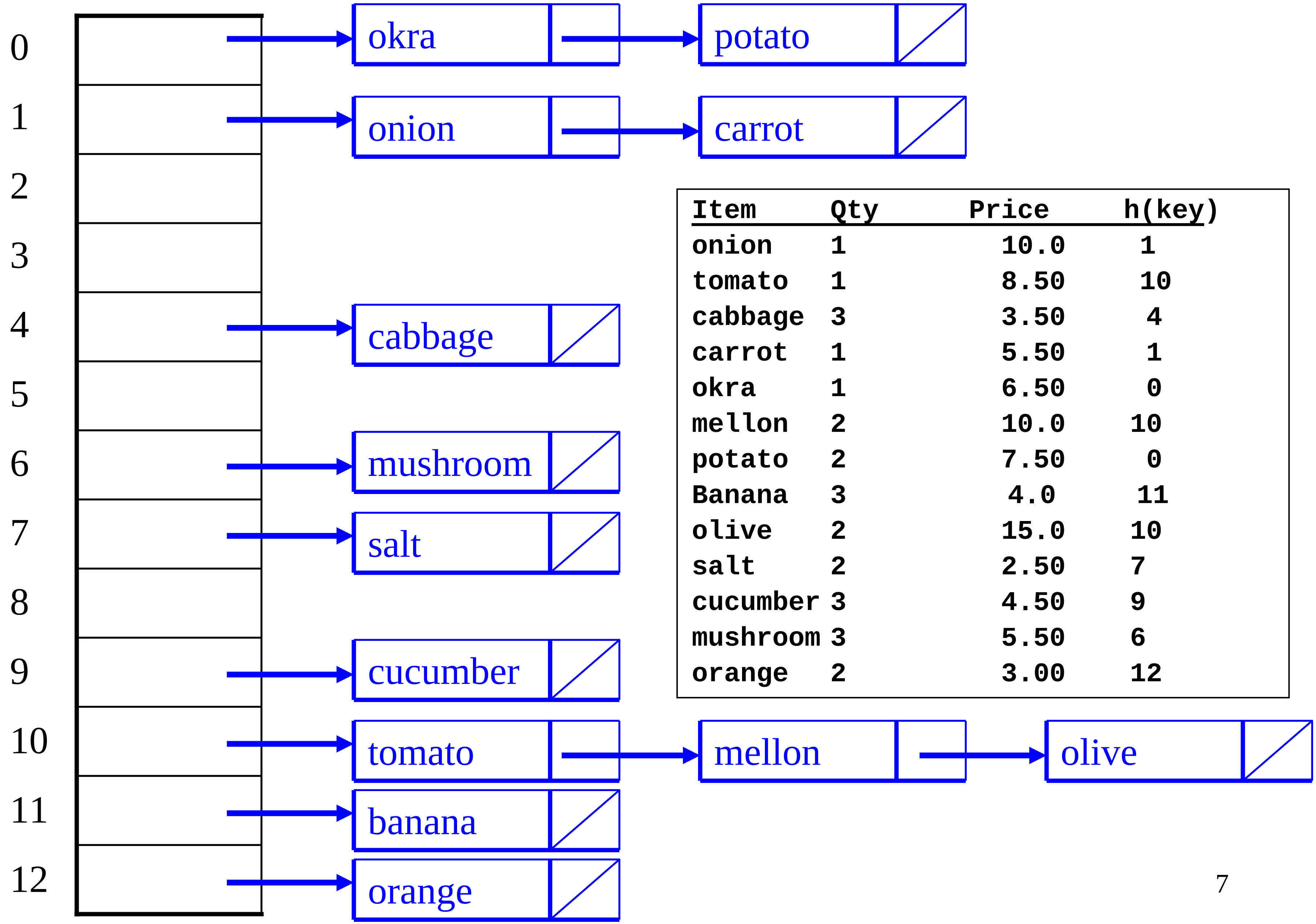
character	a	b	c	e	g	h	i	k	l	m	n	o	p	r	s	t	u	v
ASCII code	97	98	99	101	103	104	105	107	108	109	110	111	112	114	115	116	117	118

hash(onion) = (111 + 110 + 105 + 111 + 110) % 13 = 547 % 13 = 1

hash(salt) = (115 + 97 + 108 + 116) % 13 = 436 % 13 = 7

hash(orange) = (111 + 114 + 97 + 110 + 103 + 101)%13 = 636 %13 = 12

Separate Chaining with String Keys (cont'd)



Item	Qty	Price	h(key)
onion	1	10.0	1
tomato	1	8.50	10
cabbage	3	3.50	4
carrot	1	5.50	1
okra	1	6.50	0
mellon	2	10.0	10
potato	2	7.50	0
Banana	3	4.0	11
olive	2	15.0	10
salt	2	2.50	7
cucumber	3	4.50	9
mushroom	3	5.50	6
orange	2	3.00	12

Separate Chaining with String Keys (cont'd)

- Alternative hash functions for a string

  $\mathbf{s = c_0c_1c_2\ldots c_{n-1}}$

exist, some are:

- hash = $(c_0 + 27 * c_1 + 729 * c_2)$ % tableSize
- hash = $(c_0 + c_{n-1} + \text{s.length()})$ % tableSize
- hash = $[\sum_{k=0}^{s.length()-1} 26 * k + \text{s.charAt(k)} - '\ ']\%\text{tableSize}$

Separate Chaining versus Open-addressing

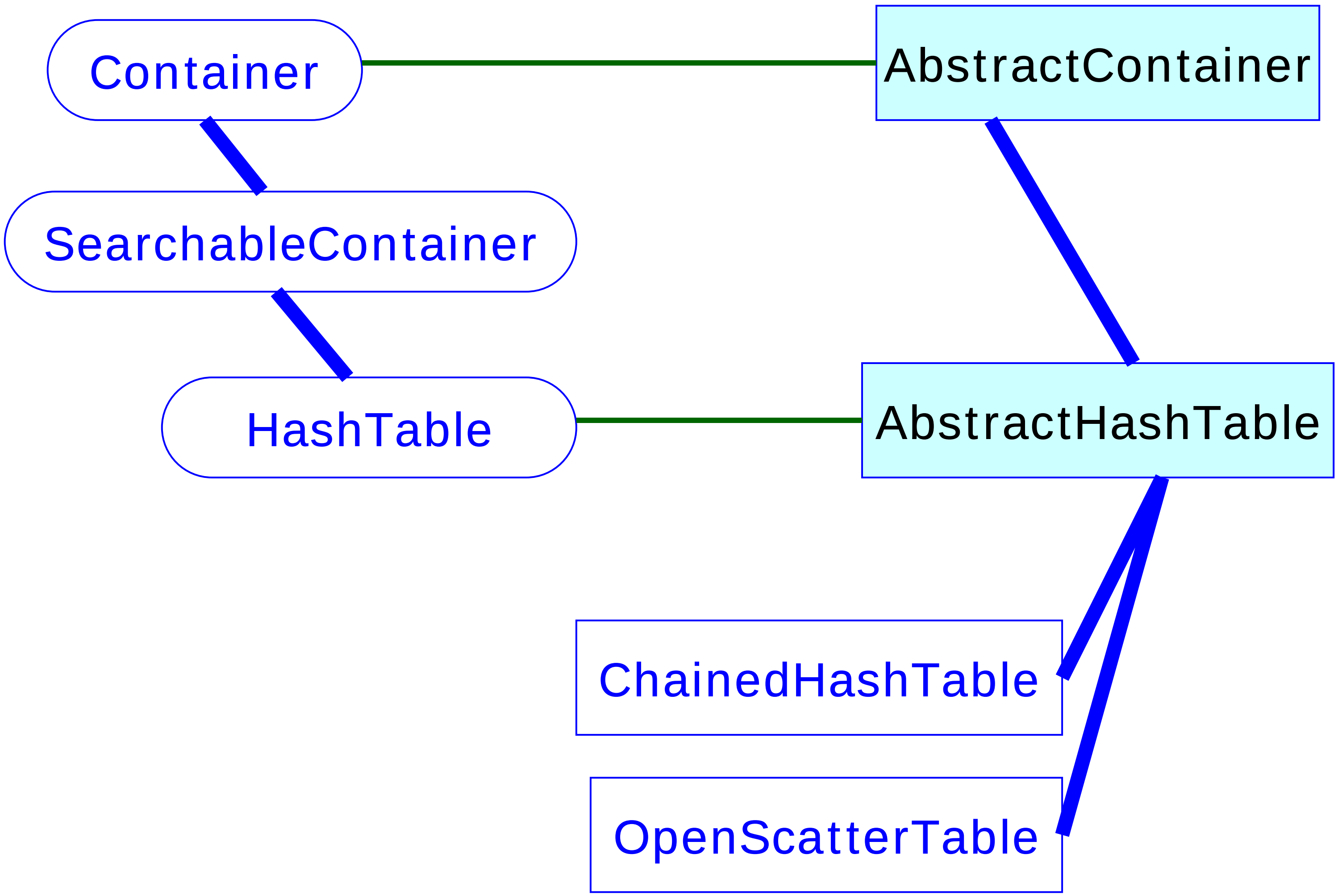
Separate Chaining has several advantages over open addressing:

- Collision resolution is simple and efficient.
- The hash table can hold more elements without the large performance deterioration of open addressing (The load factor can be 1 or greater)
- The performance of chaining declines much more slowly than open addressing.
- Deletion is easy - no special flag values are necessary.
- Table size need not be a prime number.
- The keys of the objects to be hashed need not be unique.

Disadvantages of Separate Chaining:

- It requires the implementation of a separate data structure for chains, and code to manage it.
- The main cost of chaining is the extra space required for the linked lists.
- For some languages, creating new nodes (for linked lists) is expensive and slows down the system.

Implementing Hash Tables: The Hierarchy Tree



Implementation of Separate Chaining

```
public class ChainedHashTable extends AbstractHashTable {
   protected MyLinkedList [ ] array;
   public ChainedHashTable(int size) {
      array = new MyLinkedList[size];
      for(int j = 0; j < size; j++)
         array[j] = new MyLinkedList( );
   }
   public void insert(Object key) {
      array[h(key)].append(key); count++;
   }
   public void withdraw(Object key) {
      array[h(key)].extract(key); count--;
   }
   public Object find(Object key){
      int index = h(key);
      MyLinkedList.Element e = array[index].getHead( );
      while(e != null){
         if(key.equals(e.getData()) return e.getData();
         e = e.getNext();
      }
      return null;
   }
}
```

Introduction to Open Addressing

- All items are stored in the hash table itself.
- In addition to the cell data (if any), each cell keeps one of the three states: EMPTY, OCCUPIED, DELETED.
- While inserting, if a collision occurs, alternative cells are tried until an empty cell is found.

- **Deletion**: (lazy deletion): When a key is deleted the slot is marked as DELETED rather than EMPTY otherwise subsequent searches that hash at the deleted cell will fail.

- **Probe sequence**: A probe sequence is the sequence of array indexes that is followed in searching for an empty cell during an insertion, or in searching for a key during find or delete operations.

- The most common probe sequences are of the form:

$$h_i(key) = [h(key) + c(i)] \% \ n, \quad \text{for } i = 0, 1, \ldots, n-1.$$

where **h** is a hash function and **n** is the size of the hash table

- The function **c(i)** is required to have the following two properties:

Property 1: c(0) = 0

Property 2: The set of values **{c(0) % n, c(1) % n, c(2) % n, . . . , c(n-1) % n}** must be a permutation of **{0, 1, 2,. . ., n – 1}**, that is, it must contain every integer between **0** and **n - 1** inclusive.

Introduction to Open Addressing (cont'd)

- The function **c(i)** is used to resolve collisions.

- To insert item r, we examine array location **$h_0(r) = h(r)$**. If there is a collision, array locations **$h_1(r), h_2(r), ..., h_{n-1}(r)$** are examined until an empty slot is found.

- Similarly, to find item **r**, we examine the same sequence of locations in the same order.

- **Note**: For a given hash function **h(key)**, the only difference in the open addressing collision resolution techniques (linear probing, quadratic probing and double hashing) is in the definition of the function **c(i)**.
- Common definitions of **c(i)** are:

Collision resolution technique	**c(i)**
Linear probing	**i**
Quadratic probing	**$\pm i^2$**
Double hashing	**$i * h_p(key)$**

where **$h_p(key)$** is another hash function.

Introduction to Open Addressing (cont'd)

- **Advantages of Open addressing:**
  - All items are stored in the hash table itself. There is no need for another data structure.
  - Open addressing is more efficient storage-wise.

- **Disadvantages of Open Addressing:**
  - The keys of the objects to be hashed must be distinct.
  - Dependent on choosing a proper table size.
  - Requires the use of a three-state (Occupied, Empty, or Deleted) flag in each cell.

Open Addressing Facts

- In general, primes give the best table sizes.
- With any open addressing method of collision resolution, as the table fills, there can be a severe degradation in the table performance.
- Load factors between 0.6 and 0.7 are common.
- Load factors > 0.7 are undesirable.
- The search time depends only on the load factor, *not* on the table size.
- We can use the desired load factor to determine appropriate table size:

$$\text{table size} = \text{smallest prime} \geq \frac{\text{number of items in table}}{\text{desired load factor}}$$

Open Addressing: Linear Probing

- **c(i)** is a linear function in **i** of the form **c(i) = a*i**.
- Usually **c(i)** is chosen as:

  $c(i) = i$ **for i = 0, 1, . . . , tableSize – 1**

- The probe sequences are then given by:

  $\mathbf{h_i(key) = [h(key) + i] \,\%\, tableSize}$ **for i = 0, 1, . . . , tableSize – 1**

- For **c(i) = a*i** to satisfy Property 2, **a** and **n** must be relatively prime.

Linear Probing (cont'd)

Example: Perform the operations given below, in the given order, on an initially empty hash table of size **13** using linear probing with **c(i) = i** and the hash function: **h(key) = key % 13**:

insert(18), insert(26), insert(35), insert(9), find(15), find(48), delete(35), delete(40), find(9), insert(64), insert(47), find(35)

- The required probe sequences are given by:

$$\mathbf{h_i(key) = (h(key) + i) \% 13 \qquad i = 0, 1, 2, ..., 12}$$

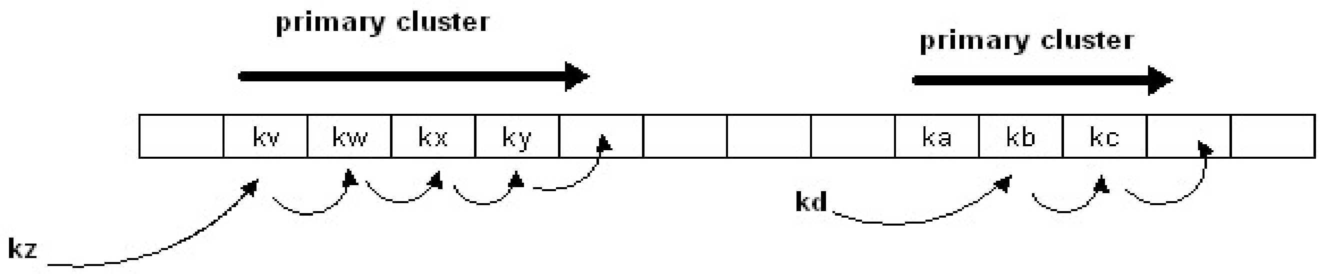
Linear Probing (cont'd)

OPERATION	PROBE SEQUENCE	COMMENT
insert(18)	$h_0(18) = (18 \% 13) \% 13 = 5$	SUCCESS
insert(26)	$h_0(26) = (26 \% 13) \% 13 = 0$	SUCCESS
insert(35)	$h_0(35) = (35 \% 13) \% 13 = 9$	SUCCESS
insert(9)	$h_0(9) = (9 \% 13) \% 13 = 9$	COLLISION
	$h_1(9) = (9+1) \% 13 = 10$	SUCCESS
find(15)	$h_0(15) = (15 \% 13) \% 13 = 2$	FAIL because location 2 has **Empty** status
find(48)	$h_0(48) = (48 \% 13) \% 13 = 9$	COLLISION
	$h_1(48) = (9 + 1) \% 13 = 10$	COLLISION
	$h_2(48) = (9 + 2) \% 13 = 11$	FAIL because location 11 has **Empty** status
withdraw(35)	$h_0(35) = (35 \% 13) \% 13 = 9$	SUCCESS because location 9 contains 35 and the status is **Occupied** The status is changed to **Deleted**; but the key 35 is not removed.
find(9)	$h_0(9) = (9 \% 13) \% 13 = 9$	The search continues, location 9 does not contain 9; but its status is **Deleted**
	$h_1(9) = (9+1) \% 13 = 10$	SUCCESS
insert(64)	$h_0(64) = (64 \% 13) \% 13 = 12$	SUCCESS
insert(47)	$h_0(47) = (47 \% 13) \% 13 = 8$	SUCCESS
find(35)	$h_0(35) = (35 \% 13) \% 13 = 9$	FAIL because location 9 contains 35 but its status is **Deleted**

Index	**Status**	**Value**
0	O	26
1	E	
2	E	
3	E	
4	E	
5	O	18
6	E	
7	E	
8	O	47
9	D	35
10	O	9
11	E	
12	O	64

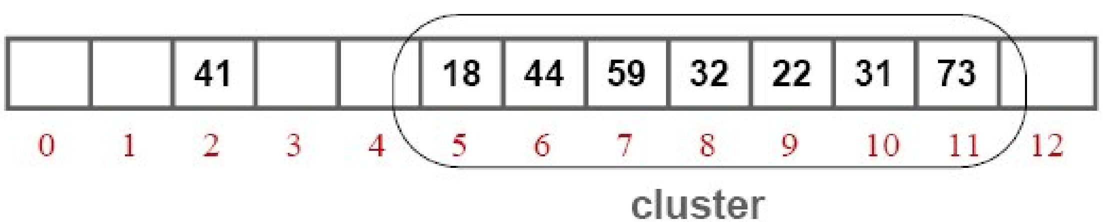
Disadvantage of Linear Probing: Primary Clustering

- Linear probing is subject to a primary clustering phenomenon.
- Elements tend to cluster around table locations that they originally hash to.
- Primary clusters can combine to form larger clusters. This leads to long probe sequences and hence deterioration in hash table efficiency.



Example of a primary cluster: Insert keys: **18, 41, 22, 44, 59, 32, 31, 73**, in this order, in an originally empty hash table of size **13**, using the hash function **h(key) = key % 13** and **c(i) = i**:

h(18) = 5
h(41) = 2
h(22) = 9
h(44) = 5+1
h(59) = 7
h(32) = 6+1+1
h(31) = 5+1+1+1+1+1
h(73) = 8+1+1+1



Exercises

1. Given that,

   $$c(i) = a*i,$$

   for `c(i)` in linear probing, we discussed that this equation satisfies Property 2 only when `a` and `n` are relatively prime. Explain what the requirement of being relatively prime means in simple plain language.

2. Consider the general probe sequence,

   $$h_i\ (r) = (h(r) + c(i))\%\ n.$$

   Are we sure that if `c(i)` satisfies Property 2, then $h_i(r)$ will cover all `n` hash table locations, `0,1,...,n-1`? Explain.

3. Suppose you are given k records to be loaded into a hash table of size `n`, with `k < n` using linear probing. Does the order in which these records are loaded matter for retrieval and insertion? Explain.

4. A prime number is always the best choice of a hash table size. Is this statement true or false? Justify your answer either way.