## King Fahd University of Petroleum and Minerals

ICS 253 - Discrete Structures I - Spring 2015 - Assignment \#3 - Due on Mon. April 13, 2015

1. Find the number of words of length 8 of letters A-Z, with no repeated letters, such that each word has equal numbers of vowels $\{\mathrm{A}, \mathrm{I}, \mathrm{E}, \mathrm{O}, \mathrm{U}\}$ and consonants.
2. Suppose $S=\{1,2, \ldots, 8\}$.
(a) Find the number of subsets $T \subseteq S$ of size six such that $T$ contains three odd numbers and three even numbers.
(b) Find the number of subsets $T \subseteq S$ of size five such that $T$ contains at least one even number.
3. How many strings of length 6 decimal digits where any of the digits appears exactly 2 times?
4. In how many ways can 11 identical books be distributed to 4 students such that
a) each student gets at least two books
b) each student gets at most five books

Hint: See part(a) in
http://faculty.kfupm.edu.sa/ics/darwish/ICS253-Spring2015/Equation_Solver/Eq_solver.htm
5. Compute the number of solutions to the equation $X+Y+Z=15$ where $X, Y, Z$ are all positive odd integers. Hint: Consider replacing each variable by some expression utilizing the definition of an odd number. For example, $X$ is replaced by $2 a+1, Y$ by $2 b+1$ and $Z$ by $2 c+1$.
6. Compute the number of solutions to the equation $X+Y+Z \leq \mathbf{1 5}$ where $X, Y, Z$ are all non-negative integers. Hint: Add one variable $Z$ (non-negative integer) to convert the given inequality to an equation.
7. Solved Problem: How many strings of length 6 decimal digits where any of the digits appears exactly 3 times?

If any digit appears exactly three times, then the string will have two distinct digits, each appearing three times. This can be approached as follows, select the two digits and then chose the positions to place one of the digits; the remaining position will be filled by the other digit.

Answer: $N=C(10,2) * C(6,3) * 1=(10 * 9 / 2) *(6 * 5 * 4 / 3 * 2)=45 * 20=900$

