Government Spending and Consumption in the Presence of Borrowing Constraints

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Abstract

Empirical estimates of the effect of government spending indicate crowding-in effect on aggregate output, consumption and labor supply, a positive co-movement between consumption of durables and non-durables and a cyclical crowding in-crowding out effect on investment. But most of the neo-classical real business cycle models fail to explain most of these empirical facts and frequently, all of them. I develop an RBC model where some agents face a binding borrowing constraint. The borrowing constraint is imposed in the form of a collateral constraint on these agents when they seek to borrow from the private debt market. Credit history is also important for borrowing. I show that once the model is properly calibrated, the impulse response functions of an unanticipated increase in government spending match all of their empirical counterparts.

Key Words: Crowding-in, crowding-out, borrowing constraint, collateral requirement, borrower-saver model, impulse response function

JEL code: E13, E2, E62, C15, H3

1 Introduction

Does government spending crowd-in or crowd-out output, private consumption and investment? This has been one of the fundamental questions in macroeconomics. Empirical analysis of the effect of government spending(Blanchard and Perotti(2002), Fatas and Mihov(2001), Burnside, Eigenbaum and Fisher(2004)) indicates that in case of an unanticipated temporary increase in government spending:

a) there is an increase in output, aggregate consumption and employment; the crowding-in effect,
b) there is a positive co-movement and crowding-in effect on durable and non-durable consumption; the co-movement effect,

c) there is a crowding-in effect on investment followed by a crowding-out; the crowding in-crowding out effect.

These empirical results are, however, at odds with theoretical results derived from standard real business cycle (RBC) models. Under reasonable parametrization, a general real business cycle model (RBC) predicts that there would be a crowding-out effect on output, consumption, employment and investment (Baxter and King (1993), Fatas and Mihov (2001)). In this paper, I developed an RBC model where some consumers face a binding borrowing constraint and the borrowing constraint relates consumers' borrowing to their durable goods purchases. I show that my model can reconcile the tension between the theoretical literature and the empirical findings. I use a log-linearized version of my model to generate impulse response functions of the macroeconomic variables for an unanticipated and temporary increase in government spending. Under reasonable parametrization, the model predicts a crowding-in effect on aggregate consumption, output and labor supply, a crowding-in effect and a positive co-movement between durable and non-durable consumption and a cyclical crowding in-crowding out effect on investment.

2 The Effect of Government Spending: Contacts with Literature

The effect of an unanticipated temporary increase in government spending on macroeconomic variables such as output, consumption and investment has been one of the most productive areas of macroeconomic research for the last 30 years. Early contributions by Barro (1981) and Bailey (1971) only focused on the effect on output while Hall (1980) argued that such policy could have significant business cycle effect on other macroeconomic variables. These authors showed that empirical effect on output was consistent with their simple version of the RBC models. Baxter and King (1993) expanded the theoretical literature on this issue by looking at effects on other macroeconomic variables such as consumption, labor supply and private investment and also by considering alternative financing of the temporary increase in government spending in a standard RBC model. The authors found a crowding-out effect on consumption and investment and a positive impact on labor supply.

These results, however, have come under serious scrutiny as a result of a recent surge in empirical papers based on Vector Auto Regressions (VAR). Using a semi-structural VAR, Blanchard and Perotti (2002) found that an unanticipated, temporary one standard deviation orthogonal shock (increase) in government spending leads to a crowding-in in both output and consumption while crowding out private investment. Fatas and Mihov (2001) expanded the empirical analysis of Blanchard and Perotti (BP from now on, 2002) by looking at a larger set of macroeconomic variables. By employing the same identification strategy used by BP, they found that that an unanticipated increase in government spending leads to a) a persistent increase in GDP, b) a persistent increase in aggregate consumption, durable and non-durable consumption, c) an immediate crowding in of investment followed by a crowding out effect (a cyclical effect), d) a persistent increase in net tax revenue, e) a persistent increase in private employment and f) a crowding-in and clear positive co-movement between durable and non-durable consumption.

The above papers gave rise to several major puzzles in the macroeconomic effects of government spending. First, the discrepancy between theoretical and empirical analysis of aggregate consumption has given risen to the crowding in-crowding out puzzle. It appears that the prediction of the standard RBC model on the effect of consumption is inconsistent with empirical findings. A large literature using different versions of the RBC model has tried to address this puzzle. There have been some recent papers that solved this puzzle by adopting alternative modeling assumptions. First, Linneman (2005) used an unconventional preference structure in an
RBC setup to solve the puzzle. Second, Bouakez and Rebei(2007) assumed complementarity between private and public consumption spending in the preference structure and habit formation. Third, Monacelli and Perotti(2009) assumed price rigidity and no capital which helped them to reconcile the crowding-in effect on aggregate consumption. Each of the above three papers made some assumptions which were in direct violation of the standard Neo-Classical RBC or even New Keynesian model. Leeper and Davig (2009), on the other hand, looked at this puzzle from a policy perspective. Using a standard New Keynesian model, they show that the effect of a temporary increase in government spending could have a crowding-in effect on aggregate consumption under alternative monetary-fiscal policy combinations. There have been several other notable successes as well. Following seminal work by Mankiw(2000), Gali, Salido and Valles(2007) used a sticky price model where a fraction of the consumers are rule-of-thumb consumers in the sense that they make no intertemporal decisions. The authors conducted both empirical (following BP) and theoretical analysis and showed that they are consistent. Two recent empirical papers have contributed to debate. Coenen and Straub(2005) developed a "new synthesis" model by combining Gali, et al. (2007) and Smets and Wouters(2003) where they added several real frictions(such as external habit persistence and investment adjustment cost) and also included an extended structure for the stochastic process for their model. They used Bayesian technique to estimate their model for the Euro area. Their results indicate a crowding out of consumption and investment. Forni, Monforte and Sessa(2006) extended Coenen and Straub(2005) by including internal habit persistence. They estimated their model by using Bayesian technique for the Euro area and showed that both their model and their estimation results indicated a crowding-in of consumption and increase in output. But the main assumption on the nature of the rule-of-thumb consumers used by the above three papers have been criticized by Yang(2007) and Rahman(2008) who argued that the assumption of the rule of the thumb consumers is quite strong and imposes considerable restriction on the theoretical results.

Second, there are concerns about the co-movement between durable and non-durable consumption in the face of an unanticipated fiscal policy experiment; the co-movement puzzle. The empirical analysis indicates that there should be a positive co-movement between the durable and non-durable goods purchase whenever there is an unanticipated increase in government spending. No paper including Fatas and Mihov(2001) has so far attempted to address this puzzle for fiscal policy in a theoretical setup

Finally, it has been difficult to show the cyclical response pattern of investment in the face of a government spending shock. None of the papers cited earlier was able to replicate this. The only notable success is Burnside, Eichenbaum and Fisher(2004). The authors used an alternative identification scheme in their VAR setup but found responses of output, consumption and labor supply similar to BP. In order to explain these results in a theoretical setup, the authors incorporated an investment adjustment cost friction and internal habit persistence in an otherwise standard RBC model. While the paper was able to match the cyclical pattern of investment, it failed to show crowding in effect on consumption.

In this paper, I address the above three puzzles. I develop a model which is an extended version of the standard two-sector RBC model incorporating elements such as investment adjustment cost and habit persistence taken from the existing literature. In my model, there is a fraction of consumers who face a binding borrowing constraint which relates private sector borrowing by the consumers to the level of durable good purchase which acts as collateral. Borrowing (credit) history is also important for current and future borrowing. I show that my model can account for all the above puzzles simultaneously.

\footnote{In monetary policy analysis, this co-movement problem has gained significant traction recently. Barsky et al. (2007) first pointed out the co-movement problem in case of a monetary policy experiment and concluded that standard sticky price model cannot account for this result. Monacelli(2009) showed that by using borrowing constraint in a new Keynesian model, one could reconcile the puzzle.}
3 The Importance of Collateral Requirement in US Economy: Theory and Empirics

In USA, almost all private sector borrowing is subject to collateral requirement. The mortgage market where the collateral requirement is determined has undergone significant structural change in the last 70 years. Prior to the Great Depression, typical home mortgage payments were only interest and homeowners refinanced their loans’ principles every few years. Consumers were also provided with installment credit through retailers. The Federal Home Loan Bank Act of 1932 and the Homeowners’ Loan Act of 1933 established a regulatory framework where mortgage markets were insulated from the fluctuations of other capital market by the federal government acting as the lender of last resort. Second, long-term amortized mortgages replaced the previous interest-only, periodically refinanced mortgages. Later, the Monetary Act of 1980 and the Garn-St.Germain Act of 1982 eliminated restrictions on mortgage lending and re-integrated it with other financial markets.

The structural change in the mortgage market had significant effect on the private sector borrowing and collateral requirements. First, there is a rising trend in the mortgaged debt and private debt accumulation. Figure 1 shows the trend in mortgaged and total household and non-profit organization’s private debt, most of which in the form of housing and automobile purchases. The data is taken from the Federal Reserve Flow of Funds Accounts. The amount of collateralized debt, as measured by the fraction of total household debt mortgaged, increased from 78% in the last quarter of 1951 to a staggering 91% in the first quarter of 2007. During the period of 1951-2007, we also see a significant increase in the share of private debt as a fraction of GDP. That ratio increased almost three fold, from 2% in 1951QIV to 6% in 2007Q1. Secondly, there appears to be a dramatic change in the volatility of private debt. Figure 2 shows the trend in the HP filtered data on private debt for the period of 1970Q1 to 2007QIV. There appears to be a dramatic decline in the volatility of private debt after 1983.

The above mentioned changes in the mortgage market also have implications for the down payment requirements. Figure 3 recreates figure 1 of Campbell and Hercowitz(2004). It plots the ratio of mortgage debt to the value of owner-occupied housing and the ratio of household debt to the value of their durable goods stocks, which includes housing. These ratios declined from 1966 to the end of 1982, and then started a dramatic increase. As Campbell and Hercowitz(2004) points out, this surge reflects the emergence of the sub prime mortgage lending market and households’ greater use of home equity loans and mortgage refinancing to cash-out previously accumulated equity and unrealized capital gains. This reflects an increased importance of good credit history which eased the process of refinancing. Greater access to refinancing and home equity loans allowed homeowners to greatly delay repayment of effective mortgage principle, and access to additional sub prime mortgage reduced effective down payment requirements.

Greater access to the mortgage market and a reduction of down payment requirement appears to have significant macroeconomic effects. There appears to be a general decline in the macroeconomic volatility for US coinciding with the changes in the mortgage market. Table 1 reports several summary statistics for the US macroeconomy. Comparing the HP filtered and logged data between sub-sample of 1970Q1-1982Q4 and 1983Q1-2007Q4, there is clear decline in the standard deviations for all the major macroeconomic variables such as real GDP, consumption, investment, government spending, public and private debt. Furthermore, the strength of the co-movement between the variables have declined, as evident by a decline in the correlation coefficient. The co-movement between government spending and other variables, although negative, appears to have weakened significantly between the two sample periods.

In summary, there appears to be an improved and easier access to the mortgage market which

\footnote{For a more detailed discussion about the US mortgage market, please see Campbell and Hercowitz(2004).}
seems to have coincided with a decline in macroeconomic volatility. Therefore, it appears that the mortgage market plays a very important role in the USA economy. The importance of the mortgage market has prompted a more thorough and accurate modeling of the borrowing constraints faced by agents in macroeconomic models. In a seminal paper, Kiyotaki and Moore (1997) used borrowing constraint to exhibit how credit constraints and collateral requirement could serve as a powerful transmission mechanism by which effects of aggregate shocks could be amplified. In their heterogenous agent model of "farmers" and "gatherers", the former can borrow from the private credit market, which is subject to collateral requirements, defined by their land holding. If the farmer has a durable goods stock of $D_t$ at date $t$, then the borrowing constraint requires that he can borrow any amount $B_t$ as long as the repayment does not exceed the market value of the durable goods. Let $q_t$ and $R$ be the market price of durable goods and the risk free interest on debt. The borrowing constraint is then specified as followed:

$$B_t R \leq q_{t+1}D_t$$  \hspace{1cm} (a)

Monacelli (2007) modified the borrowing constraint by assuming that borrowing limit cannot exceed a certain fraction of the durable goods stock, which he defined to be the fraction of the durable goods that can be collateralized:

$$B_t \leq (1 - \pi)D_t$$  \hspace{1cm} (b)

Here, $\pi$ is the fraction of durable goods that cannot be used as collateral. Monacelli (2009) and Iacoviello (2005) further modified the borrowing constraint by requiring that the repayment cannot exceed the value of the fraction of the durable goods that can be used as collateral:

$$B_t R_t \leq (1 - \pi)D_t$$  \hspace{1cm} (c)

I will, however, use the borrowing constraint specified by Campbell and Hercowitz (2004). The authors developed a heterogeneous agent model consisting of two types of agents, borrowers and savers. They used a borrowing constraint to analyze the amplification mechanism of productivity shock in an otherwise standard RBC model to explain the reduction of macroeconomic volatility in USA. The authors made a significant modification to the borrowing constraint by making borrowing not only depend on collateral requirement but also on past credit history, as observed in the data. The household in their model faces three saving opportunities, invest in physical capital, buy government/public bond and serve as a lender in the private debt market. Households cannot short-sell one asset and buy another. Households also have the opportunity to borrow from the private debt market by issuing one period state contingent bond. The state-contingent claims are assumed to be unbacked and are unenforceable. As a result, the private credit market is incomplete. Private borrowing is subject to an endogenous limit. The collateralized value of the durable goods stock is generally less than its replacement cost. For a stock of durable good $D_{t+1}$, it is given by:

$$V_{t+1} = (1 - \pi) \sum_{j=1}^{\infty} (1 - \phi) [D_{t-j+1} - (1 - \delta_D)D_{t-j}]$$  \hspace{1cm} (1)

Here $\pi$ is the fraction of a new durable good that cannot serve as collateral. $\phi$ is the rate at which a good’s collateral value depreciates and $\delta_D$ is the depreciation rate of the durable good. Campbell and Hercowitz (2004) assumed that $\phi \geq \delta_D$, so that the good’s value to a creditor declines at least as rapidly as its value to its owner. Equation (1) can be written in the following recursive form:

$$V_{t+1} = (1 - \phi)V_t + (1 - \pi) [D_t - (1 - \delta_D)D_{t-1}]$$  \hspace{1cm} (1.a)
Collateral requirement limits household borrowing. That is:

\[ B_{t+1} \leq V_{t+1} \quad (2) \]

Here \( B_{t+1} \) refers to the outstanding debts of the households at the end of period \( t \) and \( V_{t+1} \) are the collateralized value of their durable goods.

4 The Borrower-Saver Model

My model is an extended version of the two sector model developed by Leeper, Walker and Yang(2008) embedded in a Campbell and Hercowitz(2004) setup. The model combines heterogeneity across household’s rates of time preference with collateral constraint on borrowing. Household debt reflects intertemporal trade between an impatient borrower and a patient saver. In the model economy, durable goods collateralize all household debt. Without collateral constraints, the patient saver lends to the impatient borrower and the debt increases over time. Collateral constraints limit the borrower’s debt, so the economy possesses a unique steady state with positive consumption by both households. In general, the borrower’s collateral constraint may bind occasionally. However, I will show that it always binds in the steady state. So if my model remains close to the steady state, the borrowing constraint for the borrower will bind. Therefore, standard log-linearization techniques can characterize its equilibrium for small disturbances. This is the path that has been followed by Campbell and Hercowitz(2004) and Monacelli(2009). I will do the same thing.

Time is taken in discrete intervals, \( t = 1, 2, \ldots \). The economy is composed of a continuum of households in the interval of \((0,1)\). There are two types of households, named Borrowers and Savers, of measure of \((1-F)\) and \(F\). They only differ in their time preferences. More specifically, I assume that they have different time preference rate, \( \beta_b \) and \( \beta_s \), where I assume \( \beta_s > \beta_b \) which makes the saver the patient agent and the borrower the impatient agent. Each household is endowed with one unit of time. Each household derives utility from three sources: consumption of durable goods \((C_{t,b,s})\), consumption of non-durable goods\((D_{t,b,s})\) and leisure\((1-L_{t,b,s})\) where the superscript \(s\) refers to savers and \(b\) refers to borrowers. They also incur disutility from working and making decisions on a durable goods purchase. The household faces three saving opportunities, as in Campbell and Hercowitz(2004). Using (1) and (2), the borrowing constraint can be written recursively as:

\[ B_{t+1}^{b,s} \leq (1-\phi)B_{t}^{b,s} + (1-\pi)\left[ D_{t+1}^{b,s} - (1-\delta_D)D_{t}^{b,s} \right] \quad (3) \]

When collateral limits a household’s borrowing, \( \pi \) is the required down-payment rate for the durable goods purchase and \( \phi \) is the rate at which the principal is repaid. Following the existing literature, I will assume these two parameters are exogenously determined by the regulatory environment.

4.1 Utility Maximization by the Borrower

Following Becker(1980), Campbell and Hercowitz(2004) and Monacelli(2009), I will assume that the borrowers are impatience enough so that they face a borrowing constraint that always binds. Since they cannot short-sell, borrowers will not invest on physical capital or buy public debt. The within period utility function of the borrower looks like:
function. The quadratic term-

\[ U(.) = \frac{[(C_t^b)^{1-\frac{1}{\sigma}} + V^b(D_t^b)^{1-\frac{1}{\tau}}]^{\frac{1}{1-\frac{1}{\sigma}}} - \frac{\eta}{2}(D_t^b - D_{t-1}^b)^2}{1 - \frac{1}{\tau}} + \chi \frac{(1 - L_t^b)^{1-\theta} - 1}{1 - \theta} \]

Here \( C_t^b = C_t^b - bC_{t-1}^b \), with \( b \geq 0 \) indicating the degree of internal habit persistence. \( \tau \) and \( \sigma \) are the elasticities of inter-temporal and intratemporal substitution of consumption. \( \theta \) is the inverse elasticity of intertemporal substitution of leisure. \( \chi \) is the weight on leisure in the utility function. The quadratic term \( \frac{\eta}{2}(D_t^b - D_{t-1}^b)^2 \) is interpreted as the deliberation cost where \( \eta \) captures the disutility of changing durable good stock. Households are also endowed with physical capital stocks, \( k_0 \geq 0 \). Given their initial capital stock, each household chooses a sequence of consumption of durable goods, non-durable goods, private debt and leisure decisions; \( \{C_t^b, D_t^b, B_t^b, L_t^b\}_{t=1}^{\infty} \) to maximize his expected lifetime utility:

\[ E_0 \sum_{t=1}^{\infty} \beta_t^{t-1} U(C_t^b, D_t^b, L_t^b) \]

Subject to the budget constraint:

\[ C_t^b + D_t^b \leq (1 - \tau_t^L) W_t L_t^b + B_t^b - B_{t-1}^b R_{t-1} + (1 - \delta_D)D_{t-1}^b + tr_t^b \]

and the borrowing constraint:

\[ B_t^b \leq (1 - \phi)B_{t-1}^b + (1 - \pi) \left[ D_t^b - (1 - \delta_D)D_{t-1}^b \right] \]

### 4.2 Utility Maximization by the Saver

I will assume that the savers are patience enough so that their borrowing constraint never binds. The preference structure of the saver is very similar to the borrower:

\[ U(.) = \frac{[(C_t^s)^{1-\frac{1}{\sigma}} + V^s(D_t^s)^{1-\frac{1}{\tau}}]^{\frac{1}{1-\frac{1}{\sigma}}} - \frac{\eta}{2}(D_t^s - D_{t-1}^s)^2}{1 - \frac{1}{\tau}} + \chi \frac{(1 - L_t^s)^{1-\theta} - 1}{1 - \theta} \]

Here all the parameters have the same interpretation as in the case of the borrower with \( C_t^s = C_t^s - bC_{t-1}^s \). Given initial capital stock \( k_0^s \geq 0 \), the saver will therefore choose a sequence of non-durable goods, durable goods, private debt, labor supply, public debt, physical capital stock, investment in physical capital stock and capital utilization rate \( \{C_t^s, D_t^s, B_t^s, L_t^s, X_t^s, K_t^s, I_t^s, \mu_t\}_{t=1}^{\infty} \) to maximize:

\[ E_0 \sum_{t=1}^{\infty} \beta_t^{t-1} U(C_t^s, D_t^s, L_t^s) \]

Subject to the budget constraint:

\[ C_t^s + D_t^s + X_t^s + I_t^s \leq (1 - \tau_t^L) W_t L_t^s + B_t^s - B_{t-1}^s R_{t-1} + (1 - \delta_D)D_{t-1}^s + \bar{\delta} \tau_t^k K_t^s + (1 - \tau_t^k)\overline{r}_t \mu_t K_{t-1}^s + X_{t-1}^s R_{t-1} + tr_t^s \]
and the law of motion of capital stock:

$$K_t^s \leq \left\{ 1 - s \left( \frac{I_t^s}{I_{t-1}^s} \right) \right\} I_t^s + (1 - \delta_t)K_{t-1}^s$$

(11)

where:

$$S(1) = S'(1) = 0, S''(1) = \gamma > 0$$

(12)

Here $s \left( \frac{I_t^s}{I_{t-1}^s} \right)$ is defined as investment adjustment cost, taken from Burnside, Eichenbaum and Fisher(2004). Following Leeper, Walker and Yang(2008), I assume a constant depreciation rate for durables, $\delta_D$, but not for capital. As in Greenwood, Hercowitz and Huffman(1988), using capital more intensively makes capital depreciate at a faster rate. The depreciation rate of capital has the following form:

$$\delta_t = \delta \mu_t^\omega$$

(13)

where $0 < \delta < 1$ and $\omega > 0$. Leeper, Walker and Yang(2008) also pointed out that U.S. tax codes does not have depreciation allowances depending on the period-by-period capital utilization intensity; the depreciation allowance is based on a pre-determined statutory schedule. Similar to these authors, I will assume that the capital depreciation allowance($\bar{\delta} \tau^k_t K_{t-1}$) is given according to the time-invariant steady state rate of capital depreciation, $\bar{\delta}(= \delta \mu_t)$. Also, following Becker(1980), I will assume that in the initial period, $K_t^0 = I_t^0$. Finally, I assume that the utility function is strictly concave, twice differentiable and satisfies the inada condition.

### 4.3 Profit Maximization by the Firm

The production function used by the firm is defined as follows:

$$Y_t = f(L_t, K_t) = \{\mu_t K_{t-1}\}^\alpha \{L_t\}^{1-\alpha}$$

(14)

The Representative firm rents capital and hires labor from agents to maximize profit

$$\text{Profit} = \{\mu_t K_{t-1}\}^\alpha \{L_t\}^{1-\alpha} - r_t \mu_t K_{t-1} - w_t L_t$$

where $K_t$ and $L_t$ are aggregate capital stock and labor supply, to be defined later. I also assume that the production function is strictly concave, twice differentiable and satisfy the inada condition.

### 4.4 Government Budget Constraint

The government levies taxes on capital($\tau^k_t$) and labor income($\tau^l_t$) separately, sells one period government bond($X_t$) to the savers, issues a depreciation allowance for capital($\bar{\delta} \tau^k_t K_{t-1}$) and provide lump-sum transfers($TR_t$) to the consumers to balance the budget. The government budget constraint is:

$$G_t + X_{t-1} R_{2t-1} + \bar{\delta} \tau^k_t K_{t-1} + TR_t = T_t + X_t$$

(15)

where $T_t$ is the total tax collected defined as:

$$T_t = T_t^k + T_t^l$$

(16)
Finally, the total transfer in the economy, $TR_t$, is:

$$TR_t = TR_s^t + TR_b^t$$  \hspace{1cm} (19)

Here, $TR_s^t$ and $TR_b^t$ are total transfers to the saver and borrower, to be defined later. In this paper, I assume that an increase in government spending could be financed in various ways. To study the implications of alternative financing, I will posit the simplest possible rules for fiscal policy instruments that are consistent with fiscal solvency. The fiscal instruments are chosen as a function of the state of government indebtedness, as measured by the debt-output ratio. The rules adopted here are abstractions designed to capture the practice of offsetting policy: when the fiscal budget deteriorates and debt rises, explicit fiscal actions are taken to improve the budget situations. Following Leeper and Yang(2008), the fiscal policy rules that the government uses are summarized as follows:

$$\ln \left( \frac{s^T R_s^t}{s^T R_s^e} \right) = -q_{TR} \ast M \ast \ln \left( \frac{s^B}{s^B} \right) + \varepsilon_{TR}^t, q_{TR} \geq 0$$ \hspace{1cm} (20)

$$\ln \left( \frac{s^T R_b^t}{s^T R_b^e} \right) = -q_{TR} \ast N \ast \ln \left( \frac{s^B}{s^B} \right) + \varepsilon_{TR}^t, q_{TR} \geq 0$$ \hspace{1cm} (21)

$$\ln G_t = \rho_G \ln G_{t-1} + u^G_t$$ \hspace{1cm} (22)

$$\ln \left( \frac{\tau^L}{\tau^L} \right) = q_L \ln \left( \frac{s^B}{s^B} \right) + \varepsilon^L_t, q_L \geq 0$$ \hspace{1cm} (23)

$$\ln \left( \frac{\tau^K}{\tau^K} \right) = q_K \ln \left( \frac{s^B}{s^B} \right) + \varepsilon^K_t, q_K \geq 0$$ \hspace{1cm} (24)

Here $s^T R_s^t = \frac{TR_s^t}{Y_t}$, $h = s, b$ and $u^G_t$ is an AR(1) process, to be defined later. Variables without time subscript denote steady state values. The rules build in a one-year delay for the response of an offsetting policy. The $q$‘s in the rules 1-4(equations 20, 21, 23 and 24) are defined as "fiscal adjustment parameters". Sign restrictions on the $q$‘s are also straightforward. When the debt-output ratio rises above the initial steady-state level, one of the future distorting taxes are raised or transfer-output is reduced to maintain fiscal solvency. To isolate the impact of each financing instruments, one of the $q$‘s is non-zero in each experiment. For example, if transfer-output ratio adjusts, $q_{TR} > 0$ and $q_L = q_K = 0$. Furthermore, since there is income heterogeneity in this model, transfers are distributionally non-neutral by nature. This means that even if transfer-output ratio for both group of consumers adjust by the same rate, as measured by $q_{TR}$, the actual magnitudes of the change are not equal. In order to achieve equal magnitude of adjustment, I introduce two new constants, $M$ and $N$ which are defined as follows:

$$M = \frac{TR_s^t}{TR} \text{ if distributionally neutral transfer adjustment and 1 otherwise}$$

Leeper and Yang(2008) argued that longer delays such as five year do not change the results significantly.
The error terms in fiscal rules are all AR(1) process, defined as follows:

\[ u_t^G = \rho_u^G u_t^G + \varepsilon_t^G; \varepsilon_t^G \sim iid. N(0, \sigma^2_G) \] (25)

\[ \varepsilon_t^{TR^*} = \rho_{TR^*} \varepsilon_{t-1}^{TR^*} + u_t^{TR^*}; \varepsilon_t^{TR^*} \sim iid. N(0, \sigma^2_{TR^*}) \] (26)

\[ \varepsilon_t^{TR^b} = \rho_{TR^b} \varepsilon_{t-1}^{TR^b} + u_t^{TR^b}; \varepsilon_t^{TR^b} \sim iid. N(0, \sigma^2_{TR^b}) \] (26.a)

\[ \varepsilon_t^{rL} = \rho_{rL} \varepsilon_{t-1}^{rL} + u_t^{rL}; \varepsilon_t^{rL} \sim iid. N(0, \sigma^2_{rL}) \] (27)

\[ \varepsilon_t^{rK} = \rho_{rK} \varepsilon_{t-1}^{rK} + u_t^{rK}; \varepsilon_t^{rK} \sim iid. N(0, \sigma^2_{rK}) \] (28)

The government also has to maintain intertemporal fiscal solvency. First, any equilibrium must satisfy the transversality conditions (TVC):

\[ E_t \lim_{T \to \infty} \beta^{t+T} \lambda_{s,t+T} K^s_t = 0 \] (29)

\[ E_t \lim_{T \to \infty} \beta^{t+T} \lambda_{h,t+T} B^h_t = 0, h = b, s \] (29.a)

\[ E_t \lim_{T \to \infty} \beta^{t+T} \lambda_{s,t+T} X_t = 0 \] (29.b)

The TVC imply that in any optimum, the households do not over-accumulate government liabilities, or private debt or physical capital. Imposing the TVC on the flow budget constraint of the government, we derive the intertemporal budget constraint for the government:

\[ \frac{B_t}{Y_t} = s^B_t = \sum_{j=0}^\infty d_{t,j} \left[ (1 - \alpha) \frac{L^s_t}{L_{t+j}^{s}} + \alpha \frac{L^b_t}{L_{t+j}^{b}} \right] \quad \text{(30)} \]

Where \( d_{t,j} = \Pi_{i=0}^{j-1} R_{t+i-1} \frac{Y_{t+i}}{Y_{t+i}} \). In equilibrium, equation (30) determines the value of government debt. It also imposes restrictions on dynamic interaction between current debt and expected future policies. An increase in government spending raises \( \frac{B_t}{Y_t} \) which automatically requires some combination of fiscal variables and/or discount factors in the future to adjust. The above fiscal rules are only a subset of expected sequences of fiscal policies that satisfy equation (30). Feasibility will be ensured by judicious choice of response magnitude parameters- the \( q \)'s in the rules. I will use the values used in Leeper and Yang (2008).

4.5 Aggregation and Market Clearing Conditions

I will aggregate the economy as follows:

\[ I_t = FL^s_t, X_t = FX^s_t, K_t = FK^s_t \] (31)

\[ B_t = FB^s_t + (1 - F)B^b_t = 0 \] (32)
\[ L_t = F L_t^s + (1 - F)L_t^b \]  
(33)

\[ C_t = F C_t^s + (1 - F)C_t^b \]  
(34)

\[ D_t = F D_t^s + (1 - F)D_t^b \]  
(35)

\[ TR_t^s = S * tr_t^s, TR_t^b = (1 - S) * tr_t^b \]  
(36)

Finally, the goods market clearing condition or the aggregate resource constraint can be written as:

\[ C_t + I_t + G_t + D_t = Y_t + (1 - \delta_D)D_{t-1} \]  
(37)

For the purpose of comparing the variables from my model to the variables found in the National Income Accounting (NIPA) data I will define the flow of durable goods service as:

\[ \text{Durable\_Service} = D\_S_t = D_t - (1 - \delta_D)D_{t-1} \]  
(38)

Since the NIPA data reports the flow of durable goods, equation (38) will be used for calibration purpose. Also, the aggregate consumption in the economy is defined as follows:

\[ \text{Aggregate\ Consumption} = AD\_C_t = C_t + D\_S_t \]  
(39)

**Definition 1** A Rational Expectations Competitive Equilibrium is a pair of sequence of prices \( \{r_t, w_t\}_{t=1}^{\infty} \), a sequence of a set of consumers’ decisions \( \{C_t^h, D_t^h, B_t^h, K_t^h, L_t^h, \mu_t^h\}_{t=1}^{\infty} \), a sequence of firm’s decisions \( \{K_t, L_t\}_{t=1}^{\infty} \), a sequence of policy variables, \( \{X_t, G_t, \tau^K_t, \tau^L_t, TR_t\}_{t=1}^{\infty} \) such that, given initial level of capital stock \( K_{t-1} \), private and public debt, the optimization for the agents and firm’s are solved; the goods, capital, labor and the debt markets clear; the transversality conditions for capital and debts hold; the government budget constraint and at least one of the policy rules and all the aggregate conditions are satisfied. Furthermore, we will only consider the ranges of the fiscal adjustment parameters- the q’s- that are consistent with the existence of a rational expectations competitive equilibrium

Appendix A shows the first order conditions of utility and profit maximization for this economy. It also shows the steady state conditions derived for this economy. Here, I will provide a simple proof that the borrowing constraint of the borrower binds in the steady state. Assume \( \lambda_b \) and \( \psi_b \) to be the Lagrangian multipliers associated with the budget constraint and borrowing constraint of the borrower. In steady state, the Kuhn-Tucker complementary slackness (CS) condition for \( B^b \) looks like:

\[ \lambda_b (1 - \beta_b R_2) - \psi_b \{1 - \beta_b (1 - \phi)\} \leq 0, B^b \geq 0 \text{ with } B^b [\lambda_b (1 - \beta_b R_2) - \psi_b \{1 - \beta_b (1 - \phi)\}] = 0 \]

From the first part of the CS condition, we see:

\[ \psi_b \{1 - \beta_b (1 - \phi)\} \geq \lambda_b (1 - \beta_b R_2) > 0 \implies \psi_b > 0 \]

Therefore, the borrowing constraint for the borrower binds in the steady state.
4.6 Model Calibration

Table 2 reports the benchmark values of parameters and the steady state values of variables that will be used for calibrating the model to US data. The value of inter-temporal elasticity of substitution ($\tau$) is taken from Ogaki and Reinhart (1998) to be 0.447. The value of Intratemporal elasticity of substitution is taken to be 0.90 which is slightly below the value the authors reported. The steady state share of total time devoted to production for both types of consumer ($L_s$ and $L_s^t$) is assumed to be 0.20, the average weekly hours of production workers to 144 hours (24 x 7), reported by BLS. The variable capital depreciation parameter ($\omega$) is assumed to be 1.56, taken from the estimates of Burnside and Eichenbaum (1996) which implies a quarterly capital depreciation rate of 0.02. The investment adjustment cost parameter ($\gamma$) is taken from Coenen and Straub (2005). The habit persistence parameter ($b$) is assumed to be 0.80, taken from Burnside, Eichenbaum and Fisher (2004). The deliberation cost parameter ($\eta$) is taken from Leeper, Walker and Yang (2008). The steady state capital tax and labor tax rate are set at the historical average of US data (1947Q1-2008Q4). The two tax rates follow the definition of Jones (2002). The weight to leisure ($\chi$) taken from Leeper and Yang (2008). The steady state capital depreciation rate ($\delta$) is taken from Leeper, Walker and Yang (2008) while the depreciation rate for durable goods ($\delta_D$) is taken from are taken from Campbell and Hercowitz (2004). The value of $q$'s are taken from Leeper and Yang (2008). The value of the AR(1) coefficients and the value of standard deviations of various shocks have been taken from Forni, Monforte and Sessa (2006) and Coenen and Straub (2005). The parameters related to the borrowing constraints are taken from Campbell and Hercowitz (2004). They reported the value of $\pi$ and $\phi$ to be 0.16 and 0.03 for the high collateral regime that corresponds to the period 1971-1982 and 0.11 and 0.01 for the low collateral regime that corresponds to 1983-2007. I will use the value of $\pi$ and $\phi$ to be 0.15 and 0.03 for the high regime. Finally, the value for the inverse elasticity of intertemporal substitution for leisure ($\theta$) is taken from Leeper and Yang (2008). The values of the betas; $\beta_s$ and $\beta_b$ are assumed to be 0.99 and 0.97, which are similar to the values used by Campbell and Hercowitz (2004) and Monacelli (2009). The value of the fraction of savers ($F$) are taken from Rahman (2008) and Joint Committee of Taxation (2006). The fraction of transfers that goes to the savers ($S$) is taken from an earlier version of Yang (2007).

In addition to the above parameter values, I needed the value of several ratios to solve the steady state values for the variables in the model. The government spending-output ratio ($S_G$), investment-output ratio ($S_I$), aggregate consumption-output ratio ($S_C$), total transfer-output ratio ($S_TR$), public debt-output ratio ($S_X$) are set at the historical average of U.S. data (1947Q1-2007Q4). The value of private debt-output ratio ($S_B$) is also set at the historical average but for a different sample period (1951Q4 - 2008Q4). For the calibration purpose, I also needed the ratio of consumption expenditure on durables to consumption expenditures on non-durables ($\frac{D \cdot S_t}{C_t}$) which is set to be 0.149 taken from the NIPA data.

4.7 Solution Method and Stability Conditions

An analytical solution of the model is not available; the equilibrium conditions are log-linearized around the original steady state and analyzed in terms of percentage deviations from that steady state. This means I will assume that the perturbation in the log-linearized model is small enough so that the log-linear model exhibits the same dynamic behavior as the original model in the steady state. In the log-linear model, I will postulate that the borrowing constraint of the
borrower binds all the time, although during the simulation exercise I will be constantly checking whether the Lagrangian multiplier associated with borrower’s borrowing constraint is positive or not. The model is solved using Sims’s(2001) algorithm. Also, the log-linearized version of the model is complicated enough that it prevents me from analyzing its stability conditions analytically. They can only be analyzed by using a computer. I will, however, provide a brief discussion of the technical aspects associated with evaluating the stability conditions for this model, following Novales, Dominguez, Perez and Ruiz(2003) and Uhlig(2006).

The log-linearized model is first cast in its canonical form:

\[ \Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \]

Here \( y_t \) is the vector of log-linearized variables, \( z_t \) is the vector of exogenous processes (5 shocks) and \( \eta_t \) is the vector of expectational error generated as result of inclusion of forward looking variables in the model dynamics. When \( \Gamma_0 \) is invertible, stability conditions are evaluated by computing the eigenvalues of \( \Gamma_0^{-1} \Gamma_1 \) and checking for signs and magnitudes. However, Novales et. al (2003) showed that in a typical RBC model when capital stock and investment shows up simultaneously, this creates a redundancy in the model because of their contemporaneous relationship and therefore, makes the \( \Gamma_0 \) matrix singular. The problem also arises in my model. As a result \( \Gamma_0 \) is not invertible and we cannot carry out standard eigenvector - eigenvalue decomposition. The alternative is to use the QZ-decomposition to derive generalized eigenvalues, as suggested by Sims(2001). According to this method, for any pair of square matrices like \((\Gamma_0, \Gamma_1)\), there exist orthonormal matrices \( Q, Z \) \((QQ' = ZZ' = I)\) and upper triangular matrices \( \Lambda \) and \( \Omega \) such that:

\[ \Gamma_0 = Q' \Lambda Z', \Gamma_1 = Q' \Omega Z' \]

Besides, \( Q \) and \( Z \) can be chosen so that all possible zeros of \( \Lambda \) occur in the lower right corner and such that the remaining ratios \( \frac{\omega_{ii}}{\lambda_{ii}} \) of diagonal elements in \( \Lambda \) and \( \Omega \), are non-decreasing in absolute values as they move down the diagonal. These ratios are the generalized eigenvalue of the pair\((\Gamma_0, \Gamma_1)\). The stability of the model depends on the magnitude of these eigenvalues. According, to Uhlig(2006), if the dimension of \( \Lambda \) is \( m \) and if there exists exactly \( m \) generalized eigenvalues smaller than unity in absolute value, the system is said to be saddle-path stable. In my model this condition is satisfied under baseline calibration. Hence my model is also saddle-path stable\(^5\).

5 Dynamic Impact of a Temporary Change in Government Spending

This section reports the dynamic impact of an unanticipated temporary increase in government spending and shows how those impacts changes when there are, a) changes in the nature of the borrowing constraints, b) changes in the financing schemes or fiscal rules, c) changes in the various modeling assumptions, and d) changes in the collateral regime. I will focus on the experiments that derive the three main results highlighted in the introduction and in section 2. Unless otherwise mentioned, my baseline model will always assume low collateral regime and that consumption of durables and non-durables are edgeworth complements, i.e. \( \sigma = 0.90 \).

5.1 Government Spending Shock under Alternative Borrowing Constraints

Figure 4 compares the baseline model with borrowing constraint following Campbell and Hercowitz(2004) with the one that uses borrowing constraint similar to Kiyotaki and Moore(1997),

\(^5\)A list of the set of generalized eigenvalues for my model is available upon request.
which was defined in equation (a). In both case, baseline parameters defined in table 2 are used to calibrate the models and non-neutral transfers adjust. This means that the first and second fiscal rules (equation 20-21) are in effect with \( q_{TR} = 0.341 \), \( q_L = q_K = 0 \) and \( M = N = 1 \). In the Campbell and Hercowitz (2004) case, we see a crowding-in for output, aggregate consumption and labor supply, and a positive co-movement between durable and non-durable consumption and crowding-in. In case of investment, there is a crowding in- crowding-out effect. All these results are consistent with the empirical facts defined in the introduction and in section 2. An unanticipated rise in government spending to be financed by future reduction in transfers create a large negative income effect. Agents respond by working more. Habit persistence and deliberation cost increases the labor supply response. These two costs coupled with increased wage income, savings motive (borrowing constraint effect) and weak complementarity between durables and non-durables ensure a positive co-movement between consumption of durables and non-durables. This positive co-movement also ensures crowding-in in aggregate consumption. But in order to clearly understand these results, we have look at the behavior of the borrowers and the savers separately. With non-neutral transfer, the relatively poor borrower faces a larger negative income effect. He does not have any conventional saving instrument. But he can use durable goods as collateral for future borrowing which could be used for consumption purpose. Therefore he “saves” by raising the consumption of durable goods. This motive is strengthened by deliberation cost, which adds some habit to consumption of durable, making consumption decision non-separable in time and creates a complementary effect with labor supply which increases as a response to the negative wealth effect. Since non-durables are weak complementary to durables and subject to habit persistence, consumption of non-durable also goes up as well. Increased wage income allows the borrower to pay off part of the debt. On the other hand, the saver faces a smaller negative income effect with non-neutral transfer adjustment. He responds by raising his labor supply initially. Since both saver and borrower raise his labor supply, aggregate labor supply increases. This reduces wage rate and raises the marginal product of capital and hence, interest rate. This makes borrowing expensive for the borrower and he pays off his debt from their increased wage income. The saver, equipped with two sources of income (wage income and debt income), raises his consumption of durable and non-durable consumption. Part of the debt payment income is spent on buying public debt and investment in physical stock. Initial rapid increase in investment gets penalized because of adjustment cost. It goes down after a while.

For the Kiyotaki and Moore (1997) case, the co-movement problem does not appear. There is, however, crowding out of aggregate consumption and its components. This is because in their model, there is less motivation to accumulate durable goods which can be used as collateral for current consumption but has no impact on future consumption because of the absence of credit history. Both output and labor goes down upon impact and shows pattern contradicting the empirical findings. Investment shows inverse cyclical pattern.

Figure 5 compares the baseline model with borrowing constraint following Campbell and Hercowitz (2004) with the one that uses borrowing constraint similar to Monacelli (2009), which was defined in equation (c). In both case, I use baseline parameters and non-neutral transfers adjust. This time the co-movement problem arises. There is crowding out of aggregate consumption and its components. Output and labor supply shows even more contractionary effect. Investment again shows the inverse cyclical pattern. Monacelli (2009)’s model does not include past credit history. Also, borrowing is now more limited. As a result, his model suffers from the same problem as that of Kiyotaki and Moore (1997).

In summary, among alternative specifications of the borrowing constraint, only the baseline specification used in this paper taken from Campbell and Hercowitz (2004) can match all the empirical facts discussed in the introduction of the paper.

Table 3 reports the size of output, consumption and investment multipliers at different points
on the transition path from the empirical works of BP(2002), Gali et al. (2007) and Fatas and Mihov(2001) and compares them with the multipliers derived from the baseline model with borrowing constraint similar to Campbell and Hercowitz(2004) where non-neutral transfers adjust. Numbers in the parentheses indicate an approximation of the one standard deviation confidence bands. Output and consumption multipliers appear to be similar to the values derived by Fatas and Mihov(2001). The size of the multipliers are quite large in BP(2002) and in Gali et al. (2007). The shocks also appear to be more persistent in those models. Finally, investment appears to be more volatile in the baseline model than the empirical papers. But the sign and the qualitative nature of all the multipliers from the baseline model is similar to their empirical counterparts.

5.2 Government Spending Shock under Alternative Financing Schemes

Figure 6-8 shows the effect of government spending under alternative financing schemes. For all the cases, I assume that durable and non-durable goods are Edgeworth complements; $\sigma = 0.90$. Figure 6 compares the effect between neutral and non-neutral transfer adjustment. This means that the first and second fiscal rules(equation 20-21) are in effect with $q_{TR} = 0.341$, $q_L = q_K = 0$. In case of the neutral transfer adjustment(solid line), the initial impact is identical to the non-neutral case(dotted line). During the transition path, the two groups face similar decline in transfer in the neutral case. In case of the non-neutral case, the borrowers face a larger reduction in their transfers while the savers face a smaller decline. This is reflected in the transition path as the dynamic response of the borrowers in non-neutral case trail their behavior in the neutral case. For the saver, we see opposite effect.

Figure 7 and 8 shows cases when government spending is adjusted by raising labor tax and capital tax. For the labor tax adjustment case (figure 7), the third fiscal rule(equation 23) is in effect with $q_L = 0.149$, $q_{TR} = q_K = 0$. Raising labor tax to finance debt distorts the after-wage income and reduces labor supply. Output goes down. Collateral constraint encourages borrower to buy more durables which ensures an increase in consumption of non-durable through borrowing. Saver, with lower income from labor supply and debt repayment consumes less of both durable and non-durable good. In the aggregate, consumption crowds in but the co-movement problem between durable and non-durable arises again.

For the capital tax adjustment case (figure 8), the fourth fiscal rule(equation 24) is in effect with $q_K = 0.206$, $q_{TR} = q_L = 0$. Raising capital tax to finance debt distorts the return to capital. Since savers are more effected by capital tax, we see a crowding-out effect in investment. The co-movement problem between durable and non-durable consumption goods also appear. The effect of the saver dominates in this case and we see that even aggregate consumption crowds out.

In summary, among alternative financing schemes, both neutral and non-neutral transfer adjustment can match all the empirical facts explained in the introduction of the paper. The other financing schemes cannot match most of the empirical facts.

5.3 Government Spending Shock under Alternative Modeling Assumptions

Figure 9 shows the effect of government spending shock under two values of Intratemporal Elasticity of Substitution (InES) to highlight the importance of this parameter. In both case, we assume that non-neutral transfers adjust. This means that first and second fiscal rules(equation 20-21) are in effect with $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. The baseline case, the dotted line, is where durable and non-durable goods are edgeworth complements, i.e. $\sigma = 0.90$. The

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6 Only BP(2002) reports the output multiplier for different points on the transition path. No other multiplier was reported in any of the cited paper. All the cited papers only reported the impulse response functions with one standard deviation confidence bands. Therefore, the size of the multipliers and the confidence bands are approximated values derived from visual inspection of the impulse response functions.
solid line shows the case when they are edgeworth substitutes, i.e. $\sigma = 1.05$. In the substitute case, we see the co-movement between durable and non-durable goods disappears; aggregate labor supply, output and aggregate consumption all crowds-in after initial decline. Also investment appears to crowd-out. When durable and non-durable goods are substitutes, borrowing constraint alone cannot create sufficient incentive for the borrowers to accumulate non-durable. Durable consumption for the borrower does go up, but at the cost of sacrificing non-durable consumption. Smaller accumulation of private debt for him also makes him pay off debt slowly. With goods being substitute and with lower debt payment, saver spends less money on investment and more on consumption of non-durable, causing his consumption of durable to go down. This also causes investment to go down.

Figure 10 compares the dynamic effect of the baseline model with habit persistence(dotted line) with one that does not have habit persistence(solid line). Here non-neutral transfers adjust. Without habit, borrowers behave quite differently. Similar to Monacelli(2008), an increased real interest rate forces the borrower to de-accumulate their debt. This relaxation of the borrowing constraint(although they do not engage in any new borrowing) reduces the user cost of durables, which produces a substitution towards durable goods, out weighing the weak complementary effect between the durable and non-durable consumption. Their labor supply now dramatically increases. On the saver’s side, absence of habit increases the volatility of their consumption and labor decisions. Both consumption of durable and non-durable crowd out with a dramatic increase in investment. Labor supply, although more volatile, appears to show similar pattern as with the habit persistence case. Over all, the crowding of output, the response of aggregate labor supply and cyclical investment response are all retained. Although co-movement problem does not occur, crowding in effect on aggregate consumption is lost.

Figure 11 compares the baseline model with one that does not have a borrowing constraint. Here also non-neutral transfers adjust. The borrower seeks to smooth consumption by changing his labor supply decisions, which appears to be remarkably volatile. Increased labor income can now enable both the borrower and the saver to afford more durable and non-durable goods. Since there is no borrowing constraint and no collateral requirement, they consume less of durable goods. They saver raises consumption of non-durable by reducing durable consumption and by dramatically reducing investment, indicating the habit persistence effect have dominated the weak complementarity effect. Expected increased income from government bonds also enable him to reduce labor supply which appears to be as volatile as the borrowers’. In the aggregate, crowding-in effect in output and aggregate consumption, increase in aggregate labor supply are all lost and co-movement problem again emerges.

Figure 12 compares the baseline model with a representative agent model with habit persistence. Since representative agent cannot borrow, consumption of non-durable go down upon impact while durable goods remain unchanged throughout the transition path. Aggregate consumption crowds out but output increases. Labor supply increase on impact too. There appears to be an asset swap where the agent dramatically increases purchase of government bonds and reduce investment. This creates a pattern in the investment response which again contradicts the empirical findings.

In summary, only the borrower-saver model with borrowing constraint, habit persistence and edgeworth complementarity between the durable and the non-durable goods can match all the empirical facts explained in the introduction of the paper.

5.4 Government Spending Shock under Alternative Collateral Regimes

Figure 13 compares the baseline model with two alternative specification of the collateral regime, following Campbell and Hercowitz(2004) to highlight the importance of changes in the collateral regime that we have observed for the USA. The parameters for different regimes are summarized
In the high collateral regime (dotted line), the borrower can borrow less and also pays off less debt. Higher collateral requirement also makes him consume less of both durable and non-durable goods. The effect on labor supply is similar to the low regime. For the saver, however, a decline in their debt income forces them to reduce consumption. In the aggregate, there is crowding-out of consumption upon impact followed by a crowding-in phase. The positive co-movement between durable and non-durable is retained after the initial impact. Output shows similar effect while investment shows a dampened cyclical response compared to the low regime.

In summary, only the baseline borrower-saver model with a low regime specification for the borrowing constraint can match all the empirical facts explained in the introduction of the paper.

6 Conclusion

In this paper, I have analyzed the effect of an unanticipated and temporary increase in government spending on macroeconomic variables such as output, aggregate consumption, consumption of durable and non-durable goods, employment and investment. The objective of the analysis was to reconcile the differences between the results generally obtained in standard RBC models and those reported in empirical papers. Empirical results show a crowding-in effect on output, aggregate consumption and employment; a positive co-movement and crowding-in effect on durable and non-durable goods and finally, a crowding-in effect on investment followed by a crowding-out effect. Standard RBC models fail to explain most of the empirical facts and sometimes, all of them. I developed a model that combines heterogeneity across household's rates of time preference with collateral constraint on borrowing in a standard two sector RBC model. Borrowing was also subject to past credit history. The model also included several other features which has recently being used with standard RBC models such as internal habit persistence, deliberation cost to durable goods' consumption and investment adjustment cost. Once the model is properly calibrated, Impulse response functions for an unanticipated increase in government spending generated from this model seemed to match with those found in empirical literature. However, there are several limitations of the model. First, impact multipliers of the shock appeared to be smaller than what were seen in empirical models for output and aggregate consumption. Second, government spending shock appeared to have a less persistent effect in the model than what was seen in the empirical literature. Third, investment appeared to be much more volatile after a shock when comparing it with its empirical counterpart. But if we only consider the qualitative nature and patterns of the results, the model seemed to be consistent with previous empirical findings. Therefore, the model has succeeded reasonably well in meeting its objective. However, further research and more rigorous structural estimation analysis is needed to shed more light on the usefulness of this model.

References


Appendix A

A.1: Model Solutions and Steady State Conditions

Assuming all the first order condition binds except for the first order condition for private bonds for the saver, the first order conditions look like:

\[
\left\{C_t^h + D_t^h - (1 - \tau_T^h) W_t L_t^{h^*} - B_t^h + B_{t-1}^h R_{1t-1} - (1 - \delta_D)D_{t-1}^h - TR_t^h \right\} = 0, h = b, s \quad (A.1)
\]

\[
\left\{B_t^h - (1 - \phi)B_{t-1}^h - (1 - \pi) \left[ D_t^h - (1 - \delta_D)D_{t-1}^h \right] \right\} = 0, h = b, s \quad (A.2)
\]
The first order conditions for the saver looks like:

\[
\left( C_t^{sb} \right)^{-\frac{1}{\delta}} + V^b(D_t^b)^{1-\frac{1}{\delta}} \right) \left( C_t^{sb} \right)^{-\frac{1}{\delta}} - \beta_b b E_t \left\{ \left( C_{t+1}^{sb} \right)^{-\frac{1}{\delta}} + V^b(D_{t+1}^b)^{1-\frac{1}{\delta}} \right\} \left( C_{t+1}^{sb} \right)^{-\frac{1}{\delta}} = \lambda_{bt}
\]

(A.3)

\[
\left[ \left( C_t^{sb} \right)^{1-\frac{1}{\delta}} + V^b(D_t^b)^{1-\frac{1}{\delta}} \right] \frac{\tau - \delta}{\tau (\sigma - 1)} V^b \left( D_t^b \right)^{-\frac{1}{\delta}} - \eta \left( D_t^b - D_{t-1}^b \right) D_{t-1}^s
\]

(A.4)

\[
-\lambda_{bt} + \psi_{bt}(1 - \pi) + \beta_b b E_t \left\{ \left( \frac{\tau}{2} \left( \frac{D_{t+1}^b - D_t^b}{D_t^b} \right)^2 - 1 \right) + \lambda_{bt+1}(1 - \delta D) \right\} = 0
\]

(A.5)

\[
-\chi(1 - L_t^b)^{-\theta} + \lambda_{bt} (1 - \tau_t^b) W_t = 0
\]

(A.6)

The first order conditions for the saver looks like:

\[
\left\{ C_t^s + D_t^s + X_t^{sb} + I_t^s - (1 - \tau_t L_t) W_t L_t^2 - B_t^s + B_{t-1}^s R_{1t-1} - (1 - \delta D) D_{t-1}^s \\
- \delta \tau_t K_{t-1}^s - (1 - \tau_t^k) r_t \mu_t K_{t-1}^s - X_{t-1}^s R_{2t-1} - TR_t^s
\right\} = 0
\]

(A.7)

\[
\left\{ K_t^s - \left\{ 1 - s \left( \frac{I_t^s}{I_{t-1}^s} \right) \right\} I_t^s - (1 - \delta_t) K_{t-1}^s \right\} = 0
\]

(A.8)

\[
\left[ \left( C_t^{ss} \right)^{1-\frac{1}{\delta}} + V^s(D_t^s)^{1-\frac{1}{\delta}} \right] \frac{\tau - \delta}{\tau (\sigma - 1)} \left( C_t^{ss} \right)^{-\frac{1}{\delta}}
\]

(A.9)

\[
-\beta_s b E_t \left\{ \left( C_{t+1}^{ss} \right)^{-\frac{1}{\delta}} + V^s(D_{t+1}^s)^{1-\frac{1}{\delta}} \right\} \frac{\tau - \delta}{\tau (\sigma - 1)} \left( C_{t+1}^{ss} \right)^{-\frac{1}{\delta}} - \lambda_{st} = 0
\]

(A.10)

\[
-\lambda_{st} + \beta_s b E_t \left\{ \left( \frac{D_{t+1}^s - D_t^s}{D_t^s} \right)^2 - 1 \right\} + \lambda_{st+1}(1 - \delta D) = 0
\]

(A.11)

\[
\beta_s E_t \lambda_{bt+1} \left\{ \delta \tau_{t+1}^k + (1 - \tau_{t+1}^k) \mu_{t+1} \right\} + \beta_s E_t \varphi_{t+1} \left[ 1 - \delta (\mu_{t+1})^{\omega} \right] - \varphi_t = 0
\]

(A.12)

\[
-\lambda_{st} + \varphi_{st} \left\{ 1 - s \left( \frac{I_t^s}{I_{t-1}^s} \right) \right\} - \varphi_t \left\{ s \left( \frac{I_t^s}{I_{t-1}^s} \right) \frac{I_t^s}{I_{t-1}^s} \right\} - \beta_s E_t \varphi_{t+1} \left\{ s \left( \frac{I_{t+1}^s}{I_t^s} \right) \frac{I_{t+1}^s}{I_t^s} \right\} = 0
\]

(A.13)
\[ \lambda_s (1 - \tau_t^k) r_t - \varphi_t \delta \omega (\mu_t)^{\omega-1} = 0 \]  
(A.14)

\[ -\lambda_s + \beta_s E_t \{ \lambda_{s,t+1} R_{2t} \} = 0 \]  
(A.15)

\[ -\chi (1 - L_t^*)^{-\theta} + \lambda_s (1 - \tau_t^L) w_t = 0 \]  
(A.16)

Also, the first order condition of profit maximization are as follows:

\[ r_t = \frac{\alpha Y_t}{\mu_t K_{t-1}} \]  
(A.17)

\[ w_t = \frac{(1 - \alpha) Y_t}{L_t} \]  
(A.18)

In steady state, A.13 implies

\[ \lambda_s = \varphi_s \]  
(A.19)

Using A.19, A.12 could be simplified as:

\[ r = \left[ \left\{ \frac{1 - \beta_2}{\beta_2 (1 - \tau^K) \delta} \right\} + \delta \mu^\omega \right] \frac{\mu}{\mu} \]  
(A.20)

Combining A.20 with A.14 in steady state and simplifying, we can get:

\[ \mu = \left[ \frac{(1 - \tau^K) (1 - \beta_2)}{\beta_2 (\omega - (1 - \tau^K) \delta)} \right]^\frac{1}{\alpha - 1} \]  
(A.21)

If we substitute A.21 into A.20 and use the value of \( r \) into A.17, we get:

\[ K = \left[ \left\{ \frac{(1 - \beta_2)}{\beta_2 (1 - \tau^K)} \right\} + \delta \mu^\omega \right]^\frac{1}{\alpha - 1} \]  
(A.22)

Given a value for \( L \), we can calculate the value \( K, Y \) and \( W \). Now from A.16, we get:

\[ \lambda_s = \varphi_s = \frac{\chi (1 - L*)^{-\theta}}{(1 - \tau^L) w} \]  
(A.23)

Similarly, we could get:

\[ \lambda_b = \frac{\chi (1 - L^b)^{-\theta}}{(1 - \tau^L) w} \]  
(A.24)

Again, from A.15, we get: \( R_2 = \frac{1}{\beta_s} \). Combine this and A.5, we get:

\[ \varphi_b = \frac{\lambda_b (1 - \beta_b b)}{1 - \beta_b (1 - \phi)} \]  
(A.25)

Now substituting A.3 into and A.5 into A.4 and also substitute the value of \( \lambda_b \) from A.24, we can simplify and get:

\[ V^b = \frac{(1 - b_0)^{\frac{-1}{\beta}} (1 - \beta_b b) (1 - \beta_b (1 - \delta D)) \{ \pi - \beta_b (1 - \phi - R_1) \}}{\{ 1 - \beta_b (1 - \phi) \} \left( \frac{c^b}{H^2} \right)^\beta} \]  
(A.26)
Similarly, I could get:

\[
V^a = \left( \frac{D^a}{C^a} \right)^{\frac{1}{\sigma}} (1 - \beta_a) \frac{1}{\sigma} \{1 - \beta_2(1 - \delta_D)\}
\]  

(A.27)

In the model, there are 6 unknown variables that needs to be solved simultaneously, \(C^s\), \(C^b\), \(D^s\), \(D^b\), \(V^b\) and \(V^a\). In order to solve them, we will use steady state version of A.3, A.9, A.26, A.27, steady state version of the aggregate resource constraint (equation 37) and a ratio which we defined in section 4.6 and looks like:

\[
\frac{\delta_D \{FD^s + (1 - F)D^b\}}{FC^s + (1 - F)C^b} = 0.149
\]  

(A.28)

Since there are now six equations and six unknowns, I can solve the system of equations. Thus I solve the entire steady state system of equations.
<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>Public Debt</th>
<th>Private Debt</th>
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<tbody>
<tr>
<td>Y</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>C</td>
<td>0.02</td>
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<td>I</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>G</td>
<td>0.02</td>
<td>0.20</td>
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<tr>
<th>Correlation Coefficient</th>
<th>Public debt</th>
<th>Private Debt</th>
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<td>Y</td>
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<td>C</td>
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<tr>
<td>G</td>
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<td>-0.47</td>
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<tr>
<td>Public Debt</td>
<td>0.15</td>
<td>1.00</td>
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<tr>
<td>Private Debt</td>
<td>0.50</td>
<td>1.00</td>
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<th>Sample: 1983QI-2007QIV</th>
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<table>
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<th>Private Debt</th>
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<tr>
<td>C</td>
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<td>I</td>
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<tr>
<td>G</td>
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<tr>
<td>Public Debt</td>
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<td>0.17</td>
</tr>
<tr>
<td>Private Debt</td>
<td>0.15</td>
<td>1.00</td>
</tr>
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</table>

Table 1: Standard deviation and correlation coefficients for major macroeconomic variables. All variables are in real terms, logged and HP filtered at quarterly frequency. The data source for the GDP and its components are NIPA and Federal Reserve Flow of Funds Accounts, table B.100, Balance sheet of households and non-profit organizations for the private debt. Public debt is defined as the total government debt held by public, reported by NIPA.
### Table 2: Benchmark Parameter values used for model calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<td>$\eta$</td>
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<td>$\theta$</td>
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<td>$q_L$</td>
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<tr>
<td>$\phi$ for Low Regime</td>
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<td>$\sigma$</td>
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<td>$q_K$</td>
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<tr>
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<td>$S^X$</td>
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<td>$\omega$</td>
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<td>$\delta$</td>
<td>0.02</td>
<td>$F$</td>
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<td>$\delta_D$</td>
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<td>$\sigma_{TR}^e$</td>
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<td>$\rho_{sL}$</td>
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Table 3: Comparing multipliers between Blanchard and Perotti (2002), Gali et al. (2007), Fatas and Mihov (2001) and the baseline model. For all the models, numbers in the parentheses indicate approximately one standard deviation confidence band.
Figure 1: Trends in Debt: 1951Q4-2006Q4. Total debt equals total debt of the household and non-profit organization, Source of the data is the Federal Reserve Flow of Funds Accounts, table B.100, Balance sheet of households and non-profit organizations.
Figure 2: Trends in the volatility of private debt held by household and non-profit organizations: 1970Q1-2007Q3. The data is HP filtered at quarterly frequency. Source of the data is the Federal Reserve Flow of Funds Accounts, table B.100, Balance sheet of households and non-profit organizations.

Figure 3: Ratio of household’s debts to their tangible assets. Taken from Campbell and Hercowitz (2004).
Figure 4: Effect of Government spending under different kinds of borrowing constraints when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Campbell and Hercowitz(2004)’s borrowing constraint (baseline): solid line; Kiyotaki and Moore(1997)’s borrowing constraint: dotted line.

Figure 5: Effect of Government spending under different kinds of borrowing constraints when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Campbell and Hercowitz(2004)’s borrowing constraint (baseline): solid line; Monacelli(2009)’s borrowing constraint: dotted line.
Figure 6: Effect of Government spending when transfers adjust. Neutral transfers adjustment: dotted line; Non-neutral adjustment: solid line.

Figure 7: Effect of Government spending when labor tax adjust. The Model is calibrated using baseline parameters, defined in table 2. This means that $q_{TR} = 0$, $q_L = 0.149$, $q_K = 0$. 

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Figure 8: Effect of Government spending when capital tax adjust. The Model is calibrated using baseline parameters, defined in table 2. This means that $q_{TR} = 0$, $q_L = 0$, $q_K = 0.206$.

Figure 9: Effect of Government spending under different Intratemporal Elasticity of Substitution when non-neutral transfers adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline is the low INES($\sigma = 0.90$): solid line; high INES($\sigma = 1.05$): dotted line.
Figure 10: Effect of Government spending under different modeling assumptions when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model with habit persistence: solid line; Baseline model without habit persistence: dotted line.

Figure 11: Effect of Government spending under different modeling assumptions when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model with borrowing constraint: solid line; Baseline model without borrowing constraint: dotted line.
Figure 12: Effect of Government spending under different modeling assumptions when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model: solid line; Representative agent model: dotted line.

Figure 13: Effect of Government spending under different collateral regime when non-neutral transfer adjust. This means that $q_{TR} = 0.341$, $q_L = q_K = 0$ and $M = N = 1$. Baseline model with low regime: solid line; Baseline model with high regime: dotted line.