Chapter 3
Seismic Velocity

Introduction

- The objective of this chapter is to learn about the porosity-velocity relationship and types of seismic velocities and how to compute them.
- Seismic velocity of subsurface rocks is the most important parameter in seismic exploration.
- Seismic velocity is used in various stages of seismic data processing and interpretation such as:
  - NMO correction and stacking
  - Interval-velocity estimation
  - Time-to-depth conversion
  - Migration
  - General interpretation purposes

Model of a sedimentary rock

- The simplest model of a sedimentary rock consists of spheres (grains) arranged (packed) in a specific pattern. The packing pattern controls the amount of pore space.
- Porosity is the most controlling factor on seismic velocity.
- Porosity ($\phi$) is defined as the ratio between the pore volume ($V_p$) and the total rock volume ($V_b$):

$$\phi = \frac{V_p}{V_b}.$$  \hspace{1cm} (1)
Porosity-density relation: Porosity is related to the rock (overall), matrix (solid), and pore-fluid densities ($\rho_r$, $\rho_m$, and $\rho_f$) by the following volume-weighted average equation:

$$\rho_r = \phi \rho_f + (1 - \phi) \rho_m.$$  \hspace{1cm} (2)

Porosity-velocity relation: Porosity is related to the rock, matrix, and pore-fluid P-wave velocities ($\alpha_r$, $\alpha_m$, and $\alpha_f$) by the following time-average (Wyllie’s) equation:

$$\frac{1}{\alpha_r} = \frac{\phi}{\alpha_f} + \frac{(1 - \phi)}{\alpha_m}.$$  \hspace{1cm} (3)

Figure.

Density-velocity relation: Gardner’s rule is an empirical equation that relates the density and P-wave velocity in sedimentary rocks as follows:

$$\rho_r = 0.31 \alpha_r^{1/4},$$  \hspace{1cm} (4)

where $\rho_r$ is the rock (overall) density ($\text{gm/cm}^3$) and $\alpha_r$ is the rock (overall) P-wave velocity (m/s).

This figure shows plots of $\alpha_r$ and $\rho_r$ as functions of $\phi$.

The following statements are generally true about the overall P-wave velocity in sedimentary rocks:

- Velocity decreases with porosity.
- Velocity increases from sandstone to limestone to dolomite.
- In the same rock, P-wave velocity is smaller, when gas fills the pores, than when oil or water fills them.
- Velocity increases with age, depth, pressure, or cementation.
• The following is a table of matrix densities and velocities of some rocks:

<table>
<thead>
<tr>
<th>Rock</th>
<th>$\alpha_m$ (km/s)</th>
<th>$\rho_m$ (gm/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shale</td>
<td>3.4</td>
<td>2.52</td>
</tr>
<tr>
<td>Sandstone</td>
<td>5.5</td>
<td>2.65</td>
</tr>
<tr>
<td>Limestone</td>
<td>6.4</td>
<td>2.71</td>
</tr>
<tr>
<td>Dolomite</td>
<td>7.0</td>
<td>2.87</td>
</tr>
<tr>
<td>Anhydrite</td>
<td>6.1</td>
<td>2.96</td>
</tr>
<tr>
<td>Salt</td>
<td>4.6</td>
<td>2.16</td>
</tr>
</tbody>
</table>

• The following is a table of densities and velocities of some fluids:

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$\alpha_f$ (km/s)</th>
<th>$\rho_f$ (gm/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.5</td>
<td>1.00</td>
</tr>
<tr>
<td>Oil</td>
<td>1.3</td>
<td>0.80</td>
</tr>
<tr>
<td>Gas (at 1 atm, 25 ºC)*</td>
<td>0.33</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

* Density and P-wave velocity in gas increase with pressure and temperature.

Seismic velocity types

• We will study the following seismic velocity types:
  - Interval
  - Average
  - NMO
  - RMS
  - Stacking
- Dix

- **The interval velocity** ($V_i$) is the velocity in a single layer, which can be determined from sonic logs or laboratory measurements on cores from the layer.

- **The average velocity** ($V_N$) to the $N^{th}$ layer is defined in terms of the layers properties as:

  \[ V_N = \frac{\sum_{i=1}^{N} V_i \Delta T_{0i}}{\sum_{i=1}^{N} \Delta T_{0i}} \]

  where $N$: the total number of layers, $V_i$: the interval velocity in the $i$-th layer, and $\Delta T_{0i} = T_{0i} - T_{0i-1}$; where $T_{0i-1}$ and $T_{0i}$ are the zero-offset traveltimes to the top and bottom of the $i$-th layer, respectively ($T_{00} = 0$).

  - The average velocity is the velocity that we get by dividing the depth ($Z_N$) over the zero-offset one-way traveltime ($T_{0N}/2$) to the bottom of the $N^{th}$ layer:

  \[ V_N = \frac{2Z_N}{T_{0N}} \]

- **The NMO velocity** ($V_{NMO}$) to the bottom of the $N^{th}$ layer is the velocity found using the approximate NMO correction formula (equation (5) of Chapter 2).

  - $V_{NMO}$ is found practically by searching for the velocity that will align the true T-X curve horizontally using the approximate NMO correction formula.

  - This is usually done through the constant velocity stack (CVS) method during the velocity analysis phase of the seismic data processing flow (Figure).

  - It can be found directly from $\Delta T_{NMO}(X)$ as

  \[ V_{NMO} \approx \frac{X}{\sqrt{2 * T_{0N} \Delta T_{NMO}(X)}} \]
• The root-mean-square velocity ($V_{RMS}$) to the bottom of the $N^{th}$ layer is defined, in terms of layers properties, as:

$$V_{RMS} = \sqrt{\frac{\sum_{i=1}^{N} V_i^2 \Delta T_{oi}}{\sum_{i=1}^{N} \Delta T_{oi}}}$$

➢ It is defined, in terms of the true $T$-$X$ curve, as the reciprocal of the square root of the $X^2$ coefficient we get by fitting a polynomial to the true $T^2 - X^2$ curve. That is, fitting a polynomial of the form: $T_N^2 = C_{0N} + C_{1N}X^2 + C_{2N}X^4 + \ldots$ to the true $T^2 - X^2$ curve,

$$V_{RMS} = \frac{1}{\sqrt{C_{1N}}}.$$

➢ Note that $V_{RMS}$ is also the reciprocal of the square root of the slope of the tangent to the true $T^2 - X^2$ curve at $X = 0$:

$$V_{RMS} = \frac{1}{\sqrt{\left[\frac{dT_N^2}{dX^2}\right]_{X=0}}}.$$

• The stacking velocity ($V_S$) to the bottom of the $N^{th}$ layer is defined as the velocity found by fitting a hyperbola to the true $T$-$X$ curve of the form:

$$T_N^2(X) = T_{0N}^2 + \frac{X^2}{V_{S}^2}$$

➢ Note that the stacking velocity is a special case of the RMS velocity (i.e., when only the first 2 terms are used in the polynomial fit).
By fitting a hyperbola to the true *nonhyperbolic* $T$-$X$ curve, we are lumping all the layers above the $N^{th}$ reflector into a single virtual layer and assigning this virtual layer a velocity of $V_{SN}$.

- $V_S$ is determined practically by searching for the velocity that will produce the best-fit hyperbola to the true $T$-$X$ curve.

- This is usually done through the velocity spectrum method during the velocity analysis phase of the seismic data processing flow (Figure).

- At small offsets ($X/Z_N < 1$), $V_{RMSN} \approx V_{SN} \approx V_{NMO}$.

- **Dix velocity** ($V_N$) of the $N^{th}$ layer is the *interval velocity calculated* from the RMS velocities to the top and bottom of the $N^{th}$ layer ($V_{RMSN-1}$ and $V_{RMSN}$) using Dix’s following formula:

$$V_N = \sqrt{\frac{V_{RMSN}^2 T_{0N} - V_{RMSN-1}^2 T_{0N-1}}{\Delta T_{0N}}}$$

- Out of $V_{RMSN}$, $V_{SN}$, and $V_{NMO}$, only $V_{RMSN}$ can be related directly to the interval velocities of subsurface layers through Dix velocity formula.

- However, if only small offsets are used, we can use $V_{SN-1}$ and $V_{SN}$ or $V_{NMO-1}$ and $V_{NMO}$ in place of $V_{RMSN-1}$ and $V_{RMSN}$ in Dix’s velocity formula (More details).
Methods of velocity determination

1. **Time-distance methods:** This includes:

   (1) $X^2-T^2$ method:
   
   - If we assume that the time-distance curve is a hyperbola, then by plotting $T^2$ versus $X^2$, we get a straight line whose slope is $1/V^2$, where $V$ is the stacking velocity ($V_S$) to the reflector.
   
   - We can also fit higher-order polynomials to the true T-X curve to get the RMS velocity ([Figure](#)).
   
   - **Exercise:** Fit polynomials of increasing orders to the true T-X curves of the second and third layers in this sheet and estimate $V_S$ or $V_{RMS}$ and compare your results to the true model parameters.
   
   - This method is more suited for high-quality small (experimental or synthetic) datasets because it requires picking of time at many offsets, which is time consuming and inaccurate on real datasets.

   (2) **Best-fit methods:** These are the most commonly used methods for determining velocity in seismic exploration. They are carried out during the velocity analysis phase of the seismic data processing flow. They include:

   - *Velocity spectrum method:* by fitting the best-fit hyperbola to the true T-X curve and finding the corresponding stacking velocity to each reflector ([Figure](#)).

   - *Constant-velocity stack method:* by finding the NMO velocity that produces the best NMO-corrected and stacked section for each reflector ([Figure](#)).
2. **Borehole methods:** These include:

- **Check-shot survey (Shooting a well):** A shot at the wellhead is fired and receivers at specific depths in the well are activated. The time-depth data of the direct arrivals are used to determine the velocities to each depth (Figure).

- **Vertical Seismic Profiling (VSP) survey:** Several shots at known offsets near a well are shot while receivers at specific depths in the well are activated. The time-depth-offset data is used to determine velocities near the well (Figure).

- **Acoustic (Sonic) Logging:** Vertical transit time in the borehole wall is recorded continuously using closely-spaced receivers. The velocity is calculated from the transit time and receiver spacing (Figure).