Chapter 7

Seismic Migration

Introduction

- The main objective of migration is to enhance the horizontal (spatial) resolution by:
  1. Moving dipping reflectors to their true subsurface positions and
  2. Collapsing diffractions to their apexes.
- Migration is also called seismic imaging.
- This is a good site on seismic migration and seismic method in general.

2-D poststack versus 2-D prestack time migration

- Migrating the stacked data is called poststack migration, while migrating the pre-stack data is called prestack migration.
- On a stacked section, sources and receivers are coincident (why?).
- A reflector is assumed to be directly below its associated source-receiver pair on the stacked section.
- This is true for horizontal reflectors; however, dipping reflectors reflect energy at non-vertical direction. Therefore, the above assumption is not true and should be corrected. (Figure)
- In general, a reflection can come from any point on a semicircle whose center is the source-receiver location and radius equal to the TWTT to the reflection.
- Migration in this case is done by performing the following operations at every point on the stacked unmigrated section:

1. Constructing a semicircle at the point:
   - The semicircle's center is the source-receiver location of the trace on which the point lies.
   - The semicircle's radius is the TWTT to the point.
2. Distributing the amplitude at this point on the stacked section over the semicircle.

- Summing up the result of this operation at all points in the stacked unmigrated section produces the stacked migrated section through interference of semicircles (Figure).
- We can also think of every point in the subsurface as a scatterer that produces a diffraction hyperbola whose apex is at the position of the scatterer. Interference of these hyperbolas produces the stacked unmigrated section.
- Migration in this case is done by constructing a hyperbola at every point on the stacked unmigrated section and summing the amplitudes lying over each hyperbola and assigning the sum at its apex. The result of this process is the stacked migrated section (Figure).
- To summarize, there are two ways to migrate a 2-D poststack time section:
  1. Superposition of semicircular wavefronts (distribution migration).
  2. Superposition of diffraction hyperbolas (summation migration).
- 2-D prestack time migration takes into account the location of the source and receiver for each trace when determining the reflector position.
- Before stack, a reflection can come from any point on an ellipse whose foci are the source and receiver (Why?).

- Prestack time migration is done by spreading the amplitude at every point in the prestack gathers over an ellipse. Interference of these ellipses will produce the prestack migrated section.
- Figure.
- Follow this link for information on the equation and graph of an ellipse.

2-D time migration versus 2-D depth migration

- A diffraction is hyperbolic only if there are no lateral heterogeneities because they can distort the diffraction's hyperbolic shape (Figure).
- Time migration assumes hyperbolic diffractions and collapses them to their apexes.
- Depth migration assumes a known velocity model and estimates the correct shape of diffractions by ray tracing (Snell's law) or wavefront modeling (Huygens' principle).
- Time migration is valid only when lateral velocity variations are mild (10%) to moderate (30%). When this assumption fails, we have to use depth migration.

2-D migration versus 3-D migration

- In 2-D migration, we migrate the data once along the profile. This might generate misties on intersecting profiles (Figure).
- In addition, 2-D migration is prone to sideswipe effects. Sideswipes are reflections from out of the plane of the profile (Figure).
- In 3-D migration, we first migrate the data in the inline direction then take that migrated data and migrate it again in the crossline direction. This is the two-pass 3-D migration (Figure).

- One-pass 3-D migration can also be done using a downward continuation approach.
- Therefore, considering 2-D versus 3-D, prestack versus poststack, and time versus depth, we can have the following types of migrations (ordered from fastest but least accurate to slowest but most accurate):
    1. 2-D poststack time migration (fastest, least-accurate).
    2. 2-D poststack depth migration.
    3. 2-D prestack time migration.
    4. 2-D prestack depth migration.
    5. 3-D poststack time migration.
    6. 3-D poststack depth migration.
    7. 3-D prestack time migration.
    8. 3-D prestack depth migration (slowest, most accurate).

Geometrical aspects of migration

- Graphically, a linear reflector can be migrated by the following procedure:
1. Select two points on the reflector.
2. Draw two semicircles whose centers are the source-receiver pairs directly above the points and radii are equal to the TWTT to these points.
3. The common tangent to these semicircles is the migrated reflector (Figure).
- Migration of linear events can be done also quantitatively using the following relations:
    - The amount of horizontal displacement (d_x) introduced to the point by migration is:

$$d_x = (1/4)v^2 t.\frac{\Delta t}{\Delta x}. \tag{1}$$

- The vertical displacement (d_t) introduced to the point by migration is:

$$d_t = t\left\{1 - \sqrt{1 - \left(\frac{v.\frac{\Delta t}{\Delta x}}{2}\right)^2}\right\}. \tag{2}$$

- The linear event will have a dip ($\Delta\tau/\Delta x$) after migration given by:

$$\frac{\Delta\tau}{\Delta x} = \frac{\frac{\Delta t}{\Delta x}}{\sqrt{1 - \left(\frac{v.\frac{\Delta t}{\Delta x}}{2}\right)^2}}, \tag{3}$$

v: RMS velocity to the point on the unmigrated section, t: TWTT to the point on the unmigrated section, τ: TWTT to the point on the migrated section, $\Delta t/\Delta x$ is the dip of the linear segment on the unmigrated section, and $\Delta\tau/\Delta x$ is the dip of the linear segment on the migrated section

- d_x, d_t, and $\Delta\tau/\Delta x$ increase with time, velocity, and dip of reflector on the unmigrated section.
- The migrator's equation relates the dip angles of a linear segment on the unmigrated and migrated depth sections given by:

$$\boxed{\sin\beta = \tan\alpha}, \tag{4}$$

where α is dip angle on the unmigrated depth section and β is dip angle on the migrated depth section.

- From equation (4), we can see that $\beta \geq \alpha$ and that the maximum dip on the unmigrated section is 45°.
- When a dipping reflector is migrated, it is moved updip, steepened, and shortened. Therefore, anticlines get narrower after migration.
- Before migration, synclines look like bowties because of the interference among diffraction hyperbolae. These bowties are untied into synclines after migration (Figure).
- Migration of isolated noise spikes generates "migration smiles" due to the non-interference of the migration semicircles. This occurs often at the deeper part of the migrated section where noise dominates the unmigrated section (Figure).

Migration algorithms

- **F-K (Stolt) migration**
- This method makes use of the 2-D Fourier transform to convert the input data from the (x,t) domain to the (k,f) domain, where K denotes wavenumber (i.e., the Fourier transform of distance) and f denotes frequency (i.e., the Fourier transform of time).
- The method is summarized in the following steps:
  1. Transform the unmigrated time section (x_u,t_u) to the unmigrated depth section (x_u,z_u) by converting time to depth using one velocity function for the whole data set.

2. Use the 2-D Fourier transform on the unmigrated depth section (x_u,z_u) to produce the unmigrated (kx_u,kz_u) section.
3. Migrate each point on the unmigrated (kx_u,kz_u) section using these steps:
   i. Calculate the angle (α) that the point makes with the kz axis.
   ii. Move the point vertically up until it makes an angle (β) with the kz axis, where β is the angle corresponding to the above α angle in the migrator's equation (equation 4). This produces the migrated (kx_m,kz_m) section.
4. Use the inverse 2-D Fourier transform on the (kx_m,kz_m) section to produce the migrated depth section (x_m,z_m).
5. Transform the migrated depth section (x_m,z_m) to the migrated time section (x_m,t_m) by converting depth to time using the same velocity function in step (1).

- More details on the F-K migration are found here.

- **Finite-difference (downward continuation) migration**
  - This method works on a conceptual volume of information (x,z,t) rather than two information planes, namely the time (x,t) and depth (x,z) planes.
  - The method can be summarized in the following steps:
    1. Input the top surface seismic unmigrated section (x,z=0,t).
    2. Compute the entire volume (x,z,t) using a finite-difference solution of the wave equation.
  - Extract the depth migrated section (x,z,t=0). Figure.

- **<u>Kirchhoff migration</u>**
  - This method employs Huygen's principle, which states that every point on the wavefront can be regarded as a secondary source that generates seismic waves in the forward direction.
  - Huygen's secondary sources are <u>point apertures</u>, which produce waves that depend on propagation angle; unlike <u>point sources,</u> which are isotropic.
  - A Huygen's secondary source generates a <u>semicircular wavefront</u> in the (x,z) plane and a <u>diffraction hyperbola</u> in the (x,t) plane.
  - The purpose of Kirchhoff migration is to sum up the energy produced by every Huygen's secondary source and map it into its point of generation.
  - Therefore, there are two schemes for Kirchhoff migration:
    - Superposition of semicircular wavefronts (Hagedoorn or distribution migration).
    - Superposition of diffraction hyperbolas (summation migration).
  - The semicircle superposition approach was used earlier and abandoned because of:
    - its unsuitability for computer implementation and
    - its invalidity for laterally variable velocity structures.
  - Kirchhoff migration is based on the far-field solution of the 3-D scalar wave equation:

$$P_{out} \approx \frac{1}{2\pi}\int_{y_{\min}}^{y_{\max}} \frac{\cos\theta}{v^2 t_0} \frac{\partial P_{in}}{\partial t} dy \, , \tag{5}$$

where P_{out} is the migrated amplitude, P_{in} is the unmigrated amplitude, θ is the propagation angle from the vertical, v is the RMS velocity at the point to be migrated, t_0 is the vertical TWTT at the point to be migrated, and y is the horizontal distance from the point to be migrated to the point to be summed (integrated). Note also that:

- $cos\theta = \frac{t_0.v}{\sqrt{(t_0.v)^2+4y^2}}$ is *the obliquity factor*, which decreases the amplitude along the hyperbola as we get away from the apex (Figure).
- $1/v^2t_0$ is *the spherical divergence factor* due to geometrical spreading.
- $\partial P_{in}/\partial t$ is the *wavelet shaping factor* (in 3D surveys).
- The integration is carried over the width (y_{min},y_{max}) of the diffraction hyperbola whose apex is located at the point to be migrated (i.e., y = 0 corresponds to the location of the apex of the hyperbola).

- Therefore, the Kirchhoff migration algorithm amounts to applying the following steps (for each point on the unmigrated zero-offset time section):

  (1) Computing the hyperbolic traveltime as:

  $$t^2(y) = t_0^2 + 4y^2/v^2, \quad (6)$$

  where t(y) is TWTT at a distance y from the point to be migrated.

  (2) Applying the obliquity factor.

(3) Applying the spherical divergence factor *<u>if it has not been applied already</u>*.

(4) Applying the wavelet shaping factor.

(5) Summing the resultant amplitudes along the hyperbola for all possible offsets.

(6) Assigning the sum at the apex of the hyperbola (i.e., point to be migrated).

- **Practical aspects of Kirchhoff migration**
  - The main important parameters when applying Kirchhoff migration in practice are:

    (1) Aperture width.

    (2) Maximum dip to migrate.

    (3) Velocity errors.

(1) <u>Aperture width:</u>

- One definition of the migration aperture is the maximum width of the hyperbola over which the summation is carried out.
- Excessively small aperture widths produces the following undesirable effects:
  - It suppresses steeply dipping events because migration is only using the flat part of the hyperbola.
  - It organizes random noise as horizontal events, especially in the deeper part of the section for the same above reason.

- Excessively large aperture widths produce the following undesirable effects:
    - It needs more unnecessary computer time.
    - It degrades the migration quality in poor S/N ratio data because more noise from the deeper part of the section is included.
- For any given depth (Z) on the unmigrated section, an optimal value of the aperture width is equal to $1.15Z$.

(2) Maximum dip to migrate:

- The maximum dip to migrate is related to the aperture width d_x by equation (1).
- Smaller maximum dip limit means smaller aperture.
- Large maximum dip limit means more computer time.
- The maximum dip can be used to filter steeply dipping coherent noise.

(3) Velocity errors:

- Velocity is related to the aperture width d_x by equation (1).
- Using a lower velocity produces undermigration due to the incomplete collapse of diffraction hyperbolae.
- Using a higher velocity produces overmigration due to the generation of reversed diffractions.
- Steeper dips are more sensitive to velocity errors, as seen from equation (3).