THROUGHPUT OF ARQ PROTOCOLS OVER RICIAN, NAKAGAMI AND MIMO BLOCK FADING CHANNELS†

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الخلاصــة

تعتبر القنوات ذات الذاكرة من أكثر القنوات شيوعا في أجهزة الاتصالات اللاسلكية. تُقدم هذه الورقة تحليل أداء نُظم اعادة الارسال الأوتوماتيكية المعتمدة على الاعادة الاختيارية في القنوات اللاسلكية ذات الذاكرة. لقد تم تحليل أداء نظم اعادة الارسال الأوتوماتيكية المستخدمة لهوائى واحد أو عدة هوائيات لعدد من معدلات الارسال المبنية على تعديل تردد أو مرحلة الموجة اللاسلكية. لقد تم حساب التحليل الجديد لعدد من النظم مع دراسة تأثير طول الذاكرة ومدى علاقة هوائيات المرسل مع بعضها البعض على الأداء العام. تدل التجارب أن التحليل الجديد دقيق جدا في معظم الأحوال المختلفة.

ABSTRACT

Block fading is a popular channel model that approximates the behavior of different wireless communication systems. Automatic-repeat request (ARQ) protocols are used to provide reliable communication in wireless networks. In this paper the throughput of the basic selective-repeat (SR) ARQ in block fading environments is derived. Single-antenna ARQ systems employing both coherent BPSK and QPSK and noncoherent orthogonal BFSK are analyzed over block fading channels characterized by Rician and Nakagami fading distributions. Moreover, the performance of multi-input multi-output (MIMO) ARQ systems employing space-time block codes (STBCs) is derived. The effect of antenna correlation in MIMO ARQ systems is investigated analytically. Results show that longer block lengths improve the performance of basic ARQ protocols. Furthermore, the throughput gain obtained by increasing the block length is a decreasing function of the block length. As the fading severity of the channel increases, the performance improvement resulting from increasing the block length increases.

Key words: ARQ, throughput, error probability, fading channels, space-time, multiple antennas, transmit diversity, MIMO, Rician, Nakagami.

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1. INTRODUCTION

A serious challenge to have good communication quality in wireless networks is the time-varying multipath fading environments, which causes the received signal-to-noise ratio (SNR) to vary randomly. The fading distribution varies according to the environment. For example, if a line-of-sight exists between the transmitter and the receiver, the fading process is modeled by a Rician distribution [1]. Another popular model for the fading process is the Nakagami distribution [2], which provides a family of distributions that fit measurements under different propagation environments [3].

Error-free communication is often accomplished using ARQ techniques. Basic ARQ protocols are based on error detection and retransmission [4]. If a received packet is detected with error at the receiver, a retransmission of the same packet is initiated through a feedback channel [4]. The performance of an ARQ protocol is characterized by its *throughput*, which is defined as the ratio of the average number of packets accepted as error-free by the receiver to the total number of transmitted packets [5]. The throughput of "selective repeat" (SR) ARQ is known to be the highest among the basic ARQ protocols [4]. In SR ARQ the sender retransmits only the negatively acknowledged packets.

The throughput of an ARQ protocol is a function of the fading statistics affecting transmitted packets. In practice, channel models that exhibit memory are often used to model wireless systems. Of a particular interest, the *block fading* channel [6] provides an acceptable model for many wireless communication systems such as frequency-hopped spread-spectrum (FH-SS) and time-division multiple access (TDMA). In this model, a packet consists of blocks of symbols that undergo independent fading realizations, where the fading coefficient stays constant for symbols within each block.

Most research efforts have concentrated on the analysis of hybrid ARQ protocols where channel coding is used [4, 5, 7]. Particularly, in [7] the throughput of basic ARQ protocols over slow Rayleigh fading was derived. However, basic ARQ protocols are used to ensure correct delivery of data packets in many systems such as Bluetooth. In some packet formats used in Bluetooth, uncoded data is transmitted without using any channel coding [8]. Thus it is of interest to analyze the throughput of uncoded transmission (*i.e.*, basic ARQ) in wireless networks. In this paper, we analyze the throughput of basic SR ARQ in block fading environments with Rician and Nakagami fading distributions. Moreover, multiple-antenna ARQ systems employing space-time block codes (STBCs) [9] are considered. Furthermore, the effect of antenna correlation on the performance of SR ARQ is studied analytically.

The outline of the paper is as follows. The SR ARQ system model is described in the next section. In Section 3, the throughput of basic SR ARQ employing single and multiple antennas is derived for different fading statistics, and results are discussed therein. Conclusions are presented in Section 4.

2. SYSTEM MODEL

2.1. Single-Antenna System

In the basic SR ARQ, the sender retransmits the packet if it contains some errors. The process is repeated until the packet is successfully received [4]. A packet is composed of kN bits, where each k bits are mapped onto one symbol of an M-ary signal constellation. Thus each packet contains N symbols. In this paper we consider coherent BPSK and QPSK and noncoherent orthogonal BFSK. The channel affecting each packet is a block fading. In this model each packet undergoes F independent fading realizations, where each block of $m = \lceil \frac{N}{F} \rceil$ symbols are affected by the same fading realization.

In coherent receivers the channel phase and amplitude are available, and the matched filter sampled output at time l in the f^{th} fading block is given by

$$y_{f,l} = \sqrt{E_s} h_f s_{f,l} + z_{f,l} \; , \tag{2.1}$$

where $s_{f,l}$ is the transmitted signal and $z_{f,l}$ is a noise modeled as an AWGN with a $\mathcal{CN}(0, N_0)$ distribution. Note that E_s represents the average received signal energy, which is equal to the average transmitted signal energy multiplied with an attenuation coefficient that depends on the distance between the transmitter and the receiver. Here, only propagation loss due to distance is assumed with no shadowing effects. The coefficient h_f is the channel gain in fading block f given by $h_f = a_f \exp(j\theta_f)$, where θ_f is uniformly distributed phase and a_f is the amplitude. In this paper, we assume that a_f has either a Rician or a Nakagami distribution. The coherent receiver chooses the signal s that maximizes the metric $\mathrm{Re}\{y^*_{f,l} s\}$, where $\mathrm{Re}\{.\}$ represents the real part of a complex number.

In a noncoherent BFSK receiver, a square-law combining [10] is employed, whose outputs are represented by

$$\begin{aligned} r^{(I,c)}_{f,l} &= \sqrt{E_s} a_f \delta(c_{f,l}, c) \cos(\theta_f) + \eta^{(I,c)}_{f,l} \\ r^{(Q,c)}_{f,l} &= \sqrt{E_s} a_f \delta(c_{f,l}, c) \sin(\theta_f) + \eta^{(Q,c)}_{f,l}, \end{aligned} \tag{2.2}$$

where $r^{(I,c)}_{f,l}$ and $r^{(Q,c)}_{f,l}$ are defined respectively as the correlation of the received signal with the inphase and quadrature dimensions of the signal corresponding to the bit $c = 0, 1$. In (2.2), $\delta(x, y) = 1$ if $x = y$ and $\delta(x, y) = 0$ otherwise; and $\eta^{(I,0)}_{f,l}$, $\eta^{(Q,0)}_{f,l}$ $\eta^{(I,1)}_{f,l}$ and $\eta^{(Q,1)}_{f,l}$ are independent random variables with a $\mathcal{N}(0, \frac{N_0}{2})$ distribution. The receiver chooses the bit c with the maximum $|r^{(I,c)}_{f,l}|^2 + |r^{(Q,c)}_{f,l}|^2$.

2.2. MIMO System

In transmitters employing STBCs [9], every n_t BBSK (or QPSK) signals are mapped into a $n_t \times n_t$ transmission matrix $\mathbf{G}$. For $n_t = 2$ the transmission matrix is

$$\mathbf{G} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}, \tag{2.3}$$

where $(.)^*$ denotes the complex conjugate operation. For $n_t = 2$, antennas 1 and 2 transmit the signals $s_1, -s_2^*$ and s_2, s_1^*, respectively during $2T$ seconds. In general, the i^{th} column of $\mathbf{G}$ is transmitted over the i^{th} transmit antenna during a *time slot*, which is n_t times the symbol duration. The resulting STBC has a full rate, *i.e.*, one symbol is transmitted every T seconds. More examples of complex orthogonal matrices were presented in [11] for different values of n_t. Note that orthogonal matrices for STBCs do not exist for all the values of n_t.

Let $\mathbf{G}_{f,l}$ be the transmission matrix in the l^{th} time slot of fading block f, the corresponding received vector is given by

$$\mathbf{y}_{f,l} = \sqrt{E_s} \mathbf{G}_{f,l} \mathbf{h}_f + \mathbf{z}_{f,l} \; , \tag{2.4}$$

where $\mathbf{z}_{f,l}$ is a length-n_t column random vector with a distribution $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ and $\mathbf{I}$ denotes the $n_t \times n_t$ identity matrix. The vector $\mathbf{h}_f$ is the channel gain in fading block f and is modeled as $\mathcal{CN}(\mathbf{0}, \mathbf{C_h})$, where $\mathbf{C_h}$ is the $n_t \times n_t$ covariance matrix of the channel. In order to enable simple detection, the fading process should remain constant for at least one time slot, *i.e.*, n_tT seconds. This constrains the block length to be a multiple of n_t. A simple receiver for STBC was proposed in [9], whose outputs for $n_t = 2$ have the form

$$\tilde{s}_1 = (|h_1|^2 + |h_2|^2)y_1 + \tilde{z}_1$$

$$\tilde{s}_2 = (|h_1|^2 + |h_2|^2)y_2 + \tilde{z}_2, \tag{2.5}$$

which makes a n_t-branch STBC equivalent to a n_t-branch maximal-ratio combiner (MRC), where $\tilde{z}_1$ and $\tilde{z}_2$ are the noise samples at the output of the detector. Note that the rows of $\mathbf{G}$ should be orthogonal [9]. This detector was generalized for larger values of n_t in [11].

3. PERFORMANCE ANALYSIS

The performance of an ARQ protocol is characterized by its throughput η, which is defined as the ratio of the average number of packets accepted as error-free by the receiver to the total number of transmitted packets [5]. It is assumed that the probability of undetected errors is negligible. Under the assumptions of infinite buffer size at the receiver and noiseless feedback channel, the throughput of SR ARQ protocols [4] is given by

$$\eta = 1 - P_p \ , \tag{3.1}$$

where P_p is the probability of packet error, which is a function of the fading process affecting a packet.

Under the block fading model a packet of length N symbols undergoes F independent fading realizations, where $m = \lceil \frac{N}{F} \rceil$ symbols is the size of each fading block, which represents the block length. The packet error probability conditioned on the fading channel gains $\mathbf{H} = \{\mathbf{h}_f\}_{f=1}^{F}$ is the probability that at least one fading block is in error, *i.e.*,

$$P_{p|\mathbf{H}} = 1 - \prod_{f=1}^{F} (1 - P_{B|\mathbf{h}_f}), \tag{3.2}$$

where $P_{B|\mathbf{h}_f}$ is the probability that the f^{th} fading block is in error conditioned on the fading gains $\mathbf{h}_f$. It is the probability that at least one symbol in the fading block is in error, *i.e.*,

$$P_{B|\mathbf{h}_f} = 1 - (1 - P_{s|\mathbf{h}_f})^m, \tag{3.3}$$

where $P_{s|\mathbf{h}_f}$ is the conditional symbol error probability, which is a function of the modulation scheme and the receiver employed. Let $\gamma_f = a_f^2 \gamma_s$ denote the received SNR value for a symbol in the f^{th} fading block, where $\gamma_s = \frac{E_s}{N_0}$ is the average SNR. Substituting (3.3) in (3.2), the conditional packet error probability becomes

$$P_{p|\Gamma} = 1 - \prod_{f=1}^{F} (1 - P_{s|\gamma_f})^m, \tag{3.4}$$

where $\Gamma = \{\gamma_f\}_{f=1}^{F}$. The unconditional packet error probability is obtained by averaging (3.4) over the probability density function (pdf) of the fading statistics Γ. Since the fading gains affecting different fading blocks are independent, the unconditional packet error probability becomes

$$P_p = 1 - \prod_{f=1}^{F} \mathrm{E}_{\gamma_f} \left[(1 - P_{s|\gamma_f})^m \right]. \tag{3.5}$$

Since all fading blocks are affected by identical fading processes, that will be denoted with γ, the packet error probability is

$$P_p = 1 - \left\{ \mathrm{E}_\gamma \left[(1 - P_{s|\gamma})^m \right] \right\}^F . \tag{3.6}$$

Using the binomial expansion

$$\mathrm{E}_\gamma \left[(1 - P_{s|\gamma})^m \right] = \sum_{i=0}^{m} (-1)^i \binom{m}{i} \mathrm{E}_\gamma [P_{s|\gamma}^i], \tag{3.7}$$

where we have used the linearity of the expectation operation. Substituting (3.7) in (3.6), the packet error probability becomes

$$P_p = 1 - \left\{ \sum_{i=0}^{m} (-1)^i \binom{m}{i} \mathrm{E}_\gamma \left[P_{s|\gamma}^i \right] \right\}^F . \tag{3.8}$$

Let $P_{si} = \mathrm{E}_\gamma [P_{s|\gamma}^i]$ and using multi-nomial expansion to simplify (3.8) yields

$$P_p = 1 - \sum_{j_0, j_1, \ldots, j_m \in J} \frac{F!}{j_0! j_1! \ldots j_m!} \prod_{i=0}^{m} (-1)^{i j_i} \binom{m}{j_i}^{j_i} P_{si}^{j_i}, \tag{3.9}$$

where $J = \{j_0, j_1, \ldots, j_m : j_0 + j_1 + \ldots + j_m = F\}$. The throughput of SR ARQ is obtained by substituting (3.9) in (3.1) and computing (3.9) for the modulation scheme and the channel model under study. The expression in (3.9) can be simplified for modulation schemes with a conditional symbol error probability given by an exponential function such as noncoherent orthogonal BFSK, whose conditional bit error probability is given by

$$P_{s|\gamma_f} = \frac{1}{2} \mathrm{e}^{-\frac{1}{2}\gamma_f} . \tag{3.10}$$

On the other hand, (3.9) is hard to be evaluated in a closed-form for coherent BPSK and QPSK, whose conditional symbol error probability is given by

$$P_{s|\gamma_f} = \mathrm{Q}\left(\sqrt{2\gamma_f}\right) . \tag{3.11}$$

Therefore, numerical integration is used to evaluate (3.9) using (3.11) and averaging over the fading statistics.

3.1. Rician Channels

If a line-of-sight exists between the transmitter and the receiver, the amplitude of the channel is modeled as a Rician random variable [1]. In this model the received signal consists of the specular and diffuse signal components. The specular component is due to the line-of-sight reception and the diffuse component results from multipath reception. In this case, the channel gain in each fading block h_f is modeled as a complex Gaussian variable with $\mathcal{CN}(b, 1)$, where b represents the specular (line-of-sight) component of the channel. The SNR pdf of a Rician fading channel [12] is given by

$$f_\gamma(\gamma) = \frac{(1+K)}{\gamma_s} \exp\left[-K - \frac{(1+K)\gamma}{\gamma_s} \right] I_0 \left(2\sqrt{\frac{K(1+K)\gamma}{\gamma_s}} \right), \qquad \gamma \geq 0, \tag{3.12}$$

where $K = b^2$ is the energy of the specular component, $I_0(.)$ is the zero-order modified Bessel function of the first kind. In this context, K denotes the ratio of the specular component energy to the diffuse component energy. If $K = 0$ we get the Rayleigh distribution, whereas the channel approaches the no fading case (AWGN channel) as K increases. Averaging (3.10) over the channel statistics in (3.12) the symbol error probability becomes

$$P_{si} = \frac{1}{2}\frac{1+K}{1+K+i\gamma_s/2}.\exp\left(\frac{iK\gamma_s/2}{1+K+i\gamma_s/2}\right). \tag{3.13}$$

The throughput of SR ARQ employing noncoherent orthogonal BFSK over Rician block fading channels is found by substituting (3.13) in (3.9) and (3.1). The throughput of SR ARQ employing BPSK or QPSK is computed using numerical integration.

Throughout the paper, the results were generated for SR ARQ with a packet length of $N = 1024$. Figure 1 shows the throughput of the SR ARQ employing orthogonal BFSK over Rician block fading channels with $K = 0$, *i.e.*, Rayleigh fading. We observe that the throughput improves with longer block because the probability of a packet error is lower. This is because a packet is considered in error if it includes at least one symbol error. Shorter blocks result in larger number of independent fading realizations affecting a packet, which increases the probability that a symbol falls in a deep fade. Hence, the probability of error increases with shorter block lengths and therefore the throughput degrades. Also, every double in the block length results in a 2 dB gain in the SNR.

In Figure 2, the throughput of SR ARQ achieved at SNR of 15 dB is shown for BPSK and noncoherent orthogonal BFSK over different Rician channels. We observe that for the same operating SNR value, the throughput improves significantly in the short block range and a little improvement is achieved in the long

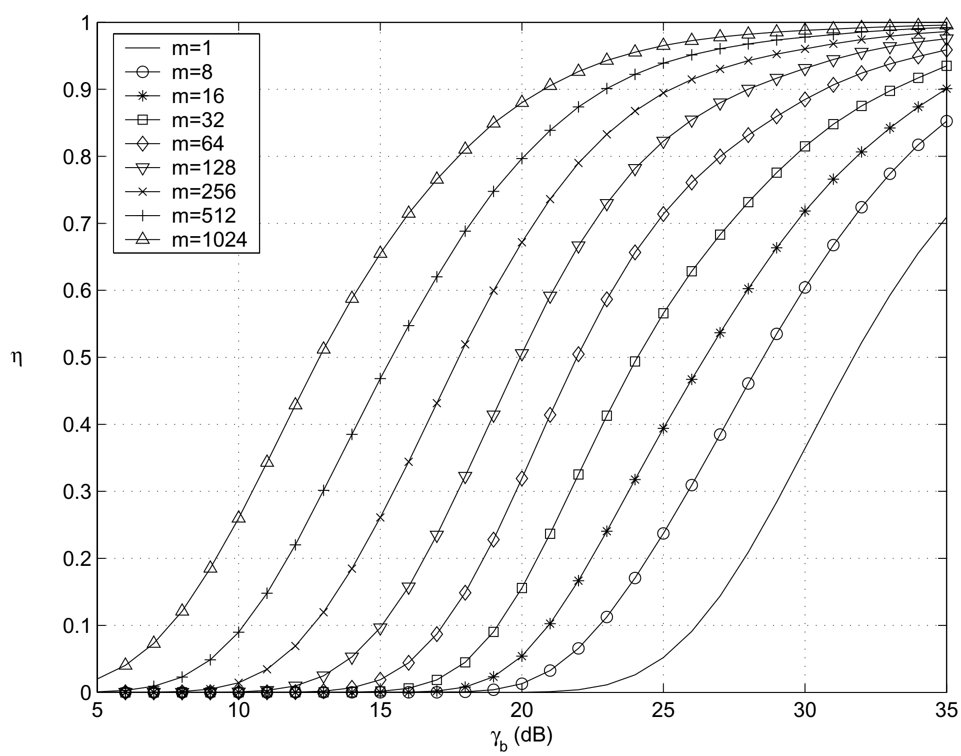

Figure 1. Throughput of SR ARQ system employing noncoherent orthogonal BFSK over Rayleigh block fading channels with a block length m and a packet length of $N = 1024$.

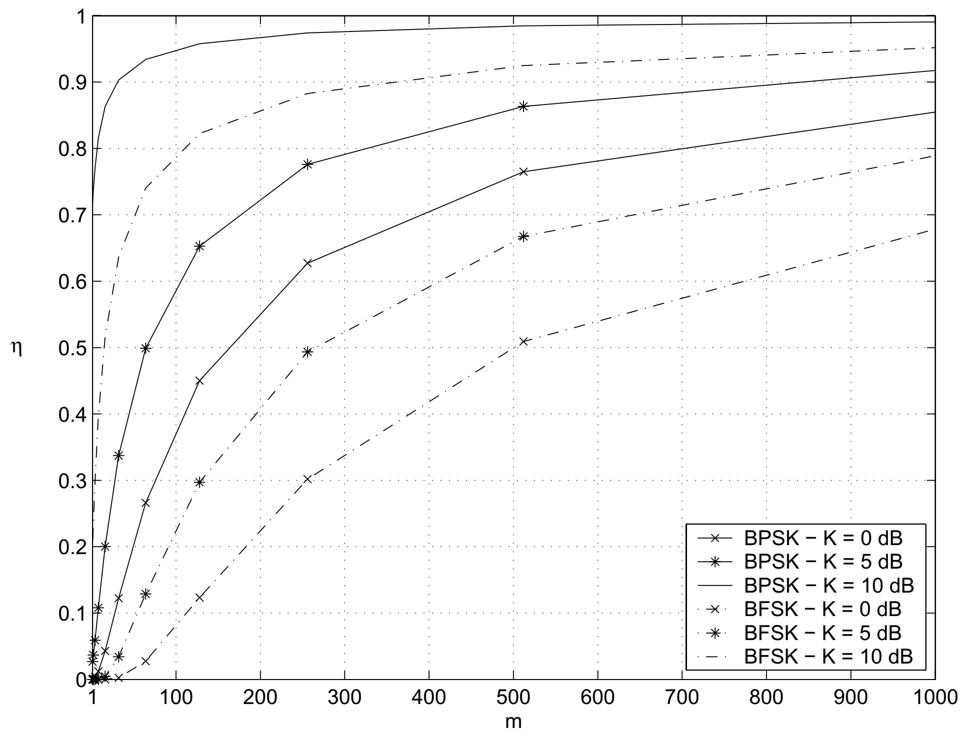

Figure 2. Throughput of SR ARQ systems employing coherent BPSK and noncoherent orthogonal BFSK over Rician block fading channels at $\gamma_s = 15$ dB versus the block length m for a packet length of $N = 1024$.

block range. For a Rician channel with $K = 10$ dB, little throughput improvement is achieved by having block lengths greater than 100 symbols. As the specular component of the channel becomes weaker (smaller K values), the block length, above which a little performance improvement is achieved, becomes larger. Moreover, the throughput gain due to longer block decreases as K increases. This is because larger specular component reduces the multipath component, which reduces the effect of longer block on improving the performance.

3.2. Nakagami Channels

Nakagami distribution was shown to fit a large variety of channel measurements [3]. Under Nakagami distribution, the pdf of the received SNR [2] is given by

$$f_\gamma(\gamma) = \left(\frac{M}{\Omega}\right)^M \frac{\gamma^{M-1}}{\Gamma(M)} \exp\left(-\frac{M\gamma}{\Omega}\right), \quad \gamma > 0, M > 0.5, \tag{3.14}$$

where $\Gamma(.)$ is the Gamma function and $M = \frac{\Omega^2}{\mathrm{Var}[\sqrt{\gamma}]}$ is the Nakagami parameter that indicates the fading severity. As M increases, the fading becomes less severe and approaches the AWGN channel when $M \to \infty$. The Nakagami distribution covers a wide range of fading scenarios including Rayleigh fading when $M = 1$. Note that the Nakagami and Rician distributions are related [12] through the relation between the Rician K-factor and the Nakagami parameter that is given by

$$M = \frac{(K+1)^2}{2K+1} \, . \tag{3.15}$$

The bit error probability for noncoherent orthogonal BFSK over Nakagami fading is found by averaging (3.10) over the statistics in (3.14) resulting in

$$P_{si} = \frac{1}{2}\left(\frac{1}{1 + i\gamma_s/2M}\right)^M. \tag{3.16}$$

Substituting (3.16) in (3.9) and (3.1) yields the throughput of SR ARQ employing orthogonal BFSK over Nakagami block fading channels.

Figure 3 shows the throughput of SR ARQ employing orthogonal BFSK over Nakagami block fading channels with $M = 2$. Comparing with Figure 1, which is the Rayleigh case ($M = 1$), we observe that the SNR gain due to longer block decreases as M increases, *i.e.*, as the channel becomes less random. The same observations noted in the Rician case apply here. In Figure 4 the throughput of SR ARQ achieved at SNR of 15 dB is shown for BPSK and noncoherent orthogonal BFSK with different Nakagami parameters. As in the Rician case, for the same operating SNR, the throughput improves significantly in the short block range and a little improvement is achieved in the long block range. Moreover, as the channel becomes less random (larger M values), the block length above which a little performance improvement is achieved becomes smaller.

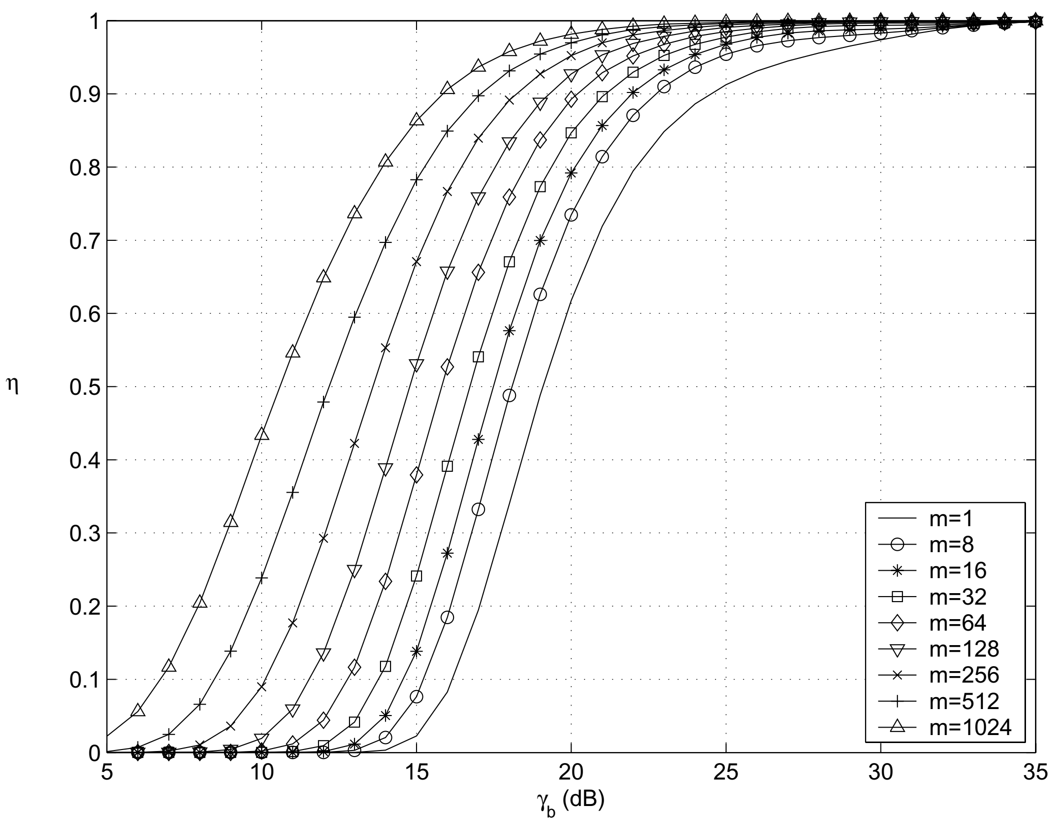

Figure 3. Throughput of SR ARQ system employing noncoherent orthogonal BFSK over Nakagami block fading channels with fading parameter $M = 2$ and a packet length of $N = 1024$.

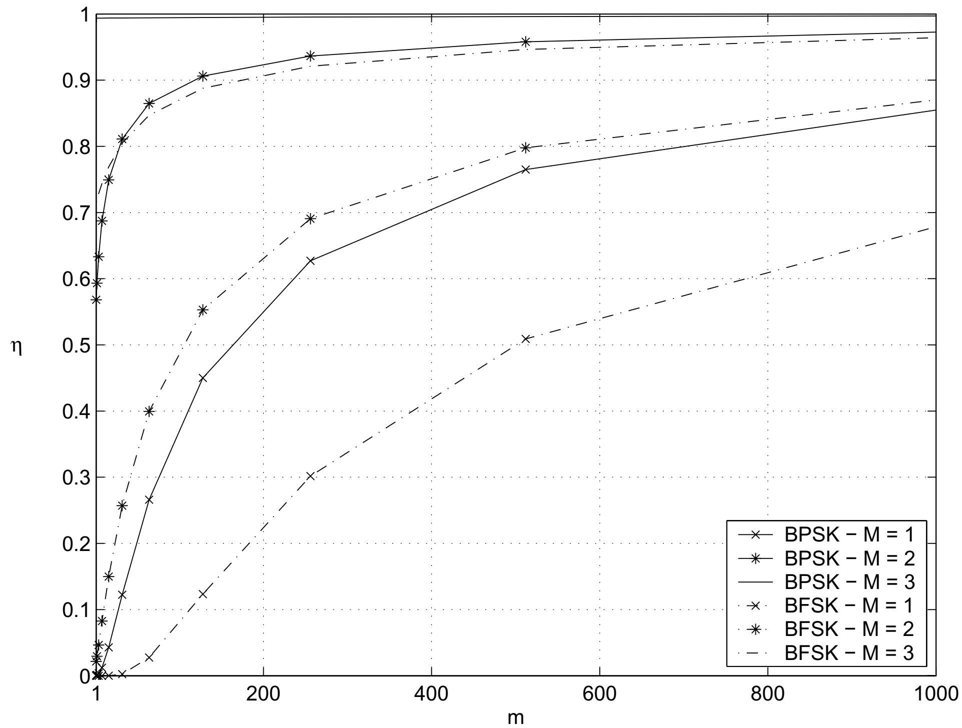

Figure 4. Throughput of SR ARQ systems employing coherent BPSK and noncoherent orthogonal BFSK modulation schemes over Nakagami block fading channels at $\gamma_s = 15$ dB versus the block length m for a packet length of $N = 1024$.

3.3. MIMO Channels

In the following the throughput of SR ARQ is derived for MIMO channels with one receive antenna. Note that the presented results can be easily extended to multiple receive antennas. In addition, the results apply for full-rate orthogonal STBCs, where they can be easily extended to the general case by taking into account the reduction in the SNR due to the reduction in the STBC rate. The conditional symbol error probability corresponding to the decision variables in (2.5) is given by

$$P_{s|\mathbf{h}} = \mathrm{Q}\left(\sqrt{2\gamma_s \sum_{i=1}^{n_t} |h_i|^2}\right). \tag{3.17}$$

In the MIMO case we assume that the channel amplitude $|h_i|$ is Rayleigh distributed. If the fading processes from different transmit antennas are uncorrelated, then the variable $q = \gamma_s \sum_{i=1}^{n_t} |h_i|^2$ has a Chi-square distribution with $2n_t$ degrees of freedom [13] given by

$$f_q(q) = \frac{1}{(n_t - 1)!} \frac{q^{n_t - 1}}{\Omega_q^{n_t}} e^{-q/\Omega_q}, \quad q > 0, \tag{3.18}$$

where $\Omega_q = \mathrm{E}[q] = n_t \gamma_s$.

When antennas are placed relatively close to each other, the fading processes from different transmit antennas will be correlated [14, 15]. In this case, the channel vector, $\mathbf{h}_f$ in (2.4) is a correlated complex Gaussian random vector with a covariance matrix $\mathbf{C_h}$ whose $(i,j)^{th}$ element is $\mathrm{E}[h_i^* h_j] = \rho_{ij}$, where ρ_{ij} is the correlation coefficient between channels from the i^{th} and j^{th} transmit antennas. Note that $\mathrm{E}[h_i^* h_i] = 1$. In order to derive the unconditional error probability from (3.17), we need to average over the pdf of the inner product $\mathbf{h}^*\mathbf{h} = \sum_{i=1}^{n_t} |h^i|^2$ is needed, which is difficult to perform.

Recall the eigenvalue decomposition of the covariance matrix $\mathbf{C_h} = \mathbf{U}\Lambda\mathbf{U}^T$, where Λ is a diagonal matrix containing the eigenvalues of $\mathbf{C_h}$, *i.e.*, $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{n_t}\}$, and $\mathbf{U}$ is a unitary matrix that contains the eigenvectors of $\mathbf{C_h}$ in its rows. Thus an uncorrelated Gaussian random vector $\mathbf{g}$ with a covariance matrix $\mathbf{C_g} = \Lambda$ is generated by applying the linear transformation $\mathbf{g} = \mathbf{U}^T\mathbf{h}$, and the conditional symbol error probability becomes

$$P_{s|\mathbf{h}} = \mathrm{Q}\left(\sqrt{2\gamma_s \sum_{i=1}^{n_t} \lambda_i |g_i|^2}\right). \tag{3.19}$$

Let $q = \gamma_s \sum_{i=1}^{n_t} \lambda_i |g_i|^2$ and assume distinct eigenvalues $\{\lambda_i\}_{i=1}^{n_t}$, the pdf of q is found using the inverse Fourier transform [16] to be

$$f_q(q) = \sum_{i=1}^{n_t} \prod_{j \neq i} \frac{1}{\lambda_i - \lambda_j}\, e^{-q/(\lambda_i \Omega_q)}, \quad q > 0. \tag{3.20}$$

Substituting (3.18) or (3.20) in (3.9) with γ is replaced by q results in the packet error probability of STBC over Rayleigh block fading channels with uncorrelated and correlated transmit branches, respectively. The throughput follows directly from substituting (3.9) in (3.1). Since the conditional symbol error probability is

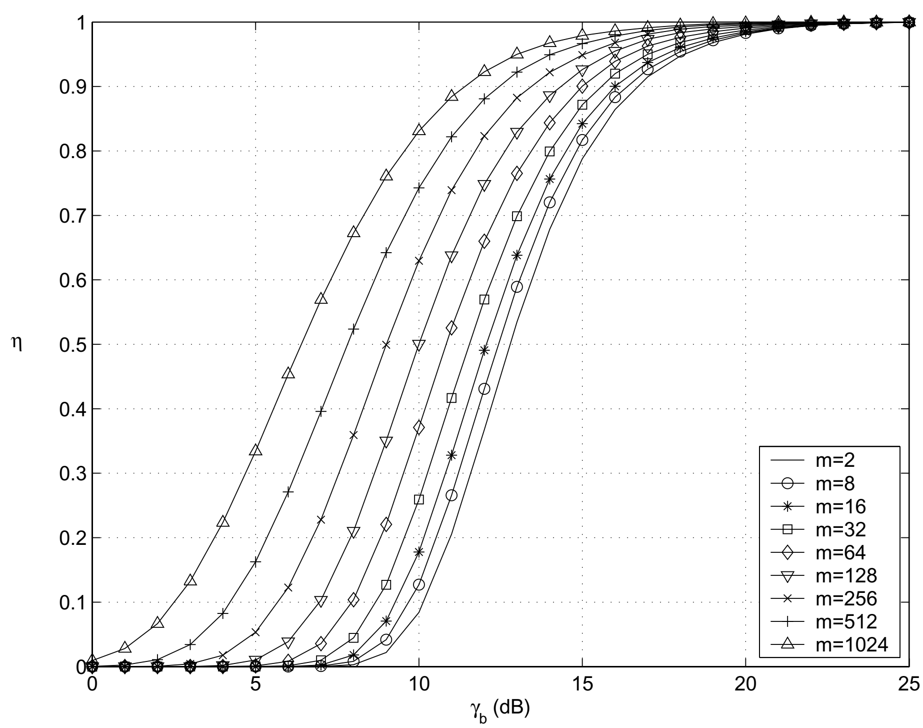

Figure 5. Throughput of SR ARQ systems employing BPSK modulated STBC with $n_t = 2$ over Rayleigh block fading channels for a packet length of $N = 1024$.

given by the Q-function, numerical integration is used to compute (3.9). Figure 5 shows the throughput of the SR ARQ employing STBC with BPSK and two transmit antennas over Rayleigh block fading channels. Note that the throughput of STBC employing QPSK is double that of the BPSK system.

3.4. **Comparisons**

In Figure 6 the throughput of SR ARQ achieved at SNR of 15 dB is shown for noncoherent orthogonal BFSK and BPSK with single and two transmit antennas. We observe that the SNR gain due to longer block decreases with increasing the number of transmit antennas. This is because having more antennas reduces the probability of packet error, which improves the throughput. As the order of space diversity increases, the block length above which a little performance improvement is achieved becomes larger. The figure also shows the effect of antenna correlation on the throughput. We observe that a correlation coefficient of 0.7 results in small degradation in the performance and a significant SNR gain compared to the single-antenna scenario.

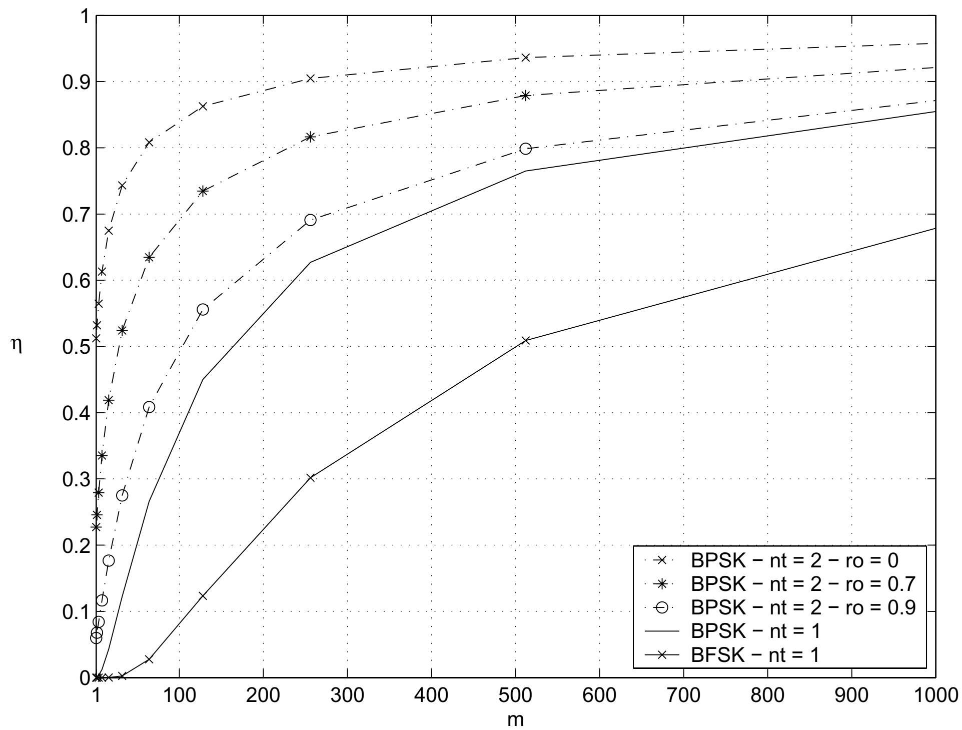

Figure 6. Throughput of SR ARQ systems employing different modulation schemes over Rayleigh block fading channels at $\gamma_s = 15$ dB versus the block length m for a packet length of $N = 1024$.

4. **CONCLUSIONS**

In this paper we derived the throughput of the SR ARQ over block fading channels with Rician and Nakagami distributions and MIMO systems. It was found that longer block improves the performance of basic SR ARQ. Furthermore, the throughput improvement obtained in basic ARQ decreases with increasing the block length. The performance improvement gained by increasing the block increases as the fading becomes more severe (random). Results shows that space diversity reduces the need for long block used in single-antenna systems to improve the performance. Moreover, a little degradation in the performance is observed when a correlation of 0.7 exists between the transmit antenna in a STBC system.

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