# Error Probability of Coded STBC Systems in Block Fading Environments

Salam A. Zummo, *Member, IEEE* and Wayne E. Stark, *Fellow, IEEE*

***Abstract*— In this letter, a union bound on the error probability of coded multi-antenna systems over block fading channels is derived. The bound is based on uniform interleaving of the coded sequence prior to transmission over the channel. Using this argument the distribution of error bits over the fading blocks is computed and the corresponding pairwise error probability (PEP) is derived. We consider coded systems that concatenate a binary code with a space-time block code (STBC). Coherent detection is assumed with perfect and imperfect channel state information (CSI) at the receiver, where imperfect CSI is obtained using pilot-aided estimation. Under channel estimation environments, the tradeoff between channel diversity and channel estimation is investigated and the optimal channel memory is approximated analytically. Results show that the performance degradation due to channel memory decreases as the number of transmit antennas is increased. Moreover, the optimal channel memory increases with increasing the number of transmit antennas.**

***Index Terms*— Convolutional, union bound, block fading, block interference, channel estimation, space-time block codes, multi-antenna, MIMO.**

## I. INTRODUCTION

THE USE OF error correction coding is among standard approaches to mitigate multipath fading by providing the receiver with channel diversity. *Channel diversity* is defined roughly as the number of independent fading realizations available at the receiver to decode a codeword. The performance of binary codes over infinitely interleaved fading channels is commonly analyzed using the union bound as in [1], [2]. However, the channel in many wireless systems, such as frequency-hopped spread-spectrum (FH-SS), time-division multiple access (TDMA) and orthogonal frequency division multiplexing (OFDM), can be modeled as a *block fading channel* [3]. In this model, a frame undergoes several independent fading realizations, each affecting a group of $m$ bits.

Another approach to diversity is to use multiple antennas at the transmitter or receiver [4], [5]. Space-time block coding (STBC) was proposed by Alamouti [6] to provide diversity at the transmitter. This idea was soon generalized by Tarokh *et al.* [7] to a general number of transmit antennas. In cellular systems base stations are frequently equipped with multiple antennas to provide receive diversity in the uplink. Since installing multiple antennas in mobile units is difficult due to size limitations, the antennas at the base station can be used along with a STBC to provide transmit diversity in the downlink. Thus it is of interest to analyze the performance of coded STBC in wireless systems that can be modeled as a block fading channel.

In practice the receiver has to estimate the channel state information (CSI). If the frame size is infinite, long channel memory permits better channel estimation. However, if the frame size is finite, there exists a fundamental tradeoff between the channel diversity and channel estimation [8], [9]. It is well known [10] that channel estimation becomes more crucial to the performance of ST codes as the number of transmit antennas increases. In this letter, we derive a union bound on the probability of error for coded STBC systems over block fading channels with perfect and imperfect CSI at the receiver. The union bound is used to investigate the tradeoff between channel diversity and estimation. Furthermore, the optimal channel memory is approximated and the effect of the space diversity on the optimal channel memory is investigated.

The letter is organized as follows. The system model is described in Section II. Then, the error probability of coded STBCs over block fading channels is derived in Section III. In Section IV, the pairwise error probability (PEP) is derived for the cases of coherent receivers with perfect and imperfect CSI at the receiver. Conclusions are discussed in Section VI.

## II. SYSTEM MODEL

The transmitter in a coded STBC system consists of a binary encoder (e.g., convolutional or turbo), an interleaver, a modulator and a STBC. In a frame duration of $NT$ seconds, a rate-$R_c$ encoder maps $K$ information bits into $N$ coded bits, where $R_c = \frac{K}{N}$. Here, $T$ represents the bit duration. Then, the coded bits are interleaved and the $i^{th}$ bit $c_i$ is modulated to a signal $s_i \in \{\pm 1\}$ using BPSK ($s_i = (-1)^{c_i}$). The frame is transmitted over a block fading channel with $F$ blocks, where each fading block of $m = \lceil \frac{N}{F} \rceil$ bits undergoes the same fading realization that is independent of the other fading blocks. This is a reasonable assumption if the channel coherence time is longer than the duration of each fading block [11]. Note that the interleaver is used to spread out burst errors in the decoder.

The transmitter is equipped with $n_t$ transmit antennas and there is a single receive antenna. Note that the results of this letter are easily generalized to multiple receive antennas. After encoding and interleaving, each group of $n_t$ signals are mapped into a $n_t \times n_t$ transmission matrix $\mathcal{G}$. For the case of $n_t = 2$, $\mathcal{G}$ is the Alamouti code [6] given by

$$\mathcal{G} = \begin{pmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{pmatrix}. \quad (1)$$

The transmission of $\mathcal{G}$ takes place in a *time slot* of duration $n_t T$ seconds, where the $i^{th}$ row of $\mathcal{G}$ is transmitted over the

Manuscript received November 12, 2003; revised December 22, 2004 and April 18, 2005; accepted April 24, 2005. The associate editor coordinating the review of this paper and approving it for publication was K. B. Lee. This work was supported in part by King Fahd University of Petroleum and Minerals (KFUPM) and the Office of Naval Research under grant N00014-03-1-0232.

S. A. Zummo is with the Electrical Engineering Department, KFUPM, Dhahran 31261, Saudi Arabia (e-mail: zummo@kfupm.edu.sa).

W. E. Stark is with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA (e-mail: stark@eecs.umich.edu).

Digital Object Identifier 10.1109/TWC.2006.05003.

$n_t$ transmit antennas in the $i^{th}$ time interval of the time slot. More examples of real and complex orthogonal matrices were presented in [7]. In this letter we limit our discussion to binary coded STBC systems such as BPSK. This is because the proposed union bound becomes very complicated and difficult to compute for complex signal constellations.

To be able to detect STBCs, the fading process from each transmit antenna should remain constant for at least one time slot, i.e., $n_t T$ seconds. Let $\mathcal{G}_{f,l}$ be the transmission matrix in the $l^{th}$ time slot of fading block $f$. The corresponding received vector is

$$\mathbf{y}_{f,l} = \sqrt{E_s}\mathcal{G}_{f,l}\mathbf{h}_f + \mathbf{z}_{f,l}, \tag{2}$$

where $\mathbf{z}_{f,l}$ is a length-$n_t$ column random vector with a distribution $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ and $\mathbf{I}$ denotes the $n_t \times n_t$ identity matrix. The vector $\mathbf{h}_f$ contains the channel gains from the transmit antennas in fading block $f$ and is modeled as $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. The decoder chooses the codeword $\mathbf{S} = \{s_{f,l}\}$ that maximizes the metric

$$\mathbf{m}(\mathbf{Y}, \mathbf{S}) = \sum_{f=1}^{F} \sum_{l=1}^{m/n_t} \mathrm{Re}\{\mathbf{y}^*_{f,l}\mathcal{G}_{f,l}\mathbf{h}_f\}, \tag{3}$$

where $(.)^*$ denotes the complex conjugate of a complex vector and $\mathbf{Y} = \{\mathbf{y}_{f,l}\}$. The union bound for block fading channels is discussed below.

## III. The Union Bound

Throughout the letter, the subscripts $c$, $u$ and $b$ are used to denote conditional, unconditional and bit error probabilities, respectively. The bit error probability for a convolutional code is upper bounded [12] by

$$P_b \leq \sum_{d=d_{\min}}^{N} w_d \; P_u(d), \tag{4}$$

where $d_{\min}$ is the minimum Hamming distance of the convolutional code and $w_d$ is the number of codewords with output weight $d$. Here, $P_u(d)$ is the unconditional PEP defined as the probability of decoding a received sequence as a weight-$d$ codeword given that the all-zero codeword is transmitted.

In block fading channels $P_u(d)$ is a function of the distribution of the $d$ nonzero error bits over the $F$ fading blocks. Denote the number of fading blocks with $v$ nonzero bits (weight $v$) by $f_v$ and define $w = \min(m, d)$. Due to the uniform interleaving of the coded bits prior to the transmission over the channel, the $d$ nonzero bits are distributed according to the pattern $\mathbf{f} = \{f_v\}_{v=0}^{w}$ that satisfies the constraints $F = \sum_{v=0}^{w} f_v and d = \sum_{v=1}^{w} v f_v$. Denote by $L = F - f_0$ the number of fading blocks with nonzero weights, then $P_u(d)$ is given by

$$P_u(d) = \sum_{L=\lceil d/m \rceil}^{d} \sum_{f_1=0}^{L_1} \sum_{f_2=0}^{L_2} \cdots \sum_{f_w=0}^{L_w} P_u(d|\mathbf{f}) p_d(\mathbf{f}), \tag{5}$$

where $P_u(d|\mathbf{f})$ is the unconditional PEP given a specific block fading pattern $\mathbf{f}$, which occurs with a probability given by

$$p_d(\mathbf{f}) = \frac{\binom{m}{1}^{f_1}\binom{m}{2}^{f_2}\cdots\binom{m}{w}^{f_w}}{\binom{mF}{d}} . \frac{F!}{f_0! f_1! \ldots f_w!}. \tag{6}$$

where

$$L_v = \min\left\{ L - \sum_{r=1}^{v-1} f_r, \frac{d - \sum_{r=1}^{v-1} r f_r}{v} \right\}, \qquad 1 \leq v \leq w. \tag{7}$$

The bit error probability of convolutional codes over a block fading channel is upper bounded by substituting (5)-(7) in (4). It should be noted that carefully designed interleavers may outperform the uniform interleaver. However, analyzing the performance of a specific interleaver is much more difficult. In addition, the number of summations involved in computing $P_u(d)$ in (5) increases as the channel memory length increases. Thus a good approximation to the union bound is obtained by truncating (4) to a distance $d_{\max} < N$. However, it should be noted that the low-weight terms in the union bound dominate the performance at high SNR values. Therefore, truncating the bound does not affect its accuracy especially at high SNR where simulation results are difficult to obtain.

## IV. Pairwise Error Probability (PEP)

The PEP, $P_u(d|\mathbf{f})$ is found by averaging the conditional PEP over the fading statistics, where the conditional PEP is given by

$$P_c(d|\mathbf{f}) = \Pr\left(\mathbf{m}(\mathbf{Y}, \mathbf{S},) - \mathbf{m}(\mathbf{Y}, \hat{\mathbf{S}}) < 0 \middle| \mathbf{H}, \mathbf{S}, \mathbf{f}\right), \tag{8}$$

where $\mathbf{H} = \{h_f\}_{f=1}^{F}$ and $\hat{\mathbf{S}}$ is a weight-$d$ codeword. Substituting the decoding metric (3) in (8) and averaging over the fading statistics yields $P_u(d|\mathbf{f})$. This is discussed in the following for the cases of perfect and imperfect CSI.

### A. Perfect CSI

It was shown in [6] that a STBC with $n_t$ transmit antennas and perfect CSI is equivalent to $n_t$-order maximal-ratio combining (MRC). Thus, the conditional PEP for a coded STBC system with perfect CSI is given by

$$P_c(d|\mathbf{f}) = \mathrm{Q}\left(\sqrt{2R_c\gamma_b \sum_{v=1}^{w} v \sum_{l=1}^{f_v} \sum_{i=1}^{n_t} |h_l^i|^2}\right). \tag{9}$$

Using the integral form of the $Q$ function, $\mathrm{Q}(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{(-x^2/2\sin^2\theta)} d\theta$ [13], $P_u(d|\mathbf{f})$ becomes

$$\begin{aligned} P_u(d|\mathbf{f}) &= \frac{1}{\pi}\mathrm{E}_{\mathbf{H}}\left[\int_0^{\frac{\pi}{2}} \exp\left(\frac{R_c\gamma_b}{\sin^2\theta} \sum_{v=1}^{w} v \sum_{l=1}^{f_v} \sum_{i=1}^{n_t} |h_l^i|^2\right) d\theta\right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left(\frac{1}{1 + vR_c\gamma_b/\sin^2\theta}\right)^{n_t f_v} d\theta. \end{aligned} \tag{10}$$

### B. Imperfect CSI

Imperfect CSI is obtained by transmitting $n_t$ known pilot sequences $\{\mathbf{p}^i\}_{i=1}^{n_t}$, each of length $n_p$, over the transmit antennas in each fading block [10], [14]. Denote by $\mathbf{y}_f^p$ the received column vector corresponding to the pilot sequence in fading block $f$. It is given by

$$\mathbf{y}_f^p = \sqrt{E_p} \sum_{i=1}^{n_t} h_f^i \mathbf{p}^i + \mathbf{z}_f, \qquad 1 \leq f \leq F, \tag{11}$$

where $E_p$ is the pilot signal energy. If pilot sequences from different transmit antennas are orthogonal, i.e., $\mathbf{p}^{i*}.\mathbf{p}^j = 0$ when $i \neq j$, then the maximum likelihood (ML) estimator of $h_f^i$ is given by

$$\hat{h}_f^i = \frac{\mathbf{y}_f^p.\mathbf{p}^{i*}}{\| \mathbf{p}^i \|^2} - \frac{\mathbf{z}_f.\mathbf{p}^{i*}}{\| \mathbf{p}^i \|^2} = h_f^i + e_f^i, \tag{12}$$

where $e_f^i = \left(\mathbf{z}_f.\mathbf{p}^{i*}/ \| \mathbf{p}^i \|^2\right)$ is the estimation error with a $\mathcal{CN}(0, \sigma_e^2)$ distribution where $\sigma_e^2 = \frac{N_0}{n_p E_p}$. In order implement a ML sequence decoding rule, the likelihood function $p(\mathbf{Y}, \hat{\mathbf{H}}|\mathbf{S})$ should be maximized, which was shown in [15] to be difficult to implement in a Viterbi-like receiver. Therefore, a suboptimal receiver that maximizes the likelihood function $p(\mathbf{Y}|\hat{\mathbf{H}}, \mathbf{S})$ is used. This suboptimal receiver chooses the codeword $\mathbf{S}$ that maximizes the metric in (3) with $\mathbf{h}_f$ being replaced by $\hat{\mathbf{h}}_f$. The received signal vector $\mathbf{y}_{f,l}$ conditioned on the estimated channel gains is a complex Gaussian random vector with a mean $\frac{\mu}{\sigma}\sqrt{E_s}\mathcal{G}_{f,l}\hat{\mathbf{h}}_f$ and a covariance matrix $(N_0 + n_t E_s(1 - \mu^2))\mathbf{I}$, where $\mu = \frac{1}{\sqrt{1+\sigma_e^2}}$. Thus the PEP conditioned on the estimated fading gains is given by

$$P_c(d|\mathbf{f}) = \Pr\left(\sum_{f=1}^{L}\sum_{l=1}^{w} \kappa_{f,l} < 0 \Big| \hat{\mathbf{H}}, \mathbf{S}\right), \tag{13}$$

where $\kappa_{f,l}$ is a Gaussian random variable with mean and variance given respectively by

$$\begin{aligned}\mathrm{E}\left[\kappa_{f,l}|\mathcal{G}_{f,l}, \hat{\mathbf{h}}_f\right] &= \frac{\mu}{\sigma}\sqrt{E_s}\mathrm{Re}\left\{\hat{\mathbf{h}}_{f,l}^* \mathcal{E}_{f,l}^T \mathcal{E}_{f,l}\hat{\mathbf{h}}_f\right\} \\ &= \frac{\mu}{\sigma}\sqrt{E_s} d_{f,l}\sum_{i=1}^{n_t}|\hat{h}_f^i|^2, \end{aligned} \tag{14}$$

$$\mathrm{Var}\left[\kappa_{f,l}|\mathcal{G}_{f,l}, \hat{\mathbf{h}}_f\right] = \left(N_0 + n_t E_s(1 - \mu^2)\right) d_{f,l}\sum_{i=1}^{n_t}|\hat{h}_f^i|^2, \tag{15}$$

where $\sigma^2 = \mathrm{Var}(\hat{h}_f^i) = 1 + \sigma_e^2$. It can be shown [15] that the PEP conditioned on the estimated fading gains is given by (9), with $\hat{\gamma}_b = \frac{\mu^2\gamma_b}{1+n_t R_c\gamma_b(1-\mu^2)}$ replacing $\gamma_b$. Here, $\hat{\gamma}_b$ represents the effective SNR taking into account the additional noise in the channel estimation. Thus the unconditional error probability is found by averaging (13) over the estimated fading gains, resulting in (10) with $\gamma_b$ being replaced by $\hat{\gamma}_b$.

Two scenarios are considered for channel estimation using pilot signals with $E_p = E_s$. The first one results from only pilot estimation (OPE) with an estimation error variance of $\sigma_e^2 = \frac{N_0}{E_s}$. The second case considers a lower bound on the performance of receivers employing iterative joint decoding and channel estimation [16], [17], [18]. In this case the correctly decoded bits can be considered as pilot signals resulting in $\sigma_e^2 = \frac{N_0}{mE_s}$. This case is referred to as correct data estimation (CDE).

In systems using pilot-aided channel estimation, $n_t$ coded bits are punctured every $m$ coded bits and replaced by a pilot sequence of length $n_t$. This reduces the error correcting capability of the code as the channel memory length becomes shorter which degrades the performance. This is in addition to energy reduction due to pilot signal insertion. The code rate of the punctured codes is given by $\tilde{R}_c = \frac{mR_c}{m-n_p}$. If no puncturing was employed the system will be affected by the energy loss only with the same error correcting capabilities of the code.

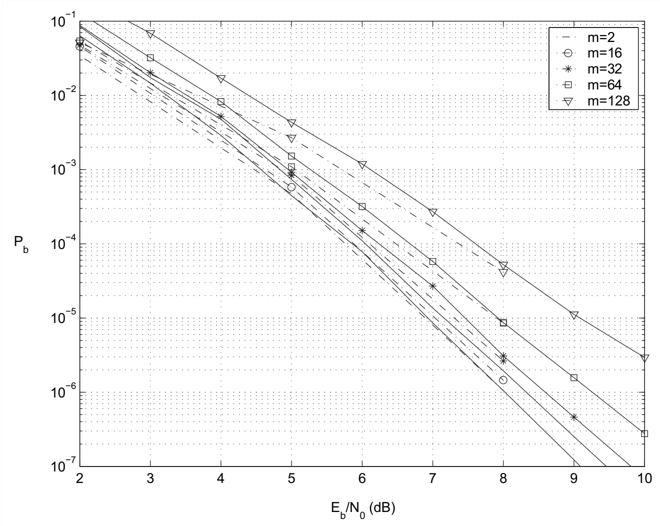


Fig. 1. Bit error probability of a convolutionally coded STBC using $n_t = 2$ with perfect CSI for channel memory lengths $m = 2, 16, 32, 64, 128$. (dashed: simulations, solid: analysis).

## C. Correlated Transmit Antennas

When the fading channels from different transmit antennas are correlated, $\mathbf{h}_f$ is a correlated complex Gaussian random vector with a covariance matrix $K_{\mathbf{h}}$, whose $(i,j)^{th}$ element is given by $K_{\mathbf{h}}(i,j) = \rho_{ij}, i \neq j$, where $\rho_{ij}$ is the correlation coefficient between channels from the $i^{th}$ and $j^{th}$ transmit antennas. When perfect CSI is available at the receiver, the conditional PEP is given by (9), which is a function of $\sum_{i=1}^{n_t}|h_f^i|^2 = \mathbf{h}_f^*\mathbf{h}_f$. Averaging the conditional PEP over $\mathbf{h}_f$ is difficult due to the complicated form of the distribution of $\mathbf{h}_f$. This problem is resolved by digonalizing $\mathbf{h}_f$ [19] resulting in an uncorrelated complex Gaussian random vector $\mathbf{g}_f$. The conditional PEP becomes

$$P_c(d|\mathbf{f}) = \mathrm{Q}\left(\sqrt{2R_c\gamma_b\sum_{v=1}^{w} v\sum_{l=1}^{f_v}\sum_{i=1}^{n_t}\lambda_i|g_f^i|^2}\right), \tag{16}$$

where $\{\lambda_i\}_{i=1}^{n_t}$ are the eigenvalues of $K_{\mathbf{h}}$. By averaging (16) over $\mathbf{g}_f$ we obtain

$$P_u(d|\mathbf{f}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{v=1}^{w}\prod_{i=1}^{n_t}\left(\frac{1}{1 + v\lambda_i R_c\gamma_b/\sin^2\theta}\right)^{f_v} d\theta. \tag{17}$$

If the receiver has imperfect CSI and does not know the channel covariance matrix [20], it estimates the channel assuming uncorrelated transmit antennas as in Section IV-B. In this case, the conditional PEP is given by (9) with $\mathbf{h}_f$ replaced by $\mathbf{q}_f = \frac{1}{\sigma}\hat{\mathbf{h}}_f$ whose covariance matrix is given by

$$K_{\mathbf{q}} = \frac{1}{1+\sigma_e^2}K_{\mathbf{h}} + \frac{\sigma_e^2}{1+\sigma_e^2}\mathbf{I}, \tag{18}$$

Let $\{\hat{\lambda}_i\}$ be the eigenvalues of $K_{\mathbf{q}}$. Then, $K_{\mathbf{q}}$ can be diagonalized as in [19]. In this case, the resulting PEP is given by (17) with $\{\lambda_i\}$ being replaced by $\{\hat{\lambda}_i\}$.

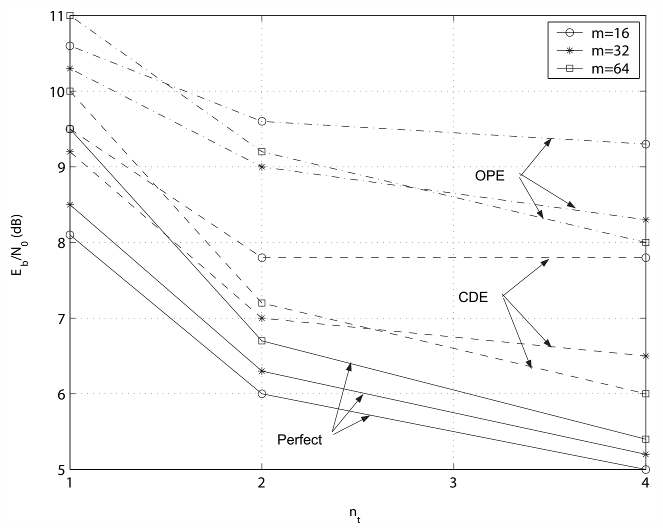


Fig. 2. Analytical SNR required for a convolutionally coded STBC to achieve $P_b = 10^{-4}$ versus $n_t$ for channel memory lengths $m = 16, 32, 64$ and different CSI assumptions.

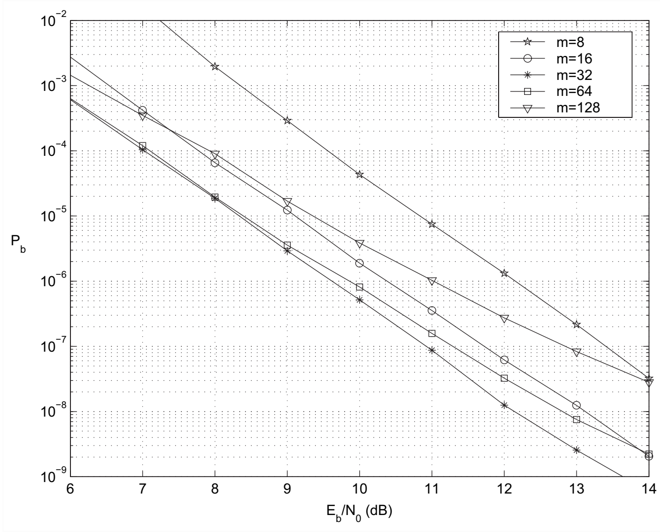


Fig. 3. Analytical approximation of bit error probability of a convolutionally coded STBC using $n_t = 2$ with imperfect CSI (CDE assumption with $E_p = E_s$) for channel memory lengths $m = 8, 16, 32, 64, 128$.

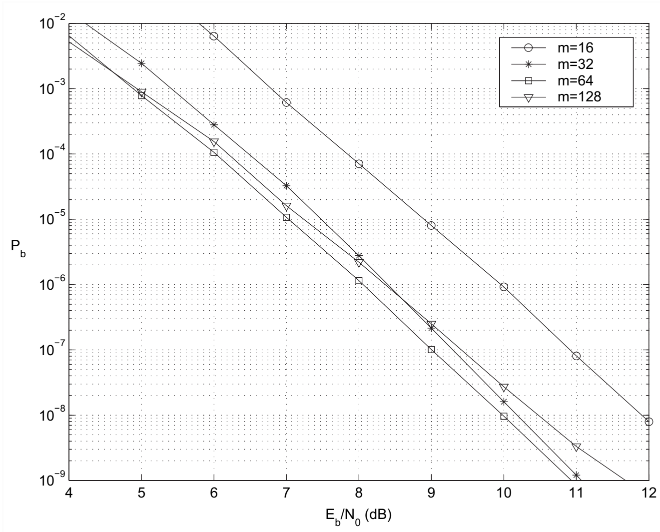


Fig. 4. Analytical approximation of bit error probability of a convolutionally coded STBC using $n_t = 4$ with imperfect CSI (CDE assumption with $E_p = E_s$) for channel memory lengths $m = 16, 32, 64, 128$.

## V. Numerical Results

To illustrate the results, a rate-$\frac{1}{2}$ (23,35) convolutional code with 4 memory elements is concatenated with a STBC employing two and four transmit antennas with a frame size of $N = 1024$ coded bits. The union bound was truncated to $d_{\max} \leq 12$. The bit error probability for the case of two transmit antennas is shown in Figure 1, where it is clear that the proposed approximation is tight to the simulation results.

Figure 2 shows the SNR required for different memory lengths to achieve $P_b = 10^{-4}$ is shown versus the number of transmit antennas. We observe that as space diversity increases, the SNR loss due to long channel memory reduces. This is expected since increasing the space diversity reduces the sensitivity of the performance to the diversity provided by the independent fading blocks. In single-antenna systems with CDE and OPE the memory length $m = 32$ performs the best, whereas $m = 32$ and $m = 64$ provide the best performance for the cases of $n_t = 2$ and $n_t = 4$, respectively. Moreover, as the channel memory length increases the gain of the CDE assumption over the OPE receiver increases because channel estimation improves under the CDE assumption as the memory length increases. Note that $E_p = E_s$ in the CDE and OPE systems. It was shown in [15] that the optimal pilot signal energy increases with increasing the channel memory equally well for different number of transmit antennas.

The results for two transmit antennas with imperfect CSI using the CDE assumption are shown in Figure 3. We observe that memory length $m = 8$ performs the worst among the other cases because the resulting code is weak due to puncturing two coded bits every 8 coded bits. Also, the case of $m = 64$ outperforms all other cases in the low SNR region, whereas the case of $m = 32$ starts to improve and becomes the best after an SNR value of 7 dB. We observe that the case of $m = 16$ outperforms the $m = 128$ case at SNR values larger than 7 dB, where the later starts to degrade due to the lack of diversity. In Figure 4 the CDE results for the case of four transmit antennas are shown. Although STBC with $n_t = 4$ and complex signals results in reduced transmission rate [7], we present this case with real signals to illustrate how the tradeoff between channel diversity and estimation is affected by the number transmit antennas. We observe that the optimal channel memory is longer and lies between $m = 64$ and $m = 128$, with a cross over at around 7 dB.

We conclude that the optimal memory increases as the number of transmit antennas increases for the following reasons. First, a larger number of transmit antennas increases the number of channels to be estimated, which requires longer observation period. Second, more space diversity reduces the effect of diversity provided by the independent fading blocks in the channel making channel estimation more crucial. Finally, the length of the pilot sequences increases as the number of transmit antennas increases, which reduces the energy efficiency and the error correcting capability of the system. If no puncturing was used for pilot signal insertion, the performance of short memory lengths would improve. In this case the optimal channel memory length is expected to be shorter than the puncturing case. This case was considered in [15] for stronger codes; namely turbo codes. It was shown that the optimal memory length becomes shorter than the case of a weak code. This is because the stronger code is less sensitive to channel estimation errors, which favors its role over that of channel estimation.

In Figure 5, we show a comparison of systems with channel memory lengths $m = 16$ and $m = 64$ for the case of $n_t = 2$. We observe that the performance degradation due to channel estimation reduces as the channel memory length increases. Furthermore, this degradation increases with the number of transmit antennas emphasizing channel estimation for larger number of transmit antennas. Note that an iterative joint decoding and channel estimation receiver has the potential to reduce the loss caused by OPE resulting in a performance close to that of the system under the CDE assumption.

Figure 6 shows the effect of antenna correlation for the case of two transmit antennas. By comparing with Figure 3, we observe that antenna correlation degrades the performance of systems with long channel memory more than it does for systems with short channel memory. This is because long channel memory reduces the channel diversity provided by the independent fading blocks causing space diversity to become more crucial. Moreover, long channel memory results in a better channel estimation, the task that becomes easier due to antenna correlation.

It is worth noting that optimizing the channel memory is often under the control of the system designer. As mentioned in the introduction, the block fading model is used frequently to model communication systems such as FH-SS and TDMA systems. In both cases, the channel memory is under the control of the system designer. More specifically, the designer can optimize the memory length by finding the most appropriate number of hops that the transmitter should hop within a codeword in FH-SS systems, and number of time slots in a TDMA frame in TDMA systems.

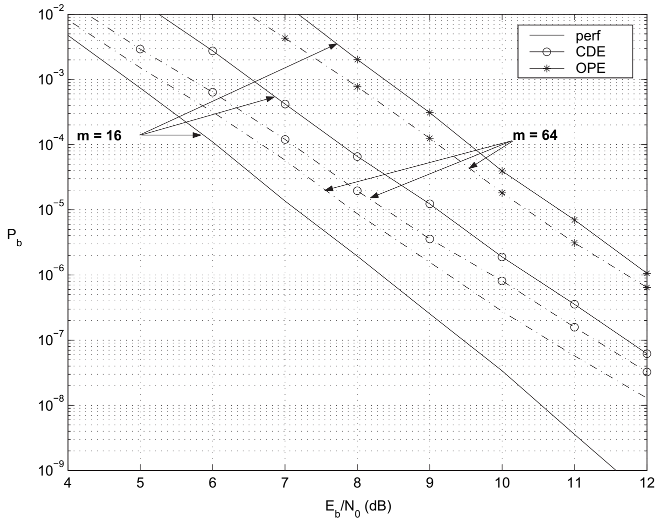


Fig. 5. Analytical approximation of bit error probability of a convolutionally coded STBC systems using $n_t = 2$ for channel memory lengths $m = 16, 64$ with perfect and imperfect CSI with $E_p = E_s$ (solid: $m = 16$, dash: $m = 64$).

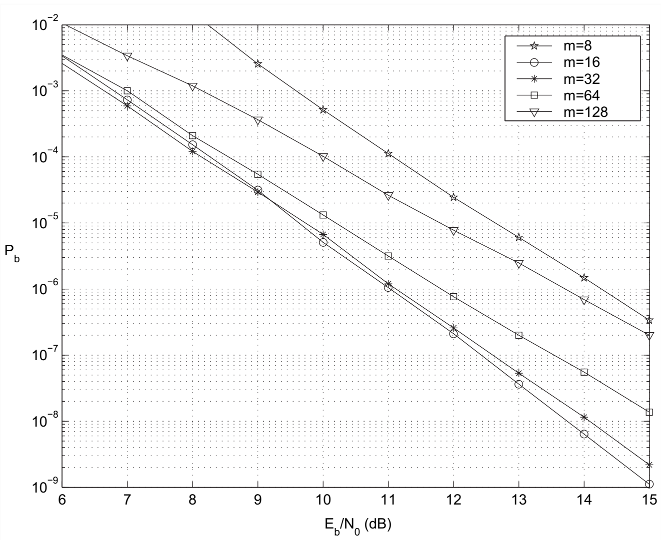


Fig. 6. Analytical approximation of bit error probability of a convolutionally coded STBC using $n_t = 2$ with antenna correlation coefficient of $\rho = 0.9$ and imperfect CSI (CDE assumption with $E_p = E_s$) for channel memory lengths $m = 8, 16, 32, 64, 128$.

## VI. CONCLUSIONS

In this letter, a union bound on the performance of binary coded systems employing STBCs over block fading channels was derived. Coherent receivers with perfect and imperfect CSI were considered. Results show that the performance degradation due to channel memory reduces as the number of transmit antennas is increased. The tradeoff between channel estimation and diversity was investigated and the optimal channel memory was approximated. Moreover, the effect of antenna correlation on the system performance and the optimal memory was studied. Results show that higher space diversity increases the optimal memory.

## REFERENCES

[1] A. Viterbi, "Convolutional Codes and Their Performance in Communication Systems," *IEEE Trans. Commun.*, vol. COM-19, pp. 751–772, October 1971.
[2] D. Divsalar and M. Simon, "Trellis Coded Modulation for 4800-9600 bits/s Transmission Over a Fading Satellite Channel," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 162–175, February 1987.
[3] R. J. McEliece and W. E. Stark, "Channels with Block Interference," *IEEE Trans. Inform. Theory*, vol. 30, pp. 44–53, January 1984.
[4] J. Guey, M. Fitz, M. Bell, and K. Wen-Yi, "Signal Design for Transmitter Diversity Wireless Communication Systems Over Rayleigh Fading Channels," *Wireless Personal Commun.*, vol. 47, pp. 527–537, April 1999.
[5] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, March 1998.
[6] S. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, October 1998.
[7] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.
[8] A. Worthen and W. Stark, *On the Channel Memory-Diversity Tradeoff in Communication Systems*, ser. in Information, Coding and Mathematics. Boston, MA, USA: Kluwer Academic Publisher, 2002.

[9] B. Hassibi and B. Hochwald, “How Much Training is Needed in Multiple-Antenna Wireless Links?” *IEEE Trans. Inform. Theory*, vol. 49, pp. 951–963, April 2003.

[10] V. Tarokh, A. Naguib, N. Seshadri, and A. Calderbank, “Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion in the Presence of Channel Estimation Errors, Mobility, and Multiple Paths,” *IEEE Trans. Commun.*, vol. 47, pp. 199–207, February 1999.

[11] J. G. Proakis, *Digital Communications*, 4th ed. New York, USA: McGraw-Hill, 2000.

[12] A. Viterbi and J. Omura, *Principles of Digital Communication and Coding*. New York, USA: McGraw-Hill, 1979.

[13] M. Simon and D. Divsalar, “Some New Twists to Problems Involving the Gaussian Probability Integral,” *IEEE Trans. Commun.*, vol. 46, pp. 200–210, February 1998.

[14] C. Budianu and L. Tong, “Channel Estimation for Space-Time Orthogonal Block Codes,” *IEEE International Conference on Communication, ICC*, pp. 1127–1131, 2001.

[15] S. A. Zummo, *Coding and Channel Estimation for Block Fading Channels*. Ann Arbor, USA: Technical Report, Communications and Signal Processing Laboratory (CSPL), University of Michigan, June 2003.

[16] S. Zummo and W. Stark, “An Algorithm for Iterative Decoding and Channel Estimation of Space-Time Coded FH Systems,” *IEEE Military Communications Conference, MILCOM*, pp. 1086–1089, October 2002.

[17] A. Tonello, “Performance of Space-Time Bit-Interleaved Codes in Fading Channels with Simplified Iterative Decoding,” *IEEE International Conference on Communication, ICC*, pp. 1357–1361, 2001.

[18] C. Xu and T. Le-Ngoc, “Channel Estimation and Decoding of Space-Time Block Codes,” *IEEE Vehicular Technology Conference, VTC/Fall*, pp. 1540–1544, 2002.

[19] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New York, USA: McGraw-Hill, 1965.

[20] P. Shamain and L. Milstein, “Detection With Spatial Diversity Using Noisy Channel Estimates in A Correlated Fading Channel,” *IEEE Military Communications Conference, MILCOM*, pp. 691–696, 2002.