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Spectrum-Sharing AF Relay Networks with Switch-and-Examine Relaying and Multiple Primary Users Using Orthogonal Spectrums

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Abstract In this paper, we propose and evaluate the behavior of a new cognitive amplify-and-forward relaying scenario where the multiple primary users utilize orthogonal spectrum bands. Using orthogonal bands aims to reduce the interference between users as in the downlink transmission in cellular networks where a base station transmits the data of different users using orthogonal frequency bands. In the proposed scenario, the spectrum of the primary user whose channel enhances the secondary system performance is shared with the secondary users. In this paper, the low-complexity switch-and-examine diversity combining relaying scheme is used to select among the secondary relays. In this scheme, the relay whose end-to-end signal-to-noise ratio (SNR) satisfies a predetermined switching threshold is selected instead of the best relay to forward the source message to destination. Approximate expressions are derived for the outage probability and average symbol error probability of the studied system assuming Rayleigh fading channels. Also, the ergodic channel capacity is numerically calculated in this paper. Furthermore, to simplify the achieved expressions and to get more insights about the system behavior, the system is studied at the high SNR values where approximate expression is derived for the outage probability in addition to the derivation of the diversity order and coding gain of the system. The achieved results are validated by Monte Carlo simulations. Main findings illustrate that the diversity order of the studied system is the same as its non-cognitive counterpart and it is independent of the primary network. In contrast to the existing systems where the same spectrum band is utilized by different primary users, increasing the number of primary users in the proposed scenario enhances the overall behavior via improving the coding gain.

Keywords Cognitive relay network · Orthogonal spectrum bands · Performance analysis · Rayleigh fading · Switching threshold

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1 Introduction

Spectrum-sharing techniques have been proposed to improve the spectrum resource utilization efficiency in wireless networks [1]. Various cognitive radio paradigms have been proposed in the literature, among which is the underlay paradigm. This scenario allows secondary or cognitive users to share the frequency bands of primary users only if the interference between them is below a certain threshold. Besides the cognitive networks, the relay networks have been presented to solve the problem of multipath fading in wireless systems [2]. Currently, the area of cognitive relay networks (CRNs) is under study by a large number of researchers.

In the area of decode-and-forward (DF) CRNs, Hong et al. studied in [3] the exact outage performance of opportunistic DF CRNs assuming Rayleigh fading channels. In [4], three relay selection scenarios were proposed, namely the relay with the best second hop, the relay with worst second hop, and the relay which satisfies the minimum level of interference with the primary user. Most recently, the outage performance of full-duplex opportunistic DF CRN was studied in [5] assuming Nakagami-m fading channels. A lot of papers can be found in the literature on the performance of amplify-and-forward (AF) CRNs. The outage and error rate performances of an underlay fixed-gain AF CRN with reactive relay selec-

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tion were evaluated in [6]. Among the relays which satisfy the interference constraint, the relay with the best end-to-end (e2e) channel is selected to forward the source message to destination. In [7], the error rate performance of an AF CRN was studied using partial-relay selection scheme where the relay which has the strongest first hop channel is selected as the best relay. Recently, in addition to the ergodic channel capacity, Bao et al. evaluated in [8] the outage and error rate performances of AF CRNs assuming Rayleigh fading channels. The outage performance of opportunistic AF and DF CRNs with multiple secondary destinations and in the presence of direct link was studied in [9]. Most recently, the performance of selection DF and AF CRNs with collaborative distributive beamforming was studied in [10]. In the area of CRNs with single secondary relay and multiple primary users, Tran et al. derived in [11] the outage probability and the channel capacity assuming Nakagami-m fading environment. The outage probability performance of CRNs with multiple secondary relays and multiple primary users was studied in [12–14].

From our reading to the papers on CRNs with multiple primary users, we noticed that the only studied scenario in such systems is the scenario where the primary users are assigned the same frequency band. Another important scenario which could be widely seen in practice is the one where the primary receivers utilize orthogonal spectrum bands as in frequency division multiple access (FDMA)-based wireless networks. This scenario has been most recently proposed by us in [15] for cognitive DF relay networks with interference from primary user. Using orthogonal spectrum bands in wireless networks aims to reduce the interference between users. The proposed scenario could be seen in long-term evolution (LTE) networks where the orthogonal FDMA (OFDMA) technique is used in the downlink transmission, and different sub-channels and bands are assigned for different users. Another application could be in IEEE 802.22 wireless regional area networks (WRANs) where the OFDMA is a candidate access method for these networks. To get rid of the heavy load of channel estimation which is required in the opportunistic and partial-relay selection schemes, the low-complexity switch-and-examine diversity combining (SEC) relaying scheme is used in this paper. In this scheme, once a checked relay satisfies a certain threshold, it is used to help the source node in sending its massage to destination. Once a relay is selected, no further channel estimations need to be conducted by the relay nodes. This reduces the channel estimation load, saves the relays power, and reduces the complexity of the system [16]. The SEC selection criterion was used in several applications before. It was firstly used to select among antennas in space diversity systems in [17]. Also, it was presented in [18] as a scheduling or user selection scheme in cellular wireless systems.

According to authors knowledge, the scenario of cognitive AF relay networks with the SEC-based relaying scheme and multiple primary users using orthogonal spectrum bands has not been presented and studied yet. In this paper: (i) We propose and study the behavior of a new cognitive AF SEC relaying scenario where the primary users utilize orthogonal spectrum bands. In the existing works where the primary users are assumed to utilize the same frequency band, the primary user which has the strongest channel with the secondary users is used in determining the interference constraint. In the proposed scenario, the spectrum of the primary user whose channel results in the best performance for the secondary system is shared with the secondary users; (ii) unlike the partial-relay selection scheme and opportunistic relaying, in the SEC relaying, the first checked relay whose composite signal-to-noise ratio (SNR) exceeds a predetermined threshold is asked to cooperate with the source node. In this scheme, the e2e SNR of a relay is first checked against a switching threshold. If it is larger, this relay is asked to cooperate with the source node, if not, other relay is examined. This process continues till a suitable relay is found or the last relay is reached. In case the last relay is reached, the selection scheme sticks to it for simplicity; and (iii) we provide a full evaluation of the system performance through the derivation of approximate expressions for the outage probability and average symbol error probability (ASEP) for the independent non-identically distributed (i.n.i.d.) generic case of relay channels and a numerical calculation of the channel capacity for the independent identically distributed (i.i.d.) case of relay channels. The switching threshold is numerically calculated to optimize the e2e outage probability. Also, a simple method for calculating approximate values for the optimum threshold is provided in this paper.

2 System and Channel Models

The considered system consists of one secondary source S, K AF secondary relays R_k ($k = 1, \ldots, K$), one secondary destination D, and M primary receivers P_m ($m = 1, \ldots, M$) using orthogonal frequency bands, as shown in Fig. 1. Each node is assumed to possess single antenna, and the communication is assumed to work in a half-duplex mode. Furthermore, the direct link is assumed to be in a deep fade, and hence, it is neglected in through the analysis. The secondary users need to share the spectrum with the primary receiver whose channel results in a best performance for the secondary system[1]. The communications take place in two phases. In the first phase, the secondary

[1] Achieving the best performance of the secondary system in the sense of selecting the primary receiver allows the secondary users to transmit at their maximum power.

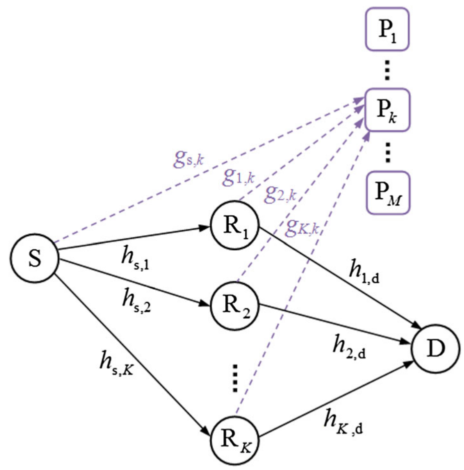

Fig. 1 Spectrum-sharing dual-hop AF network with SEC relaying and multiple primary users

source sends its message x to relays under a power constraint which ensures that the interference with the selected primary receiver $\mathsf{P}_{\mathsf{Sel}}$ does not exceed an interference temperature of $\mathcal{I}_{\mathsf{p}}$. To satisfy the primary interference constraint and result in a best performance for the secondary system, the secondary source S must transmit at a power given by $P_{\mathsf{S}} = \mathcal{I}_{\mathsf{p}}/\min\limits_{m}|g_{\mathsf{s},m}|^2$, where $g_{\mathsf{s},m}$ is the channel coefficient of the $\mathsf{S} \to \mathsf{P}_m$ link. In the second phase, R_k amplifies the received message from S with a scaling factor G_k and forwards the amplified version to the secondary destination. In order to satisfy the primary interference constraint and result in a best performance for the secondary system, the transmit power at R_k must be $P_{\mathsf{R}_k} = \mathcal{I}_{\mathsf{p}}/\min\limits_{m}|g_{k,m}|^2$, where $g_{k,m}$ is the channel coefficient of the $\mathsf{R}_k \to \mathsf{P}_m$ link. Hence, the received message at D from the k^{th} relay R_k can be given by $y_{k,\mathsf{d}} = \sqrt{P_{\mathsf{S}}}G_k h_{k,\mathsf{d}} h_{\mathsf{s},k} x + G_k h_{k,\mathsf{d}} n_{\mathsf{s},k} + n_{k,\mathsf{d}}$, where $h_{\mathsf{s},k}$ and $h_{k,\mathsf{d}}$ are the channel coefficients of the $\mathsf{S} \to \mathsf{R}_k$ and $\mathsf{R}_k \to \mathsf{D}$ links, respectively, $n_{\mathsf{s},k}$ and $n_{k,\mathsf{d}}$ represent the additive white Gaussian noise (AWGN) terms at R_k and D, respectively, with a power of N_0. It is assumed in this study that the interference from the primary user is neglected[2]. As we are using a channel-state-information (CSI)-assisted AF relaying, the scaling factor G_k can be written as $G_k^2 = 1\Big/\min\limits_{m}|g_{k,m}|^2\left(\frac{|h_{\mathsf{s},k}|^2}{\min\limits_{m}|g_{\mathsf{s},m}|^2} + \frac{N_0}{\mathcal{I}_{\mathsf{p}}}\right)$. Thus, the instantaneous SNR of the $\mathsf{S} \to \mathsf{R}_k \to \mathsf{D}$ path can be expressed as

$$\gamma_k = \frac{\frac{\bar{\gamma}|h_{\mathsf{s},k}|^2}{\min\limits_{m}|g_{\mathsf{s},m}|^2}\frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{\min\limits_{m}|g_{k,m}|^2}}{\frac{\bar{\gamma}|h_{\mathsf{s},k}|^2}{\min\limits_{m}|g_{\mathsf{s},m}|^2} + \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{\min\limits_{m}|g_{k,m}|^2} + 1} = \frac{\frac{X_{k1}}{Y}X_{k2}}{\frac{X_{k1}}{Y} + X_{k2} + 1}, \tag{1}$$

where $X_{k1} = \bar{\gamma}|h_{\mathsf{s},k}|^2$, $Y = \min\limits_{m}|g_{\mathsf{s},m}|^2$, $X_{k2} = \frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{\min\limits_{m}|g_{k,m}|^2}$, and $\bar{\gamma} = \mathcal{I}_{\mathsf{p}}/N_0$. With the SEC relaying, the first examined relay whose γ_k satisfies a predetermined switching threshold is asked to help the source node in sending its message to destination. To simplify the mathematical manipulations, an upper bound on γ_k in (1) is introduced as follows [8]

$$\gamma_k \leq \gamma_k^{\mathsf{up}} = \min\left(\underbrace{\frac{X_{k1}}{\min\limits_{m}|g_{\mathsf{s},m}|^2}}_{\gamma_{k1}}, \underbrace{\frac{\bar{\gamma}|h_{k,\mathsf{d}}|^2}{\min\limits_{m}|g_{k,m}|^2}}_{\gamma_{k2}}\right). \tag{2}$$

By assuming the links follow slow fading Rayleigh model, the channel gains $|h_{\mathsf{s},k}|^2$, $|g_{\mathsf{s},m}|^2$, $|h_{k,\mathsf{d}}|^2$, and $|g_{k,m}|^2$ will be exponentially distributed random variables with mean powers $\Omega_{\mathsf{s},k}$, $\mu_{\mathsf{s},m}$, $\Omega_{k,\mathsf{d}}$, and $\mu_{k,m}$, respectively. Assuming perfect channel information[3], the SEC relaying scheme works as follows: (1) At the beginning of each transmission time, the SNR γ_k of the current active relay is checked against the switching threshold. If it is still above the threshold, this relay sends an acknowledgment to the destination and keeps forwarding the source massage to it. (2) If the SNR of the checked relay is below the switching threshold, the relay sends a negative acknowledgment to the destination, which in turn switches to other relay. (3) Through the channel estimation techniques as the ones in [20], the second relay estimates its channels and goes to step 1. The SEC scheme keeps examining the relays till a suitable relay is found. If all relays are examined and found unacceptable, the scheme sticks to the last one for simplicity.

3 Performance Analysis

In this section, we evaluate approximate expressions for the outage probability and ASEP of the studied system. Also, the ergodic channel capacity is numerically calculated in this section.

3.1 Outage Probability

In this section, we derive the outage probability of the studied system for i.n.i.d. and i.i.d. cases of relay channels. The outage probability is defined as the probability that the SNR at D goes below a predetermined outage threshold γ_{out}, i.e., $P_{\mathsf{out}} = \Pr\left[\gamma_{\mathsf{D}} \leq \gamma_{\mathsf{out}}\right]$, where $\Pr[.]$ denotes the probability operation. The results of the outage probability are summarized in the following theorem and corollary.

[2] This assumption is feasible when the primary transmitter is far from the secondary receiver or the interference is represented in terms of noise [19].

[3] Secondary users can know the channel information of the primary user by exchange of channel information through a band manager [20].

Theorem 1 *The outage probability of cognitive CSI-assisted AF SEC relaying network with primary users using orthogonal spectrum bands and Rayleigh fading is given for i.n.i.d. relay hops as*

$$
\begin{aligned}
P_{\text{out}} \simeq & \sum_{i=0}^{K-1} \pi_i \left(\sum_{m=1}^{M} \zeta_{\mathrm{s},m}\right) \Bigg\{ \frac{\left(1-\exp\left(-\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\gamma_{\mathrm{T}}\right)\right)}{\sum_{m=1}^{M}\zeta_{\mathrm{s},m}} \\
& + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\cdots<n_k \\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1+\lambda_{n_t 2}\gamma_{\mathrm{T}}\right)^{-1}}{\Delta_1} \\
& - \left(1+\lambda_{i2}\gamma_{\text{out}}\right)^{-1} \\
& \times \Bigg[\frac{\left(1-\exp\left(-\left(\lambda_{i1}\gamma_{\text{out}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)\gamma_{\mathrm{T}}\right)\right)}{\lambda_{i1}\gamma_{\text{out}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}} \\
& + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\cdots<n_k \\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1-\exp\left(-\Delta_1\gamma_{\mathrm{T}}\right)\right)}{\left(1+\lambda_{n_t 2}\gamma_{\mathrm{T}}\right)\Delta_1} \Bigg] \Bigg\} \\
& + \sum_{l=0}^{K-1} \pi_l \left(\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right) \\
& \times \Bigg(\sum_{q=0}^{K} \frac{(-1)^q}{q!} \sum_{m_1,\ldots,m_q}^{K} \prod_{z=1}^{q} \frac{\left(1-\exp\left(-\Delta_2\gamma_{\mathrm{T}}\right)\right)}{\left(1+\lambda_{m_z 2}\gamma_{\mathrm{T}}\right)\Delta_2} \\
& + \sum_{w=0}^{K-1} \pi_{((l-w))_K} \Bigg[\left(1+\lambda_{l2}\gamma_{\mathrm{T}}\right)^{-1} \\
& \times \Bigg\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\mathrm{T}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)\gamma_{\mathrm{T}}\right)}{\lambda_{l1}\gamma_{\mathrm{T}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}} \\
& + \sum_{p=0}^{w-1}(-1)^{p+1} \sum_{v_0<\cdots<v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\Delta_3\gamma_{\mathrm{T}}\right)}{\Delta_4\Delta_3} \Bigg\} \\
& - \left(1+\lambda_{l2}\gamma_{\text{out}}\right)^{-1} \\
& \times \Bigg\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\text{out}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)\gamma_{\mathrm{T}}\right)}{\lambda_{l1}\gamma_{\text{out}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}} \\
& + \sum_{p=0}^{w-1}(-1)^{p+1} \sum_{v_0<\cdots<v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\Delta_5\gamma_{\mathrm{T}}\right)}{\Delta_4\Delta_5} \Bigg\} \Bigg] \Bigg),
\end{aligned}
\tag{3}
$$

where $\Delta_1 = \sum_{s=0}^{k}\lambda_{n_s 1}\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$, $\Delta_2 = \sum_{r=1}^{q}\lambda_{m_r 1}\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$, $\Delta_3 = \sum_{u=0}^{p}\lambda_{((l-w+v_u))_K 2} + \lambda_{l1}\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$, $\Delta_4 = 1 + \lambda_{((l-w+v_g))_K 2}\gamma_{\mathrm{T}}$, *and* $\Delta_5 = \sum_{u=0}^{p}\lambda_{((l-w+v_u))_K 2} + \lambda_{l1}\gamma_{\text{out}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$.

Proof Please see the "Appendix." □

Corollary 1 *The outage probability of cognitive CSI-assisted AF SEC relaying network with primary users using orthogonal spectrum bands and Rayleigh fading is given for i.i.d. relay hops* $(\Omega_{\mathrm{s},1} = \cdots = \Omega_{\mathrm{s},K} = \Omega_{\mathrm{s,r}})$, $(\Omega_{1,\mathrm{d}} = \cdots = \Omega_{K,\mathrm{d}} = \Omega_{\mathrm{r,d}})$, $\{\mu_{k,1} = \cdots = \mu_{k,M} = \mu_{k,\mathrm{p}}\}_{k=1}^{K}$, *and* $\{\mu_{1,\mathrm{p}} = \cdots = \mu_{K,\mathrm{p}} = \mu_{\mathrm{r,p}}\}$ *as*

$$
\begin{aligned}
P_{\text{out}} \simeq & \Bigg\{ \sum_{g=0}^{K-1} \frac{\binom{K-1}{g}(-1)^g}{\left(1+\lambda_2\gamma_{\mathrm{T}}\right)^g} \Bigg[\frac{\left(1-\exp\left(-\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\right)}{\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)} \\
& - \frac{\left(1-\exp\left(-\left(\left(\gamma_{\text{out}}+g\gamma_{\mathrm{T}}\right)\lambda_1+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\right)}{\left(1+\lambda_2\gamma_{\text{out}}\right)\left(\left(\gamma_{\text{out}}+g\gamma_{\mathrm{T}}\right)\lambda_1+M\zeta_{\mathrm{s,p}}\right)} \Bigg] \\
& + \sum_{j=0}^{K-1}\sum_{w=0}^{j} \frac{\binom{j}{w}(-1)^w}{\left(1+\lambda_2\gamma_{\mathrm{T}}\right)^w} \Bigg[\frac{\exp\left(-\left((j+1)\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{\left(1+\lambda_2\gamma_{\mathrm{T}}\right)\left((j+1)\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)} \\
& - \frac{\exp\left(-\left(\left(\gamma_{\text{out}}+j\gamma_{\mathrm{T}}\right)\lambda_1+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{\left(1+\lambda_2\gamma_{\text{out}}\right)\left(\left(\gamma_{\text{out}}+j\gamma_{\mathrm{T}}\right)\lambda_1+M\zeta_{\mathrm{s,p}}\right)} \Bigg] \\
& + \sum_{q=0}^{K} \frac{\binom{K}{q}(-1)^q}{\left(1+\lambda_2\gamma_{\mathrm{T}}\right)^q} \frac{\exp\left(-\left(q\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{\left(q\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)} \Bigg\} \left(M\zeta_{\mathrm{s,p}}\right).
\end{aligned}
\tag{4}
$$

Proof In evaluating the outage probability in (4), the conditional CDF of γ_{D} is required to be obtained first. It is given for i.i.d. relay hops as [21]

$$
F_{\gamma_{\mathrm{D}}}(\gamma|Y) = \begin{cases} \left[F_{\gamma^{\mathrm{up}}}(\gamma_{\mathrm{T}}|Y)\right]^{K-1} F_{\gamma^{\mathrm{up}}}(\gamma|Y), & \gamma<\gamma_{\mathrm{T}}; \\ \sum_{j=0}^{K-1}\left[F_{\gamma^{\mathrm{up}}}(\gamma|Y)-F_{\gamma^{\mathrm{up}}}(\gamma_{\mathrm{T}}|Y)\right] & \\ \times\left[F_{\gamma^{\mathrm{up}}}(\gamma_{\mathrm{T}}|Y)\right]^j+\left[F_{\gamma^{\mathrm{up}}}(\gamma_{\mathrm{T}}|Y)\right]^K, & \gamma\geq\gamma_{\mathrm{T}}, \end{cases}
\tag{5}
$$

where $F_{\gamma^{\mathrm{up}}}(\gamma|Y)$ is the CDF of γ_k^{up} conditioned on Y and it can be expressed for i.i.d. first and second hop channels using (17) as

$$
F_{\gamma_k^{\mathrm{up}}}(\gamma|Y) = 1 - \frac{\exp\left(-\lambda_1 Y\gamma\right)}{\left(1+\lambda_2\gamma\right)},
\tag{6}
$$

where $\lambda_1 = 1/\left(\Omega_{\mathrm{s,r}}\bar{\gamma}\right)$ and $\lambda_2 = 1/\left(M\zeta_{\mathrm{r,p}}\Omega_{\mathrm{r,d}}\bar{\gamma}\right)$. Upon substituting (6) in (5) and with the same analysis method in the Appendix, we get (4). □

3.2 Average Symbol Error Probability

In this section, we derive the ASEP of the studied system for i.n.i.d. and i.i.d. cases of relay channels. The ASEP can be expressed in terms of the CDF of γ_{D}, $F_{\gamma_{\mathrm{D}}}(\gamma) = P_{\text{out}}(\gamma_{\text{out}} = \gamma)$ as [22]

$$\mathrm{ASEP} \simeq \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{\exp\left(-b\gamma\right)}{\sqrt{\gamma}} F_{\gamma_{\mathrm{D}}}(\gamma)\mathrm{d}\gamma, \tag{7}$$

where a and b are two constant parameters and their values depend on the modulation type. The results of the ASEP are summarized in the following theorem and corollary.

Theorem 2 *The ASEP of cognitive CSI-assisted AF SEC relaying network with primary users using orthogonal spectrum bands and Rayleigh fading is given for i.n.i.d. relay hops as*

$$\begin{aligned}
\mathrm{ASEP} \simeq\ & \frac{a\sqrt{b}}{2\sqrt{\pi}} \left(\sum_{m=1}^{M} \zeta_{\mathrm{s},m}\right) \Bigg\{ \sum_{i=0}^{K-1} \pi_i \Bigg(\frac{\sqrt{\pi}}{\sqrt{b}\sum_{m=1}^{M}\zeta_{\mathrm{s},m}} \\
& + \sum_{k=0}^{K-1}(-1)^{k+1} \sum_{\substack{n_0<\cdots<n_k \\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1+\lambda_{n_t 2}\gamma_{\mathrm{T}}\right)^{-1}}{\vartheta_1} \\
& - \Bigg[\frac{\Gamma(1/2)(\lambda_{i2})^{-1/2}}{\left(-\frac{\lambda_{i1}}{\lambda_{i2}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)} \Bigg\{ \exp\left(\frac{b}{\lambda_{i2}}\right) \Bigg(\Gamma\left(1/2, b/\lambda_{i2}\right) \\
& - \frac{\exp\left(-\left(\sum_{m=1}^{M}\zeta_{\mathrm{s},m}-\frac{\lambda_{i1}}{\lambda_{i2}}\right)\gamma_{\mathrm{T}}\right)}{\left(\Gamma\left(1/2, (\lambda_{i1}\gamma_{\mathrm{T}}+b)/\lambda_{i2}\right)\right)^{-1}} \Bigg) + \frac{\exp\left(\frac{b\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{i1}}\right)}{\left(\frac{\lambda_{i2}\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{i1}}\right)^{1/2}} \\
& \times \Bigg(\Gamma\Bigg(1/2, (\lambda_{i1}\gamma_{\mathrm{T}}+b)\sum_{m=1}^{M}\zeta_{\mathrm{s},m}/\lambda_{i1} \Bigg) \\
& - \Gamma\Bigg(1/2, b\sum_{m=1}^{M}\zeta_{\mathrm{s},m}/\lambda_{i1} \Bigg)\Bigg)\Bigg\} + \sum_{k=0}^{K-1}(-1)^{k+1} \\
& \times \sum_{\substack{n_0<\cdots<n_k \\ n(.)\neq i}}^{K-1} \prod_{t=0}^{k} \frac{\left(1-\exp\left(-\vartheta_1\gamma_{\mathrm{T}}\right)\right)}{\left(1+\lambda_{n_t 2}\gamma_{\mathrm{T}}\right)\vartheta_1} \Bigg]\Bigg) \\
& + \sum_{l=0}^{K-1}\pi_l \Bigg(\sum_{q=0}^{K} \frac{(-1)^q}{q!} \sum_{m_1,\ldots,m_q}^{K} \prod_{z=1}^{q} \frac{\exp\left(-\vartheta_2\gamma_{\mathrm{T}}\right)}{\left(1+\lambda_{m_z 2}\gamma_{\mathrm{T}}\right)\vartheta_2} \\
& + \sum_{w=0}^{K-1} \pi_{((l-w))_K} \\
& \times \Bigg[\left(1+\lambda_{l2}\gamma_{\mathrm{T}}\right)^{-1} \Bigg\{ \frac{\exp\left(-\left(\lambda_{l1}\gamma_{\mathrm{T}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)\right)}{\left(\lambda_{l1}\gamma_{\mathrm{T}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)} \\
& + \sum_{p=0}^{w-1}(-1)^{p+1} \sum_{v_0,\ldots,v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\vartheta_3\gamma_{\mathrm{T}}\right)}{\left(1+\lambda_{((l-w+v_g))_K 2}\gamma_{\mathrm{T}}\right)\vartheta_3} \Bigg\} \\
& - \Bigg\{ \frac{\Gamma(1/2)(\lambda_{l2})^{-1/2}}{\left(-\frac{\lambda_{l1}}{\lambda_{l2}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)} \exp\Bigg(-\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\gamma_{\mathrm{T}} \Bigg) \\
& \times \Bigg(\exp\left(\frac{\lambda_{l1}\gamma_{\mathrm{T}}+b}{\lambda_{l2}}\right) \Gamma\left(1/2, (\lambda_{l1}\gamma_{\mathrm{T}}+b)/\lambda_{l2}\right) \\
& - \left(\frac{\lambda_{l2}\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{l1}}\right)^{-1/2} \exp\left(\frac{(\lambda_{l1}\gamma_{\mathrm{T}}+b)\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{l1}}\right)\Gamma \\
& \times \Bigg(1/2, (\lambda_{l1}\gamma_{\mathrm{T}}+b)\sum_{m=1}^{M}\zeta_{\mathrm{s},m}/\lambda_{l1} \Bigg)\Bigg)\Bigg\} + \sum_{p=0}^{w-1}(-1)^{p+1} \\
& \times \prod_{g=0}^{p} \frac{\exp\left(-\left(\sum_{u=0}^{p}\lambda_{((l-w+v_u))_K 1}\gamma_{\mathrm{T}}+\sum_{m=1}^{M}\zeta_{\mathrm{s},m}\right)\gamma_{\mathrm{T}}\right)}{1+\lambda_{((l-w+v_g))_K 2}\gamma_{\mathrm{T}}} \\
& \times \Bigg\{ \Bigg(-\frac{\lambda_{l1}}{\lambda_{l2}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m} \Bigg)^{-1} \Gamma(1/2) \\
& \times (\lambda_{l2})^{-1/2} \Bigg(\frac{\exp\left(\frac{\lambda_{l1}\gamma_{\mathrm{T}}+b}{\lambda_{l2}}\right)}{\Gamma\left(1/2, (\lambda_{l1}\gamma_{\mathrm{T}}+b)/\lambda_{l2}\right)} \\
& - \left(\frac{\lambda_{l2}\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{l1}}\right)^{-1/2} \exp\left(\frac{(\lambda_{l1}\gamma_{\mathrm{T}}+b)\sum_{m=1}^{M}\zeta_{\mathrm{s},m}}{\lambda_{l1}}\right) \\
& \times \Gamma\Bigg(1/2, (\lambda_{l1}\gamma_{\mathrm{T}}+b)\sum_{m=1}^{M}\zeta_{\mathrm{s},m}/\lambda_{l1} \Bigg)\Bigg)\Bigg\}\Bigg]\Bigg)\Bigg\},
\end{aligned} \tag{8}$$

where $\Gamma(.,.)$ *is the incomplete Gamma function defined in* [23, Eq. (8.350.2)], $\vartheta_1 = \sum_{s=0}^{k}\lambda_{n_s 1}\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$, $\vartheta_2 = \sum_{r=0}^{q}\lambda_{m_r 1}\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$, *and* $\vartheta_3 = \left(\sum_{u=0}^{p}\lambda_{((l-w+v_u))_K 1} + \lambda_{l1}\right)\gamma_{\mathrm{T}} + \sum_{m=1}^{M}\zeta_{\mathrm{s},m}$.

By replacing γ_{out} with γ in (3) and using the partial fraction expansion and the integration in (7) and using [23, Eq. (3.361.2)] and [23, Eq. (3.383.10)], we get (8).

Corollary 2 *The ASEP of cognitive CSI-assisted AF SEC relaying network with primary users using orthogonal spectrum bands and Rayleigh fading is given for i.i.d. relay hops as*

$$\begin{aligned}
\mathrm{ASEP} \simeq\ & \frac{a\sqrt{b}}{2\sqrt{\pi}} M\zeta_{\mathrm{s,p}} \Bigg\{ \sum_{g=0}^{K-1} \frac{\binom{K-1}{g}(-1)^g}{\left(1+\lambda_2\gamma_{\mathrm{T}}\right)^g} \\
& \times \Bigg[\frac{\sqrt{\pi}\left(1-\exp\left(-\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\right)}{\sqrt{b}\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)} \\
& - \Gamma(1/2)(\lambda_2)^{-1/2} \\
& \times \left(\left(-\frac{1}{\lambda_2}+g\gamma_{\mathrm{T}}\right)\lambda_1 + M\zeta_{\mathrm{s,p}}\right)^{-1} \\
& \times \Bigg\{ \Bigg(\Gamma\left(1/2, b/\lambda_2\right) - \frac{\exp\left(-\left(\left(g-\frac{1}{\lambda_2}\right)\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{\Gamma\left(1/2, (\lambda_1\gamma_{\mathrm{T}}+b)/\lambda_2\right)^{-1}} \Bigg) \\
& \times \exp\left(b/\lambda_2\right) - \Big(\Gamma\left(1/2, b\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)/\lambda_1\right)
\end{aligned}$$

$$
\begin{aligned}
& -\Gamma\left(1/2,\left(\lambda_1\gamma_{\mathsf{T}}+b\right)\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)/\lambda_1\right)\Big\}\Big]\Bigg] \\
& \times \frac{\exp\left(\frac{b\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)}{\lambda_1}\right)}{\left(\lambda_2\left(g\lambda_1+\frac{M\zeta_{\mathsf{s,p}}}{\lambda_1}\right)\right)^{1/2}}+\sum_{j=0}^{K-1}\sum_{w=0}^{j}\frac{\binom{j}{w}(-1)^w}{(1+\lambda_2\gamma_{\mathsf{T}})^w} \\
& \times\left[\frac{\sqrt{\pi}\exp\left(-\left((j+1)\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)}{\sqrt{b}\left(1+\lambda_2\gamma_{\mathsf{T}}\right)\left((j+1)\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)}\right. \\
& -\Gamma(1/2)\times\frac{\exp\left(-\left(j\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)}{(\lambda_2)^{1/2}\left(\left(-\frac{1}{\lambda_2}+j\gamma_{\mathsf{T}}\right)\lambda_1+M\zeta_{\mathsf{s,p}}\right)} \\
& \times\left\{\exp\left(\frac{\lambda_1\gamma_{\mathsf{T}}+b}{\lambda_2}\right)\Gamma\left(1/2,\left(\lambda_1\gamma_{\mathsf{T}}+b\right)/\lambda_2\right)\right. \\
& -\exp\left(\frac{\left(j\gamma_{\mathsf{T}}+\frac{M\zeta_{\mathsf{s,p}}}{\lambda_1}\right)}{\left(\lambda_1\gamma_{\mathsf{T}}+b\right)^{-1}}\right)\times\left(\lambda_2\left(j\lambda_1+\frac{M\zeta_{\mathsf{s,p}}}{\lambda_1}\right)\right)^{-1/2} \\
& \left.\left.\times\Gamma\left(1/2,\frac{\left(j\gamma_{\mathsf{T}}+\frac{M\zeta_{\mathsf{s,p}}}{\lambda_1}\right)}{\left(\lambda_1\gamma_{\mathsf{T}}+b\right)^{-1}}\right)\right\}\right] \\
& \left.+\sum_{q=0}^{K}\frac{\binom{K}{q}(-1)^q}{(1+\lambda_2\gamma_{\mathsf{T}})^q}\frac{\exp\left(-\left(q\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)}{(\sqrt{\pi})^{-1}\sqrt{b}\left(q\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)}\right\}.
\end{aligned}
\tag{9}
$$

By replacing γ_{out} with γ in (4) and using the partial fraction expansion and the integration in (7) and using [23, Eq. (3.361.2)] and [23, Eq. (3.383.10)], we get (9).

3.3 Ergodic Channel Capacity

In this section, we derive the channel capacity of the studied system for i.i.d. relay channels. The capacity can be expressed in terms of the probability density function (PDF) of γ_{D} as

$$
C=\frac{1}{2\ln(2)}\int_0^{\infty}\ln(1+\gamma)f_{\gamma_{\mathsf{D}}}(\gamma)\mathrm{d}\gamma. \tag{10}
$$

Our result on the ergodic capacity is provided in the following theorem.

Theorem 3 *The ergodic capacity of cognitive CSI-assisted AF SEC relaying network with primary users using orthogonal spectrum bands and Rayleigh fading is given for i.i.d. relay hops as*

$$
\begin{aligned}
C \simeq\ & \frac{1}{2\ln(2)}\sum_{g=0}^{K-1}\binom{K-1}{g}(-1)^g\left(1+\lambda_2\gamma_{\mathsf{T}}\right)^{-g} \\
& \times\Bigg[\alpha\ln(\lambda_2)(\lambda_2-1)^{-1}+\alpha\exp\left(-\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right) \\
& \times\left\{-\left(\lambda_1\gamma_{\mathsf{T}}+\lambda_2\right)\int_0^{\infty}\frac{\ln(1+\gamma)\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\mathrm{d}\gamma\right. \\
& \left.-\lambda_1\lambda_2\gamma_{\mathsf{T}}\int_0^{\infty}\frac{\ln(1+\gamma)\gamma\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\mathrm{d}\gamma\right\}+\lambda_1\beta \\
& \times\int_0^{\infty}\frac{\ln(1+\gamma)}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)}\mathrm{d}\gamma \\
& +\beta\exp\left(-\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right) \\
& \times\left\{\int_0^{\infty}\frac{\ln(1+\gamma)\gamma\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)^2}\mathrm{d}\gamma\right. \\
& \left.\times\left(-(\lambda_1)^2\gamma_{\mathsf{T}}\right)-\left(g(\lambda_1)^2(\gamma_{\mathsf{T}})^2+\lambda_1M\zeta_{\mathsf{s,p}}\gamma_{\mathsf{T}}+\lambda_1\right)\right\}\Bigg] \\
& +\sum_{j=0}^{K-1}\sum_{w=0}^{j}\frac{\binom{j}{w}(-1)^w}{(1+\lambda_2\gamma_{\mathsf{T}})^w}\Bigg[-\exp\left(-\left(j\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right) \\
& \times\Bigg(\alpha'\left\{-\left(\lambda_1\gamma_{\mathsf{T}}+\lambda_2\right)\int_0^{\infty}\frac{\ln(1+\gamma)\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\mathrm{d}\gamma\right. \\
& \left.-\lambda_1\lambda_2\gamma_{\mathsf{T}}\int_0^{\infty}\frac{\ln(1+\gamma)\gamma\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\mathrm{d}\gamma\right\} \\
& +\beta'\left\{-(\lambda_1)^2\gamma_{\mathsf{T}}\int_0^{\infty}\frac{\ln(1+\gamma)\gamma\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{\left(\lambda_1\gamma+j\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)^2}\mathrm{d}\gamma\right. \\
& -\left(j(\lambda_1)^2(\gamma_{\mathsf{T}})^2+\lambda_1M\zeta_{\mathsf{s,p}}\gamma_{\mathsf{T}}+\lambda_1\right)\int_0^{\infty}\ln(1+\gamma) \\
& \left.\times\frac{\exp\left(-\lambda_1\gamma_{\mathsf{T}}\gamma\right)}{\left(\lambda_1\gamma+j\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)^2}d\gamma\right\}\Bigg)\Bigg],
\end{aligned}
\tag{11}
$$

where α, β, α', and β' are as given in the proof below. According to authors knowledge, the integrations in (11) have no closed-form solution. Hence, the ergodic channel capacity of the system is numerically evaluated.

Proof We need first to derive the PDF of γ_{D}. By replacing γ_{out} with γ in (4) and using the partial fraction, the CDF $F_{\gamma_{\mathsf{D}}}(\gamma)$ can be written as

$$
\begin{aligned}
F_{\gamma_{\mathsf{D}}}(\gamma)\simeq\ & M\zeta_{\mathsf{s,p}}\left\{\sum_{g=0}^{K-1}\frac{\binom{K-1}{g}(-1)^g}{(1+\lambda_2\gamma_{\mathsf{T}})^g}\right. \\
& \times\left[\left(1-\exp\left(-\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)\right)\left(g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)^{-1}\right. \\
& -\alpha\frac{\left(1-\exp\left(-\left((\gamma+g\gamma_{\mathsf{T}})\lambda_1+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)\right)}{(1+\lambda_2\gamma)} \\
& \left.-\beta\frac{\left(1-\exp\left(-\left((\gamma+g\gamma_{\mathsf{T}})\lambda_1+M\zeta_{\mathsf{s,p}}\right)\gamma_{\mathsf{T}}\right)\right)}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathsf{T}}+M\zeta_{\mathsf{s,p}}\right)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=0}^{K-1}\sum_{w=0}^{j}\binom{j}{w}\\
&\times\frac{(-1)^w}{(1+\lambda_2\gamma_{\mathrm{T}})^w}\Bigg[\frac{\exp\left(-\left((j+1)\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{(1+\lambda_2\gamma_{\mathrm{T}})\left((j+1)\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)}\\
&-\exp\left(-\left((j\lambda_1\gamma_{\mathrm{T}})+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\Bigg(\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)\\
&\times\left\{\frac{\alpha'}{(1+\lambda_2\gamma)}+\frac{\beta'}{\left((\gamma+j\gamma_{\mathrm{T}})\lambda_1+M\zeta_{\mathrm{s,p}}\right)}\right\}\Bigg)\Bigg]\\
&+\sum_{q=0}^{K}\frac{\binom{K}{q}(-1)^q}{(1+\lambda_2\gamma_{\mathrm{T}})^q}\frac{\exp\left(-\left(q\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)}{\left(q\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)}\Bigg\}.
\end{aligned}\tag{12}
$$

Solving for α, α', β, and β', we get $\alpha=\left(-\frac{\lambda_1}{\lambda_2}+g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)^{-1}$, $\beta=\left(1-\frac{(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}})\lambda_2}{\lambda_1}\right)^{-1}$, and $\alpha'=\alpha$, $\beta'=\beta$ with replacing g by j. By taking the derivative of (12) with respect to γ, the PDF $f_{\gamma_{\mathrm{D}}}(\gamma)$ can be obtained as

$$
\begin{aligned}
f_{\gamma_{\mathrm{D}}}(\gamma)&\simeq M\zeta_{\mathrm{s,p}}\Bigg\{\sum_{g=0}^{K-1}\frac{\binom{K-1}{g}(-1)^g}{(1+\lambda_2\gamma_{\mathrm{T}})^g}\Bigg[\alpha\Bigg(\frac{\lambda_2}{(1+\lambda_2\gamma)^2}\\
&-\exp\left(-\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\Bigg\{\frac{\lambda_1\gamma_{\mathrm{T}}\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{(1+\lambda_2\gamma)}\\
&+\frac{\lambda_2\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\Bigg\}\Bigg)+\beta\Bigg(\frac{\lambda_1}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)^2}\\
&-\exp\left(-\left(g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\Bigg\{\frac{\lambda_1\gamma_{\mathrm{T}}\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)}\\
&+\frac{\lambda_1\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{\left(\lambda_1\gamma+g\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)^2}\Bigg\}\Bigg)\Bigg]+\sum_{j=0}^{K-1}\sum_{w=0}^{j}\frac{\binom{j}{w}(-1)^w}{(1+\lambda_2\gamma_{\mathrm{T}})^w}\\
&\times\Bigg[-\exp\left(-\left((j\lambda_1\gamma_{\mathrm{T}})+M\zeta_{\mathrm{s,p}}\right)\gamma_{\mathrm{T}}\right)\\
&\times\Bigg(\alpha'\Bigg\{-\frac{\lambda_1\gamma_{\mathrm{T}}\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{(1+\lambda_2\gamma)}-\frac{\lambda_2\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{(1+\lambda_2\gamma)^2}\Bigg\}\\
&+\beta'\Bigg\{-\frac{\lambda_1\gamma_{\mathrm{T}}\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{\left(\lambda_1\gamma+j\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)}\\
&-\frac{\lambda_1\exp\left(-\lambda_1\gamma_{\mathrm{T}}\gamma\right)}{\left(\lambda_1\gamma+j\lambda_1\gamma_{\mathrm{T}}+M\zeta_{\mathrm{s,p}}\right)^2}\Bigg\}\Bigg)\Bigg]\Bigg\}.
\end{aligned}\tag{13}
$$

Upon substituting (13) in (10) and after some mathematical arrangements and using [23, Eq. (4.291.15)], we get (11). □

4 Asymptotic Performance Analysis

Due to complexity of the derived expressions, we see it is important to derive an approximate but simple expression for the outage probability which helps in achieving more insights about the system behavior. This can be achieved by studying the system performance at the high SNR regime where the outage probability can be expressed as $P_{\mathrm{out}}\approx(G_{\mathrm{c}}\mathrm{SNR})^{-G_{\mathrm{d}}}$, where G_{c} and G_{d} are the coding gain and diversity order of the system, respectively. Obviously, G_{c} represents the horizontal shift in the outage probability performance relative to the benchmark curve $(\mathrm{SNR})^{-G_{\mathrm{d}}}$, and G_{d} refers to the increase in the slope of the outage probability versus SNR curve [21, Ch.14]. The parameters on which the diversity order depends will affect the slope of the outage probability curves, and the parameters on which the coding gain depends will affect the position of the curves. In the upcoming analysis, the relays are assumed to have identical first hop channels ($\lambda_{\mathrm{s},1}=\cdots=\lambda_{\mathrm{s},K}=\lambda_{\mathrm{s,r}}$) and identical $\mathrm{S}\to\mathrm{P}_m$ links ($\zeta_{\mathrm{s},1}=\cdots=\zeta_{\mathrm{s},M}=\zeta_{\mathrm{s,p}}$). Furthermore, the relays are assumed to have identical second hop channels ($\lambda_{1,\mathrm{d}}=\cdots=\lambda_{K,\mathrm{d}}=\lambda_{\mathrm{r,d}}$) and identical $\mathrm{R}\to\mathrm{P}_m$ links ($\zeta_{1,1}=\cdots=\zeta_{1,M}=\zeta_{1,\mathrm{p}}$), ($\zeta_{1,\mathrm{p}}=\cdots=\zeta_{K,\mathrm{p}}=\zeta_{\mathrm{r,p}}$).

As $\bar{\gamma}\to\infty$, the CDF in (21) simplifies to $F_{\gamma_k^{\mathrm{up}}}(\gamma|Y)\approx\lambda_1 Y\gamma$. Upon substituting this CDF in (5) and following the same procedure as in the Appendix, the asymptotic outage probability can be obtained using [23, Eq. (3.351.1)] and [23, Eq. (3.351.2)] as

$$
\begin{aligned}
P_{\mathrm{out}}^{\infty}&=(\gamma_{\mathrm{T}})^{K-1}\left(\frac{\lambda_1}{M\zeta_{\mathrm{s,p}}}\right)^K\gamma_{\mathrm{out}}\,\gamma\left(K+1,M\zeta_{\mathrm{s,p}}\gamma_{\mathrm{T}}\right)\\
&+\sum_{j=0}^{K-1}(\gamma_{\mathrm{T}})^j\left(\frac{\lambda_1}{M\zeta_{\mathrm{s,p}}}\right)^{j+1}\Gamma\left(j+2,M\zeta_{\mathrm{s,p}}\gamma_{\mathrm{T}}\right)\\
&\times(\gamma_{\mathrm{out}}-\gamma_{\mathrm{T}})\\
&+(\gamma_{\mathrm{T}})^K\left(\frac{\lambda_1}{M\zeta_{\mathrm{s,p}}}\right)^K\Gamma\left(K+1,M\zeta_{\mathrm{s,p}}\gamma_{\mathrm{T}}\right).
\end{aligned}\tag{14}
$$

One can notice that (14) is dominated by the second term which is still dominant when $j=0$. Hence, the outage probability can be evaluated at the high SNR values as

$$
P_{\mathrm{out}}^{\infty}=\left(\frac{\lambda_1}{\lambda_{\mathrm{s,p}}}\right)\Gamma\left(2,M\zeta_{\mathrm{s,p}}\gamma_{\mathrm{T}}\right)(\gamma_{\mathrm{out}}-\gamma_{\mathrm{T}}).\tag{15}
$$

Recalling that for the case of identical first hops $\lambda_1=1/\Omega_{\mathrm{s,r}}\bar{\gamma}$, (15) can be further simplified as

$$
P_{\mathrm{out}}^{\infty}=\left\{\frac{\Omega_{\mathrm{s,r}}M\zeta_{\mathrm{s,p}}}{\Gamma\left(2,M\zeta_{\mathrm{s,p}}\gamma_{\mathrm{T}}\right)(\gamma_{\mathrm{out}}-\gamma_{\mathrm{T}})}\bar{\gamma}\right\}^{-1}.\tag{16}
$$

As can be seen from (16), the coding gain of the system is affected by several parameters including $\Omega_{\mathrm{s,r}}$, M, $\zeta_{\mathrm{s,p}}$, γ_{out}, and γ_{T}, while the diversity order is constant at 1. This is clear in the numerical examples where all curves of different K asymptotically converge to the same behavior and result in a diversity order of 1. Also, as the asymptotic analysis is done at the high SNR values, it is expected to have most of the relays being acceptable the whole time, and thus, the first examined relay is being selected by the relaying scheme. Furthermore, it can be noticed from (16) that the diversity order of the proposed cognitive system with SEC relaying is similar to that of the non-cognitive one, see, e.g., [16] for the case of no direct link. Specifically, it is equal to 1 and is independent of the primary network. Again, it is worthwhile to mention here that the importance of the SEC relaying scheme is in the low number of channel estimations and it requires each transmission time compared to the opportunistic and partial-relay selection schemes. Furthermore, from the asymptotic results, we conclude that the SEC relaying is inefficient at the range of high SNR values where the diversity order becomes 1. Due to its features and performance, the switching threshold-based relaying scheme is actually an attractive option for practical implementation in emerging mobile broadband communication systems [24].

As we have mentioned before, the optimum switching threshold can be numerically calculated to minimize the e2e outage probability. Due to complexity of the achieved expressions, any further manipulations with them to find the optimum switching threshold will increase the system complexity. Alternatively, we present here a simple method that can be used to get approximate values for the optimum switching threshold. In the region of low SNR values, the optimum switching threshold can be calculated using $\min\left(\frac{\bar{\gamma}\Omega_{\mathrm{s},k}}{(M\zeta_{\mathrm{s,p}})^{-1}}, \frac{\Omega_{k,\mathrm{d}}}{(M\zeta_{\mathrm{r,p}})^{-1}}\right)$. In general, a good choice of the switching threshold is to have it near the average value of the e2e SNR which can be upper bounded by the average value of minimum of its two hops. Unfortunately, as we go further in increasing SNR, the approximate values of the optimum switching threshold get far from the actual optimum values which are numerically calculated by minimizing the e2e outage probability of the system. To deal with this issue, we use the asymptotic expressions that we derived at high SNR values to calculate the approximate values of the optimum threshold. As the asymptotic expression of the outage probability in (15) is simple to deal with, it can be used to get approximate values of the optimum switching threshold in the region of high SNR values.

5 Simulation and Numerical Results

In this section, we validate the achieved analytical and asymptotic results via a comparison with Monte Carlo simulations. Also, we provide some numerical examples to illustrate the effect of some system parameters such as the number of relays, number of primary users, and switching threshold on the system performance. In generating the simulation results, 40,000 samples/SNR value have been used. Also, the BPSK modulation scheme was assumed in generating the error probability curves.

It is clear from Fig. 2 that the asymptotic results perfectly converge to the analytical results as well as the exact ones. Also, it is obvious that the used bound on the e2e SNR is indeed very tight, especially, at high SNR region. Furthermore, we can see from this figure that the SEC relaying scheme has almost the same behavior as the opportunistic relaying or best-relay selection scheme for very low SNR region [5], whereas as we go further in increasing SNR, the best-relay selection scheme is clearly outperforming the SEC relaying, as expected. The figure also compares the SEC scheme with the partial-relay selection scheme where the relay with the best second or even first hop channel is selected [7]. Clearly, the SEC scheme outperforms the partial-relay selection scheme over the whole range of SNR. That is expected as in the partial-relay selection, only one hop of relay channels is considered in selecting among relays, whereas in the SEC scheme, the two hop links affect the selection process. In addition, we can see from this figure that for the

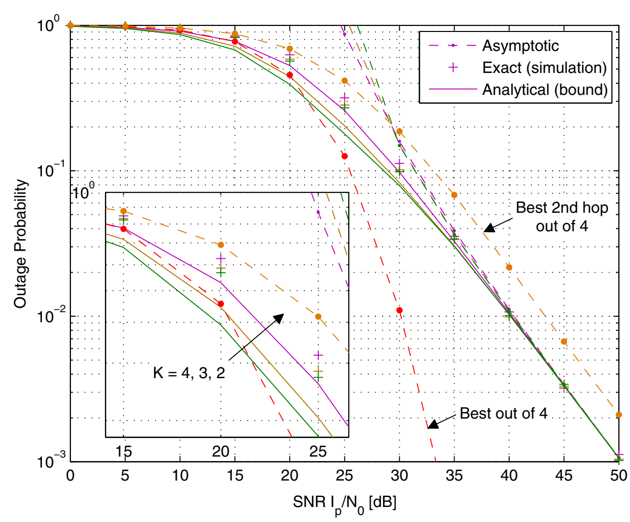

Fig. 2 P_{out} versus SNR for different values of K and $M = 1$, $\gamma_{\mathrm{out}} = 7.78$ dB, $\gamma_{\mathrm{T}} = 5$ dB, $\mu_{\mathrm{s,p}} = 30$, and $\Omega_{\mathrm{s},k} = 0.8$, $\Omega_{k,\mathrm{d}} = 0.7$, $\mu_{k,\mathrm{p}} = 0.1$ for $k = 1, \ldots, 4$

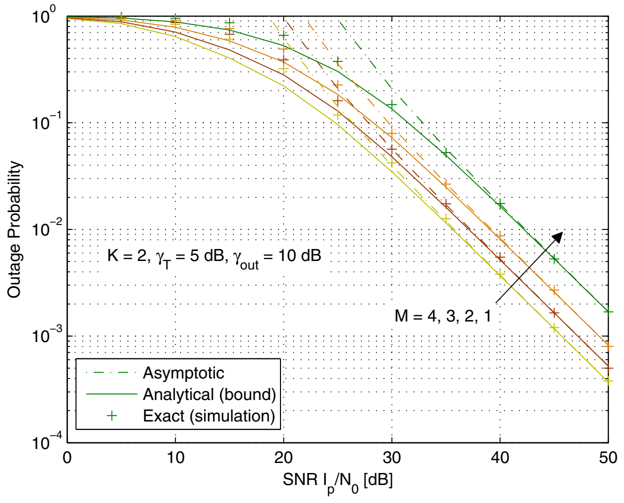

Fig. 3 P_{out} versus SNR for different values of M and $\mu_{\text{s},1} = \cdots = \mu_{\text{s},10} = 20$, and $\Omega_{\text{s},k} = 0.8$, $\Omega_{k,\text{d}} = 0.9$, $\mu_{k,1} = \cdots = \mu_{k,10} = 0.01$ for $k = 1, 2$

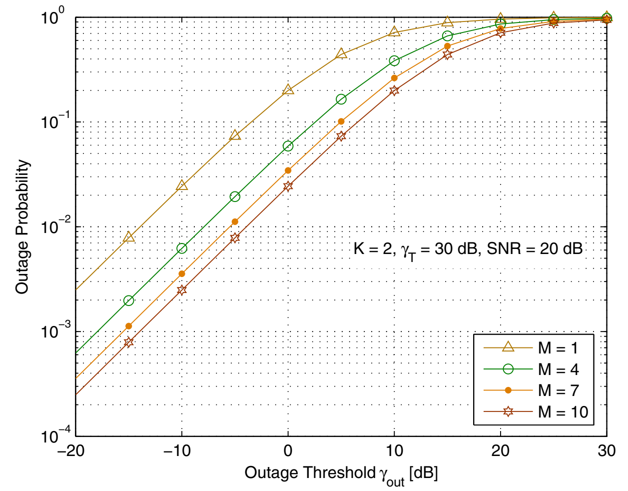

Fig. 4 P_{out} versus γ_{out} for different values of M and $\mu_{\text{s},1} = \cdots = \mu_{\text{s},10} = 20$, and $\Omega_{\text{s},k} = 0.8$, $\Omega_{k,\text{d}} = 0.9$, $\mu_{k,1} = \cdots = \mu_{k,10} = 0.01$ for $k = 1, 2$

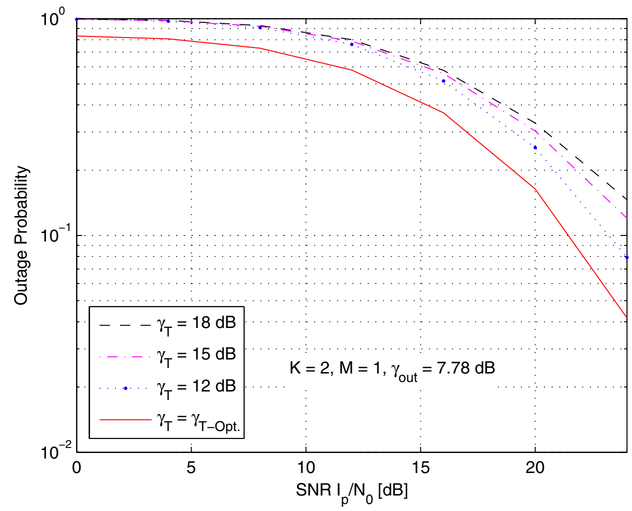

Fig. 5 P_{out} versus SNR for different values of γ_{T} and $\mu_{\text{s,p}} = 7$, and $\Omega_{\text{s},k} = 0.91$, $\Omega_{k,\text{d}} = 0.82$, $\mu_{k,\text{p}} = 0.73$ for $k = 1, 2$

SEC relaying as K increases, the system performance becomes more enhanced, especially, at the SNR values that are comparable to γ_{T}. More importantly, for $K = 2$, 3, and 4, it is obvious that the system behavior does not achieve any gain when more relays are used at the two regions where the average SNR is much smaller or larger than γ_{T}. This makes sense as when γ_{T} is much smaller or larger than the average SNR, the system asymptotically converges to the case of two relays and having more relays adds no gain to the system behavior. Finally, it is worthwhile to mention here that the reduction in number of channel estimations offered by the SEC relaying scheme compared to opportunistic and partial-relay selection schemes happens with a reasonable loss in the system behavior. This makes the SEC relaying scheme more suitable and desirable for systems where the complexity issue is of a highest priority. Examples on these systems are the sensor and ad hoc networks where once the minimum requirements of the system performance are achieved, no more operations that increase the system complexity need to be done.

Again, we can see from Fig. 3 that the asymptotic results perfectly converge to the analytical results as well as the exact ones. Also, it is obvious that the used bound on the e2e SNR is indeed very tight, especially, at high SNR region. More importantly, we can see from this figure that as M increases, the achieved performance becomes better. This is because increasing M increases the probability to find primary users of weaker channels and hence the higher the transmit power of secondary transmitters. Clearly, the parameter M affects the coding gain of the system and not the diversity order. This fact was also proved by the asymptotic results. It is worthwhile to mention here that in systems where the multiple primary receivers utilize the same spectrum band as in [25], increasing the number of primary receivers degrades the system performance and the coding gain of the system. This is because in those systems, the transmit power of secondary users is determined by the best link between the secondary users and the primary receivers. Therefore, having more primary receivers increases the probability of having better links and hence the less the transmit power of secondary users.

Figure 4 shows the outage performance versus outage threshold for different values of M. Obviously, as γ_{out} increases, the achieved performance worsens. Moreover, the best behavior is obtained at the largest value of M.

We can see from Fig. 5 that the value of the switching threshold γ_{T} which gives the best behavior is the optimum one $\gamma_{\text{T}-\text{Opt.}}$, as expected. To get rid of the complexity of finding the optimum switching threshold using a numerical way,

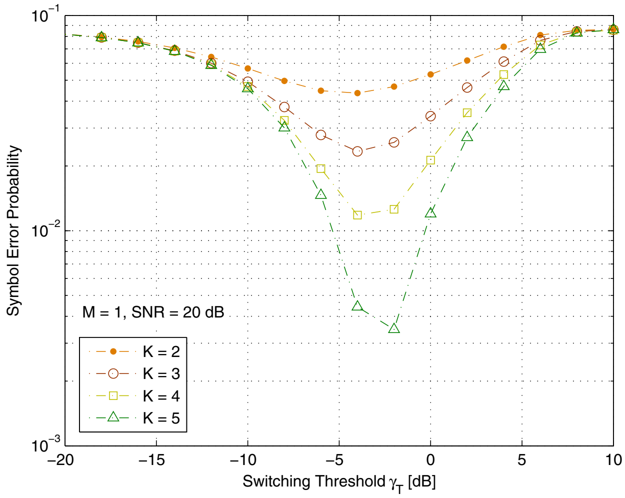

Fig. 6 ASEP versus γ_{T} for different values of K and $\mu_{\mathrm{s,p}} = 20$, and $\Omega_{\mathrm{s},k} = 0.1$, $\Omega_{k,\mathrm{d}} = 0.2$, $\mu_{k,\mathrm{p}} = 0.3$ for $k = 1, \ldots, 5$

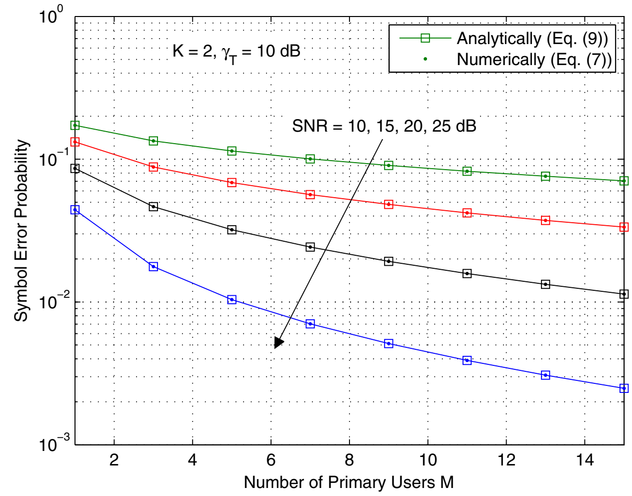

Fig. 7 ASEP versus M for different values of SNR $= \mathcal{I}_{\mathrm{p}}/N_0$ and $\mu_{\mathrm{s},1} = \cdots = \mu_{\mathrm{s},15} = 20$, and $\Omega_{\mathrm{s},k} = 0.1$, $\Omega_{k,\mathrm{d}} = 0.2$, $\mu_{k,1} = \cdots = \mu_{k,15} = 0.3$ for $k = 1, 2$

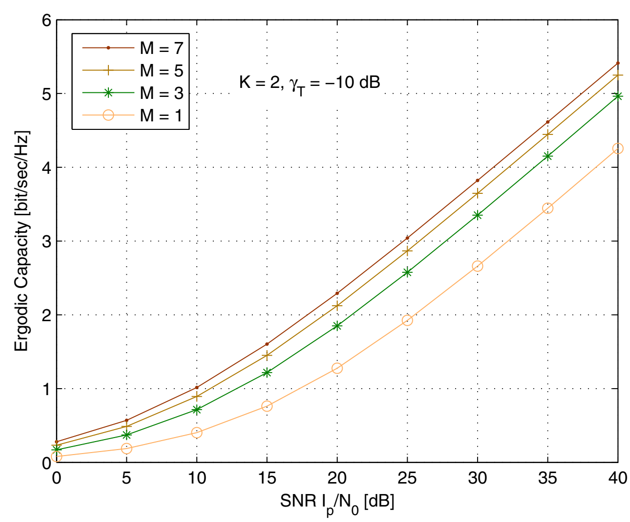

Fig. 8 Channel capacity versus SNR for different values of M and $\mu_{\mathrm{s},1} = \cdots = \mu_{\mathrm{s},7} = 20$, and $\Omega_{\mathrm{s},k} = 0.8$, $\Omega_{k,\mathrm{d}} = 0.9$, $\mu_{k,1} = \cdots = \mu_{k,7} = 0.3$ for $k = 1, 2$

it is calculated using the simple method explained at the end of Sect. 4. One more thing to mention here is that in order to not lose much in the system diversity, it is preferred to run the SEC relaying scheme using the optimum switching threshold or its approximate values. This guarantees that a maximum gain is achieved by the system when more relays are added.

The gain achieved in system behavior due to adding more relays is shown in Fig. 6. This gain is obvious in the range of γ_{T} values that are close to the average value of γ_k^{up}. As mentioned before, as γ_{T} becomes much smaller or larger than the average value of γ_k^{up}, the improvement in the performance decreases, as all curves asymptotically converge to the case of two relays. Therefore, in order to get benefit of the simplicity of the SEC relaying scheme and at the same time not losing much in the system behavior compared to the opportunistic relaying, the optimum values of switching threshold need to be used when the SEC relaying scheme is implemented.

It can be seen from Fig. 7 that the studied system still achieves behavior gain and the error performance enhances as the number of primary users M increases, but the slope of the curves depends on the SNR values. Also, this figure validates the derived analytical expression of the symbol error probability where a perfect matching can be seen between the analytical result in (9) and its numerical solution that is achieved using (7).

The effect of number of primary users M on the system ergodic capacity is illustrated in Fig. 8. Due to complexity of the derived expression of the channel capacity, it is numerically calculated and plotted in this figure. As expected, the ergodic channel capacity enhances as M increases.

Figure 9 portrays the ergodic capacity versus M for several values of SNR $\mathcal{I}_{\mathrm{p}}/N_0$. Clearly, as M increases, the channel capacity becomes higher. Also, we can see that the best performance can be achieved at the highest value of SNR, as expected. Again, it can be noticed from the last three figures that increasing the number of primary receivers enhances the system performance. This is because the more the primary receivers, the higher the transmit power of secondary users and hence better the achieved performance. This enhancement in the behavior happens in the coding gain of the system and not the diversity order.

Figure 10 compares the SEC relaying scheme with the opportunistic and partial-relay selection schemes from a complexity-wise. The figure portrays the average number of channel estimations that are required in the three schemes

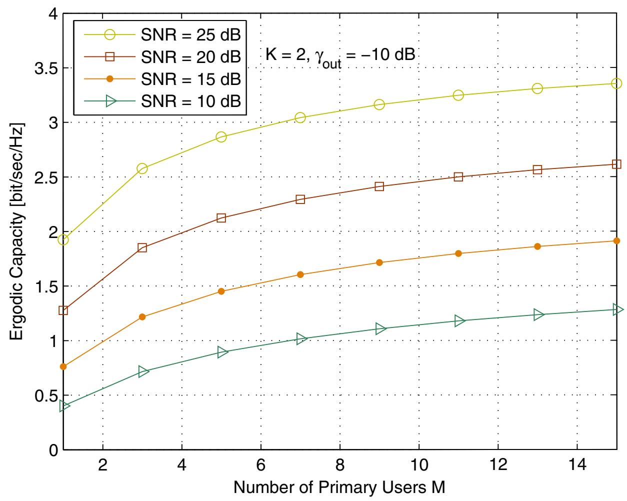

Fig. 9 Channel capacity versus M for different values of SNR $= \mathcal{I}_{\mathsf{p}}/N_0$ and $\mu_{\mathsf{s},1} = \cdots = \mu_{\mathsf{s},15} = 20$, and $\Omega_{\mathsf{s},k} = 0.8$, $\Omega_{k,\mathsf{d}} = 0.9$, $\mu_{k,1} = \cdots = \mu_{k,15} = 0.3$ for $k = 1, 2$

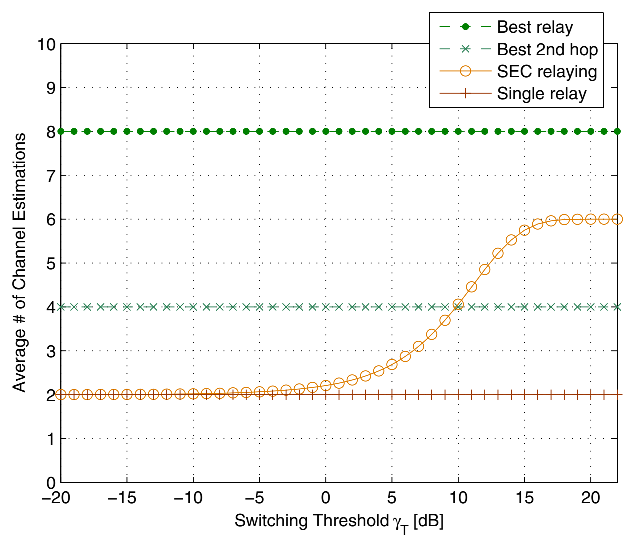

Fig. 10 Average number of channel estimations of the SEC relaying scheme in comparison with the opportunistic and partial-relay selection schemes with $K = 4$ and an average power/relay path = 10 dB

versus switching threshold γ_{T} for the case of four relays. In this comparison, we assume that the links between the source and primary users and those between the relays and primary users are all known to the relays and that the relays are selected according to their first and second hop channels. Also, it is assumed that in the partial-relay selection scheme, the relay is selected according to its second hop channel. We can see from this figure that as all channels are needed to be estimated for its operation, the opportunistic or best-relay selection scheme is always of need for eight channel estimations [15]. On the other hand, the partial-relay selection scheme requires only four channel estimations as in this scheme only the second hop channels of relays are used to select among them. Also, we can notice from this figure that as γ_{T} increases, the average number of channel estimations in the SEC relaying scheme increases since it is more difficult to find a relay with an acceptable quality. Again, the reduction in channel estimation load and system complexity offered by the SEC relaying scheme compared to the best-relay or partial-relay selection schemes is achieved on the expense of a reasonable loss in the system complexity. Therefore, in order not to lose much in the diversity gain of the system, it is preferable to have the switching threshold close to the average SNR of relay paths. It is worthwhile to mention here that an attractive research issue which could be investigated in future is the delay effect on the performance of systems such as the studied one.

6 Conclusion

In this paper, we proposed and evaluated the performance of a new cognitive AF SEC relaying network with primary users using orthogonal spectrum bands where the outage and error probabilities were obtained assuming Rayleigh fading environment. Also, the channel ergodic capacity was numerically calculated in this paper. Moreover, the behavior was evaluated at the high SNR values where the diversity order and coding gain were derived. Monte Carlo simulations proved the accuracy of the achieved analytical and asymptotic results. Main results illustrated that the diversity order of the studied system is the same as its non-cognitive counterpart, and it is independent of the number of primary users. Also, findings showed that only the coding gain is affected by the number of primary users. Finally, results illustrated that in contrast to the existing papers where the same spectrum band is assumed to be used by the primary users, increasing the number of primary users in the considered scenario enhances the system performance.

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Appendix: Proof of Theorem 1

In this "Appendix," we derive the outage probability for i.n.i.d. relay hops. In order to evaluate the outage probability, the cumulative distribution function (CDF) of γ_{D} is required to be obtained first. The CDF of γ_k^{up} conditioned on $Y = \min_m |g_{\mathsf{s},m}|^2$ can be written as

$$F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) = 1 - \left(1 - F_{\gamma_{k1}}(\gamma|Y)\right)\left(1 - F_{\gamma_{k2}}(\gamma|Y)\right). \tag{17}$$

It is easy to see that

$$F_{\gamma_{k1}}(\gamma|Y) = 1 - \exp(-\lambda_{k1} Y\gamma), \tag{18}$$

$$F_{\gamma_{k2}}(\gamma|Y) = \int_0^\infty F_{|h_{k,\mathsf{d}}|^2}\left(\frac{\gamma}{\bar{\gamma}}x\right) f_{\min\limits_m |g_{k,m}|^2}(x)\,\mathrm{d}x = 1 - (1+\lambda_{k2}\gamma)^{-1}, \tag{19}$$

where $\lambda_{k1} = 1/\left(\Omega_{\mathsf{s},k}\bar{\gamma}\right)$, $\lambda_{k2} = 1\Big/\left(\sum_{m=1}^{M}\zeta_{k,m}\Omega_{k,\mathsf{d}}\bar{\gamma}\right)$, and

$$f_{\min\limits_m |g_{k,m}|^2}(w) = \sum_{m=1}^{M}\zeta_{k,m}\exp\left(-\sum_{m=1}^{M}\zeta_{k,m}w\right), \tag{20}$$

where $\zeta_{k,m} = 1/\mu_{k,m}$.
Upon substituting (18) and (19) in (17), we get

$$F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) = 1 - \frac{\exp(-\lambda_{k1}Y\gamma)}{(1+\lambda_{k2}\gamma)}. \tag{21}$$

The conditional CDF at the output of the SEC relay selection scheme is given by [21]

$$F_{\gamma_{\mathsf{D}}}(\gamma|Y) = \begin{cases} \sum_{i=0}^{K-1}\pi_i F_{\gamma_i^{\mathsf{up}}}(\gamma|Y)\prod_{\substack{k=0\\k\neq i}}^{K-1} F_{\gamma_k^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y), & \gamma < \gamma_{\mathsf{T}}; \\ \sum_{l=0}^{K-1}\pi_l\Big(\prod_{q=1}^{K} F_{\gamma_q^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y) & \\ +\sum_{w=0}^{K-1}\pi_{((l-w))_K}\Big[F_{\gamma_l^{\mathsf{up}}}(\gamma|Y) - F_{\gamma_l^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\Big] & \\ \times\prod_{p=0}^{w-1} F_{\gamma^{\mathsf{up}}_{((l-w+p))_K}}(\gamma_{\mathsf{T}}|Y)\Big), & \gamma \geq \gamma_{\mathsf{T}}, \end{cases} \tag{22}$$

where K is the number of relays and γ_{T} is a predetermined switching threshold, π_i, $i = 0, \ldots, K-1$ are the stationary distribution of a K-state Markov chain and it is the probability that the i^{th} relay is chosen, and $((l-w))_K$ denotes $l-w$ modulo K. It is given by

$$\pi_i = \left[\sum_{j=0}^{K-1}\left(\frac{F_{\gamma_{K-1}^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\left(1-F_{\gamma_j^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right)}{F_{\gamma_j^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\left(1-F_{\gamma_{K-1}^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right)}\right)\right]^{-1} \times \frac{F_{\gamma_{K-1}^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\left(1-F_{\gamma_i^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right)}{F_{\gamma_i^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\left(1-F_{\gamma_{K-1}^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right)}. \tag{23}$$

Upon substituting (21) in (22), we get

$$F_{\gamma_{\mathsf{D}}}(\gamma|Y) = \begin{cases} \underbrace{\sum_{i=0}^{K-1}\pi_i\left(1-\frac{\exp(-\lambda_{i1}Y\gamma)}{(1+\lambda_{i2}\gamma)}\right)\prod_{\substack{k=0\\k\neq i}}^{K-1}\left(1-\frac{\exp(-\lambda_{k1}Y\gamma_{\mathsf{T}})}{(1+\lambda_{k2}\gamma_{\mathsf{T}})}\right)}_{\mathcal{P}_1}, & \gamma < \gamma_{\mathsf{T}}; \\ \sum_{l=0}^{K-1}\pi_l\Bigg(\underbrace{\prod_{q=1}^{K}\left(1-\frac{\exp(-\lambda_{q1}Y\gamma_{\mathsf{T}})}{(1+\lambda_{q2}\gamma_{\mathsf{T}})}\right)}_{\mathcal{P}_2} & \\ +\sum_{w=0}^{K-1}\pi_{((l-w))_K}\left[\left(1-\frac{\exp(-\lambda_{l1}Y\gamma)}{(1+\lambda_{l2}\gamma)}\right) -\left(1-\frac{\exp(-\lambda_{l1}Y\gamma_{\mathsf{T}})}{(1+\lambda_{l2}\gamma_{\mathsf{T}})}\right)\right] & \\ \underbrace{\times\prod_{p=0}^{w-1}\left(1-\frac{\exp\left(-\lambda_{((l-w+p))_K 1}Y\gamma_{\mathsf{T}}\right)}{\left(1+\lambda_{((l-w+p))_K 2}\gamma_{\mathsf{T}}\right)}\right)}_{\mathcal{P}_3}\Bigg), & \gamma \geq \gamma_{\mathsf{T}}. \end{cases} \tag{24}$$

Using the product identities [26, Eq. (A2)] and [8, Eq. (6)], the terms $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ can be simplified as follows

$$\mathcal{P}_1 = 1 + \sum_{k=0}^{K-1}(-1)^{k+1}\sum_{\substack{n_0<\cdots<n_k\\n_{(.)}\neq i}}^{K-1}\prod_{t=0}^{k}\left(1+\lambda_{n_t 2}\gamma_{\mathsf{T}}\right)^{-1} \times\exp\left(-\sum_{s=0}^{k}\lambda_{n_s 1}Y\gamma_{\mathsf{T}}\right), \tag{25}$$

where $\sum_{\substack{n_0<\cdots<n_k\\n_{(.)}\neq i}}^{K-1}$ is a short-hand notation for $\sum_{\substack{n_0=0\\n_0\neq i}}^{K-k-1}\sum_{\substack{n_1=n_0+1\\n_1\neq i}}^{K-k}\cdots\sum_{\substack{n_k=n_{k-1}+1\\n_k\neq i}}^{K-1}$.

$$\mathcal{P}_2 = \sum_{q=0}^{K}\frac{(-1)^q}{q!}\sum_{m_1,\ldots,m_q}^{K}\prod_{z=1}^{q}\left(1+\lambda_{m_z 2}\gamma_{\mathsf{T}}\right)^{-1} \times\exp\left(-\sum_{r=1}^{q}\lambda_{m_r 1}Y\gamma_{\mathsf{T}}\right), \tag{26}$$

where $\sum_{m_1,\ldots,m_q}^{K}$ is a short-hand notation for $\sum_{\substack{m_1=\cdots=m_q=1\\m_1\neq\ldots\neq m_q}}^{K-q-1}$.

$$\mathcal{P}_3 = 1+\sum_{p=0}^{w-1}(-1)^{p+1}\sum_{v_0<\cdots<v_p}^{w-1}\prod_{g=0}^{p}\left(1+\lambda_{((l-w+v_g))_K 2}\gamma_{\mathsf{T}}\right)^{-1} \times\exp\left(-\sum_{u=0}^{p}\lambda_{((l-w+v_u))_K 1}Y\gamma_{\mathsf{T}}\right), \tag{27}$$

where $\sum_{v_0<\cdots<v_p}^{w-1}$ is a short-hand notation for $\sum_{v_0=0}^{w-p-1} \sum_{v_1=v_0+1}^{w-p} \cdots \sum_{v_p=v_{p-1}+1}^{w-1}$.

Up to now, the outage probability can be expressed as

$$P_{\mathsf{out}} \simeq \int_0^\infty F_{\gamma_{\mathsf{D}}}(\gamma|Y)\, f_Y(y)\mathrm{d}y, \tag{28}$$

where $f_Y(y)$ is given by

$$f_Y(y) = \sum_{m=1}^{M} \zeta_{\mathsf{s},m} \exp\left(-\sum_{m=1}^{M} \zeta_{\mathsf{s},m} w\right), \tag{29}$$

where $\zeta_{\mathsf{s},m} = 1/\mu_{\mathsf{s},m}$.

Upon substituting (25), (26), and (27) in (24) and using (28), the outage probability can be evaluated as in (3).

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