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# On the Performance of Multiuser Switched Diversity Systems with Co-channel Interference 

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#### Abstract

We investigate the effect of co-channel interference on the performance of multiuser switched diversity (MUSwiD) systems. In such systems, the first checked user whose channel quality is greater than a predetermined switching threshold is assigned the system resources and allowed to conduct its uplink transmission. In this paper, we derive closed-form expressions for the end-to-end outage probability and symbol error probability (SEP) for the independent non-identically distributed and independent identically distributed cases of desired users channels. In the analysis, all system links are assumed to follow Rayleigh distribution with the interference is being considered at the base station. Approximate expressions for the outage probability and SEP are derived in the high signal-to-noise (SNR) regime which allow for an easy evaluation of the system performance. Monte Carlo simulations are provided to validate the derived analytical and asymptotic expressions. Results show that the system can still achieve performance gain when more desired users are available and the interference power is fixed. This happens at the values of switching threshold that are comparable to average SNR. Asymptotic results illustrate that the system has a diversity order of 1 and a coding gain that is affected by several parameters such as the switching threshold, number of interferers, and the outage threshold. Finally, findings show that the MUSwiD user selection scheme is efficient for systems which operate at the range of low SNR values and this makes it an attractive candidate to be implemented in the emerging mobile broadband communication systems.


[^0]Keywords Multiuser switched diversity • Switching threshold • Rayleigh fading • Co-channel interference

## الخلاصـة

تمت - في هذه الورقة العلمية ـ دراسة نأثير ظاهرة تداخل قنوات المستخدمين في آداء أنظمة الإتصالات اللاسلكية متعدّدة المستخدمين. وفي مثل هذه الأنظمة ، أول مستخدم يتم إيجاده بحيث تكون جودة قناته أعلى من

قيمة عتبة معرّفة مسبقاً ، يُسمح له بالبدء بإرسال معلوماته.
لقد تّمّ حساب صيَيغ جبرية لإحتمال سقوط النظام وإحتمال حدوث خطأ في إستقبال الإشارة للحالتين التاليتين: قنوات مستخدمين مستقلة وغير متجانسة وقنوات مستخدمين مستقلة ومتجانسة. وخلال تحليل آداء النظام تم فرض كاقة قنوات النظام وفقاً لنموذج التنلاشي من نوع Rayleigh ، مع إعتبار وجود تداخل بين القنوات عند المستقبل. بالإضافة إلى ذلك تمت دراسة الآداء عند القيم العالية جدأ لـ SNR ، حيث تمّ حساب صيَيغ جبرية بسيطة وتقريبية لإحتمال سقوط النظام وإحتمال حدوث خطأ في إستقبال Monte- الإشارة المرغوبة. أيضاً ، تمّ التأكدا من صحّة النتائج بمقارنتها بالـ

Carlo simulations
وأظهرت النتائج أن آداء النظام يتحسن بزيادة عدد المستخدمين
المر غوبين ، وأنّ هذا التحسّن بظهر بشكل واضح عنـ وند قيم ال SNR القريبة diversity order من قيمة العتبة المعرّفة مسبقاً. كما أظهرت النتائج أنّ الـ الـ الـ الـ للالنظام هي 1 و أنّ الـ coding gain يعتمد على عدّة عناصر منها: قيمة
 طريقة إختيار المستخدمين المدروسة مناسبة للتطبيق في أنظمة الإتصالات

الدنتقلة الناشئة ذات الطيف العريض.

## 1 Introduction

Multiuser diversity (MUD) is an efficient way in which the system users are allowed to access a shared air-link resources of wireless systems in a dynamic way [1]. Several MUD schedulers that arrange the way the users can access the system resources were presented in the literature, among which is the opportunistic scheduler. In this scheme, the user with

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the best instantaneous channel quality is always assigned the system resources [2]. The design of such scheme is widely studied in the literature, e.g., [3].

In [4], the authors extended the study of [2] to the case of downlink where the opportunistic scheduling was employed. In order to avoid monopolizing the resources by the users having the strongest channel most of the time, a proportional fair scheduling that is based on the relative channel strength was proposed in [5] to exploit multiuser diversity while maintaining fairness among users. A new scheduling scheme was presented in [6] where a trade-off between the multiuser diversity gain and the mobility of users is considered in selecting between users. The channel capacity and average fairness of MUD systems with opportunistic scheduling were derived in [7].

The performance of MUD systems with multiple antennas and opportunistic scheduling where the best user is selected to conduct its downlink transmission was studied in [8,9]. Recently, the opportunistic scheduling along with various adaptive modulation schemes was studied [10]. In that study, the transmission rate or power or both are adapted according to the SNR of the best user. Wang et al. employed in [11] the opportunistic scheduling in which a multi-antenna user selects the best receiver among several receivers to conduct its transmission. In [12, 13], some MUD techniques were recently employed to select among users in cognitive radio networks where the spectrum is shared between primary and secondary users. A paper that studies the fairness, power allocation, and channel-state-information (CSI) quantization in block fading multiuser systems was presented by Makki et al. [14]. The system throughput was derived for a given set of schedulers with different power allocation strategies and various fading distributions.

As a way for reducing the amount of feedback signals from the users to their base stations (BSs), some lowcomplexity scheduling schemes were presented in [15-17]. These schemes are based on the switched diversity combining techniques and known as multiuser switched diversity (MUSwiD) schemes. They proved themselves as efficient techniques in reducing the system complexity compared to the opportunistic scheduling. In such schemes, the users are probed by their BS in a sequential way where the first checked user whose channel quality exceeds a certain switching threshold is assigned the system resources [15]. In [16], Nam et al. studied the performance of MUSwiD schemes where instead of comparing the users channels with one feedback threshold, a set of feedback thresholds was presented with which the users channels are compared. The BS probes the users one after another via a comparison with a set of feedback thresholds, and only a single user has an opportunity to send a feedback at one time and hence being scheduled by the BS to start its data transmission. A study on the performance of such schemes and their effectiveness in reducing
the amount of required CSI feedback load between the users and the central unit was recently proposed in [17]. In [18], the authors proposed an approach to maximize the sum capacity in multiuser switched diversity systems. The per user threshold rate was optimized with some prior estimation of CSI to maximize the system sum capacity. The switched diversity selection scheme was used to select among secondary users in spectrum-sharing networks in [19]. A system similar to that in [19] was also studied in [20] but with adaptive modulation. Most of the existing papers on multiuser switching schemes assumed noise-limited environments and ignored co-channel interference (CCI). As known, the interference is a crucial problem in wireless systems and inherently existed. This motivates us to contribute in this area of research by addressing the effect of CCI on the performance of MUSwiD systems.

In this paper, we study the effect of interference on the behavior of MUSwiD systems. In such systems, the first checked user whose channel quality satisfies a predetermined switching threshold is assigned the system resources by the BS. The paper presents exact closed-form expressions for the outage and symbol error probabilities where the effect of interference and other system parameters such as the switching threshold and number of desired users on the system behavior is provided. Another contribution of the paper is the derivation of approximate expressions for the outage probability, symbol error probability, diversity order, and coding gain at high SNR regime. In this paper, both the independent non-identically distributed (i.n.i.d.) and independent identically distributed (i.i.d.) cases of desired users channels are considered. Also, the scheme of MUSwiD with post-examine selection (MUSwiDps) is presented in this paper. In such scheme, in the case where all users are probed by the BS and found of unacceptable quality, the BS assigns the system resources to the best user among all checked users. Finally, a simple method to calculate approximate but accurate values for the optimum switching threshold is presented in this paper.

## 2 System and Channel Models

The considered system consists of $L$ desired users and a BS where each node is assumed to be equipped with a single antenna. We assume that the signal at the BS is corrupted by interfering signals from $I_{d}$ co-channel interferers $\left\{x_{i}^{I}\right\}_{i=1}^{I_{d}}$. Therefore, the signal at the BS from the $j$ th desired user can be written as
$y_{j}=h_{j} x_{0}+\sum_{i=1}^{I_{d}} h_{i}^{I} x_{i}^{I}+n_{\mathrm{d}}$,
where $h_{j}$ is the channel coefficient between the $j$ th desired user and the BS, $x_{0}$ is the transmitted symbol with $\mathbb{E}\left\{\left|x_{0}\right|^{2}\right\}=$
$P_{0}, h_{i}^{I}$ is the channel coefficient between the $i$ th interferer and the BS, $x_{i}^{I}$ is the transmitted symbol from the $i$ th interferer with $\mathbb{E}\left\{\left|x_{i}^{I}\right|^{2}\right\}=P_{i}^{I}, n_{\mathrm{d}} \sim \mathcal{C N}\left(0, N_{0}\right)$ is an additive white Gaussian noise (AWGN), and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. All the channel gains are assumed to follow Rayleigh distribution. That is, the channel powers denoted by $\left\{\left|h_{j}\right|^{2}\right\}_{j=1}^{L}$ and $\left\{\left|h_{i}^{I}\right|^{2}\right\}_{i=1}^{I_{d}}$ are exponentially distributed random variables ( RVs ) with average values $\sigma_{j}^{2}$ and $\sigma_{I, i}^{2}$, respectively. Using (1), the e2e signal-to-interference plus noise ratio (SINR) at the BS output can be written as
$\gamma_{\mathrm{d}} \triangleq \frac{\frac{P_{0}}{N_{0}}\left|h_{\mathrm{sel}}\right|^{2}}{\sum_{i=1}^{I_{d}} \frac{P_{i}^{I}}{N_{0}}\left|h_{i}^{I}\right|^{2}+1}$,
where $h_{\text {sel }}$ is the channel coefficient between the selected desired user and the BS or the destination. Let $\bar{\gamma}_{j}=\frac{P_{0}}{N_{0}} \sigma_{j}^{2}$, $\bar{\gamma}_{i}^{I}=\frac{P_{i}^{I}}{N_{0}} \sigma_{I, i}^{2}, \lambda_{j}=1 / \bar{\gamma}_{j}$, and $\lambda_{i}^{I}=1 / \bar{\gamma}_{i}^{I}$ denote the average values and parameters of the $j$ th desired user and the $i$ th interferer, respectively.

The MUSwiD scheme works as follows: The SNR of a desired user is compared with a predetermined switching threshold. If it is larger, this user is selected by the BS to conduct its transmission. Otherwise, the examining process continues until a suitable user is found or the last user is reached. In this case, the selection scheme sticks to the last user regardless of its channel quality. In the MUSwiDps scheme, if no desired user is found satisfying the switching threshold and the last user is reached, the scheme goes back to select the best user among all checked users. Again, the main advantage of these schemes is to reduce the required amount of feedback signals from the desired users and the BS each transmission time. In the linear diversity combining techniques such as the opportunistic scheduling, the CSI of all users is required during each transmission time to be able to select among these users. Whereas, in the MUSwiD and MUSwiDps selection schemes, once a user whose SNR satisfies the predetermined switching threshold is found, no need for more feedback signals to be sent from the users to the BS and this reduces the system complexity. To guarantee fairness among users, a centralized feedback collection method can be used to organize the users to be orthogonal when they send feedbacks, such as time division multiplexing (TDM) where users are separated over time [21]. This access method re-arranges the user sequence every scheduling opportunity, so that every user can have the same chance of taking the first place in user sequence over an extended period of time.

## 3 Exact Performance Analysis

In this section, we derive exact closed-form expressions for the outage probability and symbol error probability of the
studied system. In Sect. 3.1, we consider the general case of i.n.i.d. desired users channels in MUSwiD scheme, where the special case of i.i.d. is considered briefly in Sect. 3.2. Also, the MUSwiDps scheme is considered in Sect. 3.3. Section 3.4 provides a simple method to calculate the value of the optimum switching threshold.

### 3.1 MUSwiD with i.n.i.d. Desired Users Channels

In this section, we consider the general case in which the cumulative distribution functions (CDFs) of desired users channels are non-identical. The outage probability is defined as the probability that the e2e SINR goes below a certain threshold $\gamma_{o u t}$ and it is given by

$$
\begin{align*}
P_{\text {out }} & \triangleq \mathrm{P}_{\mathrm{r}}\left[\log _{2}\left(1+\gamma_{\mathrm{d}}\right)<R\right] \\
& =\mathrm{P}_{\mathrm{r}}\left[\gamma_{\mathrm{d}}<\gamma_{\text {out }}\right] . \tag{3}
\end{align*}
$$

Theorem 1 The outage probability for MUSwiD systems with interference is givenfor the case of non-identical desired users channels $\left(\lambda_{j}, j=1, \ldots, L\right)$ and non-identical interferers $\left(\lambda_{i}^{I}, i=1, \ldots, I_{d}\right)$ as

$$
\begin{align*}
& P_{\text {out }}=\prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \quad \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{k=0 k \neq i}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I}}\right. \\
& \quad \times\left(1-\exp \left(-\lambda_{i} \gamma_{\top}\right)\right)+\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}} \\
& \quad \times \prod_{m=0}^{l-1}\left(1-\exp \left(-\lambda_{\left.\left.((j-l+m))_{L} \gamma_{\top}\right)\right)}^{\lambda_{j} \gamma_{o u t}+\lambda_{g}^{I}}\right)\right]
\end{align*}
$$

Proof In evaluating the outage probability, the e2e SINR given in (2) can be first written as a ratio of two RVs $\gamma_{\mathrm{d}}=$ $Y_{1} / Z_{1}$. The CDF of $\gamma_{\mathrm{d}}$ is given by
$\operatorname{Pr}\left[\gamma_{\mathrm{d}}<\gamma_{\mathrm{out}}\right]=\int_{1}^{\infty} f_{Z}(z) \int_{0}^{\gamma_{\mathrm{out}} z} f_{Y}(y) \mathrm{d} y \mathrm{~d} z$.
First, we evaluate the probability density function (PDF) of $Z_{1}=\sum_{i=1}^{I_{d}} \frac{P_{i}^{I}}{N_{0}}\left|h_{i}^{I}\right|^{2}+1=X_{1}+1$. The PDF of $X_{1}$ is given by $f_{X_{1}}(x)=\prod_{i=1}^{I_{d}} \lambda_{i}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(-\lambda_{g}^{I} x\right)}{\prod_{\substack{I_{d}=1 \\ m \neq g}}\left(\lambda_{m}^{I}-\lambda_{g}^{I}\right)}$.

Using the transformation of RVs, we get
$f_{Z_{1}}(z)=f_{X_{1}}(z+1)=\prod_{i=1}^{I_{d}} \lambda_{i}^{I} \sum_{g=1}^{I_{d}} \exp \left(\lambda_{g}^{I}\right) \frac{\exp \left(-\lambda_{g}^{I} z\right)}{\prod_{\substack{m=1 \\ m \neq g}}^{I_{d}}\left(\lambda_{m}^{I}-\lambda_{g}^{I}\right)}$.

The PDF of $Y_{1}=\frac{P_{0}}{N_{0}}\left|h_{\text {sel }}\right|^{2}$ can be written as in [22]
$f_{Y_{1}}(y)=\left\{\begin{array}{l}\sum_{i=0}^{L-1} \pi_{i} f_{\gamma_{i}}(y) \prod_{\substack{k=0 \\ k \neq i}}^{L-1} F_{\gamma_{k}}\left(\gamma_{\top}\right), y<\gamma_{\top} ; \\ \sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}}^{L} \\ \times \prod_{m=0}^{l-1} F_{\gamma_{((j-l+m))_{L}}}\left(\gamma_{\top}\right) f_{\gamma_{j}}(y), y \geq \gamma_{\top},\end{array}\right.$
where $L$ is the number of desired users, $\gamma_{\top}$ is a predetermined switching threshold, $\pi_{i}, i=0, \ldots, L-1$ are the stationary distribution of a $L$-state Markov chain, and it is the probability that the $i$ th user is chosen as given in [22], and $((j-l))_{L}$ denotes $j-l$ modulo $L$. For the detailed derivation of (7), one can refer to [22].

For Rayleigh fading, the $\operatorname{PDF} f_{\gamma_{i}}(y)$ and the $\operatorname{CDF} F_{\gamma_{j}}(y)$ are, respectively, given by $\lambda_{i} \exp \left(-\lambda_{i} y\right)$ and $1-\exp \left(-\lambda_{j} y\right)$. Upon substituting (7) and (6) in (5), we get

$$
\begin{align*}
P_{\mathrm{out}}= & \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{I_{d=1} \\
q \neq g}}^{I_{q}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \\
& \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{\substack{k=0 \\
k \neq i}}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right) \lambda_{i} \int_{0}^{\gamma^{\top}} \exp \left(-\lambda_{i} y\right) \mathrm{d} y\right. \\
& +\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}} \prod_{m=0}^{l-1}\left(1-\exp \left(-\lambda_{((j-l+m))_{L}}^{l} \gamma_{\top}\right)\right) \lambda_{j} \\
& \left.\times \int_{\gamma_{\top}}^{u z} \exp \left(-\lambda_{j} y\right) \mathrm{d} y\right] \mathrm{d} z . \tag{8}
\end{align*}
$$

Upon solving the integrals in (8), we get

$$
\begin{align*}
P_{\mathrm{out}}= & \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{I_{d} \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \\
& \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{\substack{k=0 \\
k \neq i}}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right)\left(1-\exp \left(-\lambda_{i} \gamma_{\top}\right)\right)\right. \\
& +\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}} \prod_{m=0}^{l-1}\left(1-\exp \left(-\lambda_{\left.\left.((j-l+m))_{L} \gamma_{\top}\right)\right)}\right.\right. \\
& \left.\times\left(\exp \left(-\lambda_{j} \gamma_{\top}\right)-\exp \left(-\lambda_{j} u z\right)\right)\right] \mathrm{d} z \tag{9}
\end{align*}
$$

Arranging (9), we get

$$
\begin{align*}
P_{\mathrm{out}}= & \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{\substack{k=0 \\
k \neq i}}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right)\left(1-\exp \left(-\lambda_{i} \gamma_{\top}\right)\right)\right. \\
& \times \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \mathrm{d} z \\
& +\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}}^{\prod_{m=0}^{l-1}\left(1-\exp \left(-\lambda((j-l+m))_{L} \gamma_{\top}\right)\right)} \\
& \times\left(\exp \left(-\lambda_{j} \gamma_{\top}\right) \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \mathrm{d} z\right. \\
& \left.\left.-\int_{1}^{\infty} \exp \left(-\left(\lambda_{j} u+\lambda_{g}^{I}\right) z\right) \mathrm{d} z\right)\right] \tag{10}
\end{align*}
$$

Upon solving the integrals in (10), we get (4).
The symbol error probability can be written as

$$
\begin{align*}
P_{S}= & \int_{0}^{\infty} a Q(\sqrt{2 b \gamma}) f_{\gamma_{D}}(\gamma) \mathrm{d} \gamma \\
& =\frac{a \sqrt{b}}{2 \sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-b \gamma}}{\gamma^{1 / 2}} F_{\gamma_{D}}(\gamma) \mathrm{d} \gamma \tag{11}
\end{align*}
$$

where $Q(\cdot)$ is the Gaussian $Q$-function and $a, b$ are modulation specific constants which are given for the BPSK modulation scheme as $a=0.5$ and $b=1$.

Upon substituting $\gamma_{o u t}=\gamma$ in (4) and then substituting it in (11), we get

$$
\begin{align*}
P_{s}= & \frac{a \sqrt{b}}{2} \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{I_{d} \\
q \neq 1 \\
q \neq g}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{\substack{k=0 \\
k \neq i}}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I}}\right. \\
& \times\left(1-\exp \left(-\lambda_{i} \gamma_{\top}\right)\right) \underbrace{\int_{0}^{\infty} \frac{e^{-b \gamma}}{\gamma^{1 / 2}} \mathrm{~d} \gamma}_{I_{1}}+\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}}^{L} \prod_{m=0}^{l-1} \\
& \times\left(1-\exp \left(-\lambda_{\left.\left.((j-l+m))_{L} \gamma_{\top}\right)\right)\left(\exp \left(-\lambda_{j} \gamma_{\top}\right)\right.}^{\exp \left(-\lambda_{g}^{I}\right)} \lambda_{g}^{I}\right.\right. \\
& -\frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{j}} \underbrace{\left.\left.\int_{0}^{\infty} \gamma^{-1 / 2} \frac{\exp \left(-\left(\lambda_{j}+b\right) \gamma\right)}{\gamma+\lambda_{g}^{I}} \mathrm{~d} \gamma\right)\right] .}_{I_{2}} \tag{12}
\end{align*}
$$

The integrals $I_{1}$ and $I_{2}$ in (12) can be solved with the help of [23, Eq. (3.381.4)] and [23, Eq. (3.383.10)], respectively. Up to now, the symbol error probability for the studied system with non-identical desired users channels $\left(\lambda_{j}, j=1, \ldots, L\right)$ and non-identical interferers $\left(\lambda_{i}^{I}, i=1, \ldots, I_{d}\right)$ can be obtained in a closed-form expression as

$$
\begin{align*}
P_{s}= & \frac{a \sqrt{b}}{2} \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[\sum_{i=0}^{L-1} \pi_{i} \prod_{\substack{k=0 \\
k \neq i}}^{L-1}\left(1-\exp \left(-\lambda_{k} \gamma_{\top}\right)\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I} \sqrt{b}}\right. \\
& \times\left(1-\exp \left(-\lambda_{i} \gamma_{\top}\right)\right)+\sum_{j=0}^{L-1} \sum_{l=0}^{L-1} \pi_{((j-l))_{L}} \\
& \times \prod_{m=0}^{l-1}\left(1-\exp \left(-\lambda_{\left.\left.((j-l+m))_{L} \gamma \top\right)\right)}\right.\right. \\
& \times\left(\exp \left(-\lambda_{j} \gamma_{\top}\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I} \sqrt{b}}-\left(\lambda_{g}^{I} \lambda_{j}\right){ }^{-\frac{1}{2}}\right. \\
& \left.\left.\times \exp \left(\frac{b \lambda_{g}^{I}}{\lambda_{j}}\right) \times \Gamma\left(\frac{1}{2},\left(1+\frac{b}{\lambda_{j}}\right) \lambda_{g}^{I}\right)\right)\right] \tag{13}
\end{align*}
$$

### 3.2 MUSwiD with i.i.d. Desired Users Channels

The outage probability for MUSwiD systems with interference and identical desired users channels can be simply obtained by letting $\left(\lambda_{j}=\lambda_{p}, j=1, \ldots, L\right)$ in (4) as follows

$$
\begin{align*}
& P_{\text {out }}=\prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{I_{d=1} \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \quad \times\left[\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{L} \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I}}+\sum_{j=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{j}\right. \\
& \left.\quad \times\left(\exp \left(-\lambda_{p} \gamma_{\top}\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I}}-\frac{\exp \left(-\left(\lambda_{p} \gamma_{\mathrm{out}}+\lambda_{g}^{I}\right)\right)}{\lambda_{p} \gamma_{\mathrm{out}}+\lambda_{g}^{I}}\right)\right] \tag{14}
\end{align*}
$$

Upon substituting $\gamma_{\text {out }}=\gamma$ in (14) and then substituting it in (11), and with the help of [23, Eq. (3.381.4)] and [23, Eq. (3.383.10)], the symbol error probability for the studied system with identical desired users channels ( $\lambda_{j}=\lambda_{p}, j=1$, $\ldots, L$ ) and non-identical interferers $\left(\lambda_{i}^{I}, i=1, \ldots, I_{d}\right)$ can be obtained in a closed-form expression as

$$
\begin{align*}
P_{s}= & \frac{a \sqrt{b}}{2} \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{L} \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I} \sqrt{b}}\right. \\
& +\sum_{j=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{j} \\
& \times\left(\exp \left(-\lambda_{p} \gamma \top\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I} \sqrt{b}}-\left(\lambda_{g}^{I} \lambda_{p}\right)^{-\frac{1}{2}}\right. \\
& \left.\left.\times \exp \left(\frac{b \lambda_{g}^{I}}{\lambda_{b}}\right) \Gamma\left(\frac{1}{2},\left(1+\frac{b}{\lambda_{p}}\right) \lambda_{g}^{I}\right)\right)\right] \tag{15}
\end{align*}
$$

### 3.3 MUSwiD with Post-Examine Selection

In this section, we evaluate the outage probability and symbol error probability when MUSwiDps is employed by selecting the user with largest SNR for the case of identical desired users channels.

Theorem 2 The outage probability for MUSwiDps systems with interference is given for the case of identical desired users channels $\left(\lambda_{j}=\lambda_{p}, j=1, \ldots, L\right)$ and non-identical interferers $\left(\lambda_{i}^{I}, i=1, \ldots, I_{d}\right)$ as

$$
\begin{align*}
P_{\mathrm{out}} & =\prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[L \sum_{j=0}^{L-1} \frac{\binom{L-1}{j}(-1)^{j}}{(j+1)} \frac{\exp \left(-\lambda_{g}^{I}\right)\left(\lambda_{g}^{I}\right)^{-1}}{\left(1-\exp \left(-(j+1) \lambda_{p} \gamma_{\top}\right)\right)^{-1}}\right. \\
& +\sum_{i=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{i} \\
& \left.\times\left(\exp \left(-\lambda_{p} \gamma_{\top}\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I}}-\frac{\exp \left(-\left(\lambda_{p} \gamma_{\text {out }}+\lambda_{g}^{I}\right)\right)}{\lambda_{p} \gamma_{\mathrm{out}}+\lambda_{g}^{I}}\right)\right] \tag{16}
\end{align*}
$$

Proof In deriving the outage probability, the e2e SINR is first written as $Y_{3} / Z_{1}$, where $Z_{1}$ is as defined in Theorem 1 and $Y_{3}$ is now having a PDF given by [22]
$f_{Y_{3}}(y)= \begin{cases}\sum_{j=0}^{L-1}\left[F_{\gamma}\left(\gamma_{\top}\right)\right]^{j} f_{\gamma}(y), & y \geq \gamma_{\top} ; \\ L\left[f_{\gamma}(y)\right]^{L-1}, & y<\gamma_{\top} .\end{cases}$
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Upon substituting (17) in (5), we get

$$
\begin{align*}
& P_{\mathrm{out}}=\prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \\
& \quad \times\left[L \lambda_{p} \sum_{j=0}^{L-1}\binom{L-1}{j}(-1)^{j} \int_{0}^{\gamma_{\top}} \exp \left(-(j+1) \lambda_{p} y\right) \mathrm{d} y\right. \\
& \left.\quad+\sum_{i=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{i} \lambda_{p} \int_{\gamma_{\top}}^{u z} \exp \left(-\lambda_{p} y\right) \mathrm{d} y\right] \mathrm{d} z \tag{18}
\end{align*}
$$

Solving the integrations, we get

$$
\begin{align*}
& P_{\mathrm{out}}=\prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \\
& \quad \times\left[L \sum_{j=0}^{L-1} \frac{\binom{L-1}{j}(-1)^{j}}{(j+1)}\left(1-\exp \left(-(j+1) \lambda_{p} \gamma_{\top}\right)\right)\right. \\
& \left.\quad+\sum_{i=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma_{\top}\right)\right)^{i}\left(\exp \left(-\lambda_{p} \gamma_{\top}\right)-\exp \left(-\lambda_{p} u z\right)\right)\right] \mathrm{d} z \tag{19}
\end{align*}
$$

Arranging (19), we get

$$
\begin{align*}
P_{\mathrm{out}}= & \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[L \sum_{j=0}^{L-1} \frac{\binom{L-1}{j}(-1)^{j}}{(j+1)}\left(1-\exp \left(-(j+1) \lambda_{p} \gamma \top\right)\right)\right. \\
& \times \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \mathrm{d} z \sum_{i=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma \gamma_{\mathrm{T}}\right)\right)^{i} \\
& \times\left(\exp \left(-\lambda_{p} \gamma \gamma_{\mathrm{T}}\right) \int_{1}^{\infty} \exp \left(-\lambda_{g}^{I} z\right) \mathrm{d} z\right. \\
& \left.\left.-\int_{1}^{\infty} \exp \left(-\left(\lambda_{p} u+\lambda_{g}^{I}\right) z\right) \mathrm{d} z\right)\right] \tag{20}
\end{align*}
$$

Upon solving the integrals in (20), we get (16).

Upon substituting $\gamma_{\text {out }}=\gamma$ in (16) and then substituting it in (11), and with the help of [23, Eq. (3.381.4)] and [23, Eq. (3.383.10)], the symbol error probability for the studied system with identical desired users channels ( $\lambda_{j}=\lambda_{p}$,
$j=1, \ldots, L)$ and non-identical interferers $\left(\lambda_{i}^{I}, i=\right.$ $1, \ldots, I_{d}$ ) can be obtained in a closed-form expression as

$$
\begin{align*}
P_{s}= & \frac{a \sqrt{b}}{2} \prod_{n=1}^{I_{d}} \lambda_{n}^{I} \sum_{g=1}^{I_{d}} \frac{\exp \left(\lambda_{g}^{I}\right)}{\prod_{\substack{q=1 \\
q \neq g}}^{I_{d}}\left(\lambda_{q}^{I}-\lambda_{g}^{I}\right)} \\
& \times\left[\frac{L}{\sqrt{b}} \sum_{j=0}^{L-1} \frac{\binom{L-1}{j}(-1)^{j}}{(j+1)} \frac{\exp \left(-\lambda_{g}^{I}\right)\left(\lambda_{g}^{I}\right)^{-1}}{\left(1-\exp \left(-(j+1) \lambda_{p} \gamma_{\top}\right)\right)^{-1}}\right. \\
& +\sum_{i=0}^{L-1}\left(1-\exp \left(-\lambda_{p} \gamma \tau\right)\right)^{i}\left(\exp \left(-\lambda_{p} \gamma \tau\right) \frac{\exp \left(-\lambda_{g}^{I}\right)}{\lambda_{g}^{I} \sqrt{b}}\right. \\
& \left.\left.-\exp \left(\frac{b \lambda_{g}^{I}}{\lambda_{p}}\right) \Gamma\left(\frac{1}{2},\left(1+\frac{b}{\lambda_{p}}\right) \lambda_{g}^{I}\right)\right)\right] \tag{21}
\end{align*}
$$

### 3.4 Simple Method to Calculate Optimum Switching Threshold

Finding the optimum switching threshold in a numerical way is hard to be implemented practically as it increases the complexity of the system and reduces the battery life of mobile stations. Alternatively, we present here a simple method to calculate approximate values for the optimum switching threshold. As the selection process among users is conducted in MUSwiD and MUSwiDps schemes based on the average values of the desired users channels, approximate values for the optimum switching threshold can be calculated by maximizing the average value of SNR at the selection scheme combiner output. In this way, the optimum switching threshold is calculated to maximize the numerator of SINR in (2), and hence, maximizing the e2e SINR. This makes sense as having information about the interferers channels is usually hard to be achieved in wireless systems. The average SNR at the output of the MUSwiD user selection scheme with assuming identical desired users channels $\left(\bar{\gamma}_{j}=\bar{\gamma}, j=1, \ldots, L\right)$ is given by

$$
\begin{align*}
\bar{\gamma}_{\text {MUSwiD }}= & \int_{0}^{\infty} x f_{Y_{2}}(x) \mathrm{d} x \\
= & \sum_{j=0}^{L-2}\left[F_{\gamma}\left(\gamma_{\top}\right)\right]^{j} \int_{\gamma_{T}}^{\infty} x f_{\gamma}(x) \mathrm{d} x \\
& +\left[F_{\gamma}\left(\gamma_{\top}\right)\right]^{L-1} \int_{0}^{\infty} x f_{\gamma}(x) \mathrm{d} x . \tag{22}
\end{align*}
$$

Using Leibnitz's rule, the derivative of (22) with respect to $\gamma_{T}$ is set to zero $\frac{d \bar{\gamma}_{\text {MUSwiD }}}{d \gamma_{T}}=0$, which can be written under the assumption of Rayleigh fading channels as

$$
\begin{align*}
& \sum_{j=0}^{L-2}\left[1-\exp \left(-\frac{2 \gamma_{T}}{\bar{\gamma}}\right)\right]^{j-1} \\
& \quad \times\left\{j \exp \left(-\frac{2 \gamma_{T}}{\bar{\gamma}}\right)-\frac{2 \gamma_{T}}{\bar{\gamma}}+2(j+1) \frac{\gamma_{T}}{\bar{\gamma}} \exp \left(-\frac{2 \gamma_{T}}{\bar{\gamma}}\right)\right\} \\
& \quad+(L-1)\left[1-\exp \left(-\frac{2 \gamma_{T}}{\bar{\gamma}}\right)\right]^{L-2}=0 \tag{23}
\end{align*}
$$

where finding $\gamma_{\top}$ as a function of $\bar{\gamma}$ and $L$ is now our goal. Unfortunately, the solution of (23) cannot be obtained as a closed form, but it has only a single root and a simple numerical search is possible to find the root. With representing $\gamma \top$ as $\alpha \bar{\gamma}$, the goal now can be rephrased as finding $\alpha$ as a function of $\bar{\gamma}$ and $L$. Substituting $\gamma_{\top}=\alpha \bar{\gamma}$ into (22), the average output SNR based on the switching threshold maximizing the output SNR can be given in a simple closed form as
$\bar{\gamma}_{\text {MUSwiD }}=\bar{\gamma}\left\{\alpha+1-\alpha(1-\exp (-\alpha))^{L-1}\right\}$.
Now, dealing with the result in (24) is very simple. For any values of $\bar{\gamma}$ and $L$, the value of $\alpha$, and hence, $\gamma_{\top}$ that maximizes $\bar{\gamma}_{\text {MUSwiD }}$ can be easily obtained. The same approach can be followed in the case of MUSwiDps selection scheme. For more details on this method of finding approximate values for optimum switching threshold, one can refer to [24].

## 4 Asymptotic Performance Analysis

Due to complexity of the derived expressions, it is hard to get more insights on the system performance. To simplify these expressions and to get more insights on the system performance, we study the system behavior at high SNR regime. In the following analysis, we assume the diversity branches as well as the interferers channels to be identical. Furthermore, the number of interferers $I_{d}$ and the interferers power $\bar{\gamma}^{I}$ are assumed to be constant, i.e., we assume high SNR not SINR since the interference power is fixed.

### 4.1 Outage Probability

At high SNR values, the outage probability can be expressed as $P_{\text {out }} \approx\left(G_{\mathrm{C}} \mathrm{SNR}\right)^{-G_{\mathrm{d}}}$, where $G_{\mathrm{C}}$ denotes the coding gain of the system and $G_{\mathrm{d}}$ is the diversity order of the system. Obviously, $G_{\mathrm{C}}$ represents the horizontal shift in the outage probability performance relative to the benchmark curve $(\mathrm{SNR})^{-G_{\mathrm{d}}}$ and $G_{\mathrm{d}}$ refers to the increase in the slope of the outage probability versus SNR curve [25, Ch. 14]. Therefore, having an approximate expression for the outage probability at high SNR values allows to derive the system diversity order and coding gain and allows for an easy evaluation of the system performance. The parameters on which the diversity
order depends will affect the slope of the outage probability curves, and the parameters on which the coding gain depends will affect the position of the curves. At high SNR values, the exponential CDF and PDF can be, respectively, approximated by $F_{\gamma}(\gamma) \approx \lambda_{p} \gamma$ and $f_{\gamma}(\gamma) \approx \lambda_{p}$.

In deriving the outage probability for the MUSwiD scheme, the e2e SINR is first written as $Y_{2} / Z_{2}$, where $Y_{2}$ is now having a PDF given by [22]
$f_{Y_{2}}(y)= \begin{cases}{\left[F_{\gamma}\left(\gamma_{\top}\right)\right]^{L-1} f_{\gamma}(y),} & y<\gamma_{\top} ; \\ \sum_{j=0}^{L-1} f_{\gamma}(y)\left[F_{\gamma}\left(\gamma_{\top}\right)\right]^{j}, & y \geq \gamma_{\top} .\end{cases}$
and $Z_{2}$ is now consisting of i.i.d. exponential RVs, i.e., $Z_{2}=$ $\sum_{i=1}^{I_{d}} \frac{P^{I}}{N_{0}}\left|h_{i}^{I}\right|^{2}+1=X_{2}+1$ with a PDF of $X_{2}$ given by
$f_{X_{2}}(x)=\frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} x^{I_{d}-1} \exp \left(-\lambda^{I} x\right)$.
Using the transformation of RVs and then the Binomial rule, the PDF of $Z_{2}$ can be obtained as

$$
\begin{align*}
f_{Z_{2}}(z)=- & \frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} \exp \left(\lambda^{I}\right)(-1)^{I_{d}} \\
& \times \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g} z^{g} \exp \left(-\lambda^{I} z\right) . \tag{27}
\end{align*}
$$

Upon substituting the approximated statistics of the exponential PDF and CDF in (25) and then substituting the resulting PDFs of $Y_{2}$ and $Z_{2}$ in (5), and after some algebraic manipulations, the outage probability can be obtained at high SNR as

$$
\begin{align*}
P_{\text {out }} \approx- & \frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} \exp \left(\lambda^{I}\right)(-1)^{I_{d}} \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g} \lambda_{p} \\
& \times\left[\left(\lambda_{p}\right)^{L-1}\left(\gamma_{\top}\right)^{L} \frac{\Gamma\left(g+1, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+1}}+\sum_{j=0}^{L-1}\left(\lambda_{p} \gamma_{\top}\right)^{j}\right. \\
& \left.\times\left(\frac{\Gamma\left(g+2, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+2}} \gamma_{\text {out }}-\frac{\Gamma\left(g+1, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+1}} \gamma_{\top}\right)\right] \tag{28}
\end{align*}
$$

From (28), we can notice that the second term in the large bracket is dominating the total result and it is still dominant for the first term of summation $j=0$, and hence, (28) simplifies to

$$
\begin{align*}
P_{\text {out }} \approx- & \frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} \exp \left(\lambda^{I}\right)(-1)^{I_{d}} \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g} \lambda_{p} \\
& \times\left(\frac{\Gamma\left(g+2, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+2}} \gamma_{\text {out }}-\frac{\Gamma\left(g+1, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+1}} \gamma_{\top}\right) \tag{29}
\end{align*}
$$

With noticing that $\mathrm{SNR}=1 / \lambda_{p}$, the outage probability in (29) can be written in a more compact form as

$$
\begin{align*}
P_{\mathrm{out}} & \approx\left(\left\{\Xi_{1} \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g}\right.\right. \\
& \left.\left.\times\left(\frac{\Gamma\left(g+2, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+2}} \gamma_{\mathrm{out}}-\frac{\Gamma\left(g+1, \lambda^{I}\right)}{\left(\lambda^{I}\right)^{g+1}} \gamma_{\top}\right)\right\}^{-1} \mathrm{SNR}\right)^{-1} . \tag{30}
\end{align*}
$$

where $\Xi_{1}=-\frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} \exp \left(\lambda^{I}\right)(-1)^{I_{d}}$.
Following the same procedure with the MUSwiDps scheme, the same result in (30) has been achieved. As can be seen from (30), the coding gain of the MUSwiD and MUSwiDps schemes is affected by several parameters as $\lambda^{I}$, $I_{d}, \gamma_{\top}$, and $\gamma_{\text {out }}$ Also, it is clear that the diversity order of the two schemes is fixed and equals 1.

### 4.2 Symbol Error Probability

Upon substituting $\gamma_{\text {out }}=\gamma$ in (29) and then substituting it in (11), and with the help of [23, Eq. (3.351.3)], the symbol error probability for MUSwiD systems can be obtained at high SNR as

$$
\begin{align*}
P_{s} & \approx-\frac{a \sqrt{b}}{2 \sqrt{\pi}} \frac{\left(\lambda^{I}\right)^{I_{d}}}{\left(I_{d}-1\right)!} \exp \left(\lambda^{I}\right)(-1)^{I_{d}} \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g} \lambda_{p} \\
& \times\left(\frac{\Gamma\left(\frac{3}{2}\right) \Gamma\left(g+2, \lambda^{I}\right)}{b^{\frac{3}{2}}\left(\lambda^{I}\right)^{g+2}}-\frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(g+1, \lambda^{I}\right)}{b^{\frac{1}{2}}\left(\lambda^{I}\right)^{g+1}} \gamma \tau\right) . \tag{31}
\end{align*}
$$

With noticing that $\operatorname{SNR}=1 / \lambda_{p}$, the symbol error probability in (31) can be written in a more compact form as

$$
\begin{align*}
P_{s} & \approx\left(\left\{\Xi_{2} \sum_{g=0}^{I_{d}-1}\binom{I_{d}-1}{g}(-1)^{g}\right.\right. \\
& \left.\left.\times\left(\frac{\Gamma\left(\frac{3}{2}\right) \Gamma\left(g+2, \lambda^{I}\right)}{b^{\frac{3}{2}}\left(\lambda^{I}\right)^{g+2}}-\frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(g+1, \lambda^{I}\right)}{b^{\frac{1}{2}}\left(\lambda^{I}\right)^{g+1}} \gamma^{\top}\right)\right\}^{-1} \mathrm{SNR}\right)^{-1} . \tag{32}
\end{align*}
$$

where $\Xi_{2}=-\frac{a \sqrt{b}}{2 \sqrt{\pi}} \Xi_{1}$.
Following the same procedure with the MUSwiDps scheme, the same result in (32) has been achieved. Again, it is obvious from (32) that the coding gain of the MUSwiD and MUSwiDps schemes is affected by several parameters as $\lambda^{I}, I_{d}, \gamma_{\mathrm{T}}$, and $\gamma_{\text {out }}$ and that the diversity order of the two schemes is fixed and equals 1 . Referring to the way the MUSwiD and MUSwiDps selection schemes work, the gain achieved in system performance due to increasing $L$
and having more desired users happens at the values of SNR that are comparable to $\gamma_{\top}$. As in the case where the values of SNR that are comparable to $\gamma_{\Gamma}$, the switching rate will increase and the probability of having users with better channels increases also. In other words, when the SNRs of users are much smaller than the switching threshold, all the users are unacceptable most of the time and hence having more users will add no gain to the system performance. Also, when the SNRs of users are much larger than the switching threshold, all the users are acceptable most of the time, and hence, the first checked user will be assigned the channel, and thus, having more users will have no effect on the system performance. At the same time, as the asymptotic analysis is done at high SNR values which means the SNRs of users are much greater than $\gamma_{\top}$, it is expected to have most of the users being acceptable the whole time, and thus, the first selected user is being selected in the two selection schemes. It is worthwhile to mention here that the MUSwiDps scheme outperforms the conventional MUSwiD scheme only in the case where more users are available with $\gamma_{\top}$ is much larger than the average SNR. This is because, when $\gamma_{T}$ is much larger than the average SNR, all the available users will be examined and the MUSwiD scheme selects the last checked user; whereas the MUSwiDps scheme selects the best user among the examined ones. Finally, having the diversity order of the MUSwiD and MUSwiDps schemes being 1 makes the implementation of these schemes inefficient at the systems which operate at the range of high SNR values. Therefore, in order not to loose much in the diversity order of the system and at the same time to get benefit from the simplicity of these two schemes when they are implemented, these schemes are seen to be efficient for systems that work at the range of low SNR values. This makes them attractive options for practical implementation in emerging mobile broadband communication systems.

## 5 Numerical Results

In this section, we illustrate the validity of the derived analytical results via a comparison with Monte Carlo simulations. Also, we give some numerical examples to show the effect of interference and other parameters such as number of desired users on the system performance. Furthermore, a figure is assigned to illustrate the effectiveness of the studied MUSwiD and MUSwiDps user selection schemes in reducing the system complexity compared to the opportunistic scheduling.

It is clear from Fig. 1 that the achieved analytical and asymptotic results perfectly fit with Monte Carlo simulations. Also, it can be seen that the MUSwiD scheme has nearly the same performance as the opportunistic or best user selection scheme for very low SNR region; whereas, as we go further in increasing SNR, the best user selection is clearly


Fig. 1 Outage probability versus SNR for MUSwiD scheme with interference for different values of $L$
outperforming the MUSwiD scheme, as expected. In addition, it is obvious from this figure that for the MUSwiD as $L$ increases, the system performance becomes more enhanced; especially, at the range of SNR values that are comparable to $\gamma_{\top}$. More importantly, for $L=2,4$, and 6 , it is clear that at both low and high SNR values, all curves asymptotically converge to the same behavior and no gain is achieved in system performance with increasing $L$. This is expected since when $\gamma_{\top}$ takes values much greater than SNR, the channels of most users will be unacceptable most of the time and having more desired users will not affect the performance. Also, when $\gamma_{\top}$ takes values much smaller than SNR, the channels of most users will be acceptable most of the time and the first examined user will be chosen to start its data transmission. This exactly matches the asymptotic or high SNR results where the diversity order of the system is constant at 1 and the coding gain is independent of the number of users $L$. Finally, it is worthwhile to mention here that the only value of outage threshold that results in a zero outage probability is $\gamma_{\text {out }}=0$. Otherwise, any continue in decreasing $\gamma_{\text {out }}$ will lead to a lower value in the outage probability but will never make it zero.

Figure 2 studies the effect of interference on the error probability performance of MUSwiD scheme for different numbers of interferers $I_{d}$. It is clear from this figure that as $I_{d}$ increases, the system performance is more degraded with the worst performance achieved at the highest value of $I_{d}$. Also, one can notice from this figure that increasing $I_{d}$ degrades the system performance through affecting its coding gain without affecting the diversity order. Finally, as the interference power is assumed to scale with SNR, a noise floor appears in all curves of this figure, and hence, a zero diversity gain is achieved by the system. This is a


Fig. 2 Symbol error probability versus SNR for MUSwiD scheme with interference for different values of $I_{d}$ when interference power scales with SNR


Fig. 3 Outage probability versus $\bar{\gamma}^{I}$ for MUSwiD and MUSwiDps schemes with interference for different values of $\gamma$ out
normal result for the effect of interference on the system performance.

Figure 3 compares the outage performance of the MUSwiD and MUSwiDps schemes. It is clear from this figure that the MUSwiD and MUSwiDps user selection schemes have exactly the same performance and that there is no performance gain achieved when the MUSwiDps scheme is used compared to MUSwiD scheme. This is because of the effect of interference on the behavior of these two selection schemes. Also, the effect of outage threshold $\gamma_{\text {out }}$ on the system performance is obvious in this figure where as $\gamma_{o u t}$ increases, worse the achieved performance.

The outage performance of MUSwiDps user selection scheme is plotted versus SNR in Fig. 4 for different values of

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Fig. 4 Outage probability versus SNR for MUSwiDps scheme with interference for different values of $\bar{\gamma}^{I}$


Fig. 5 Outage probability versus $\gamma_{o u t}$ for MUSwiD and MUSwiDps schemes with interference for different values of SNR
interference power $\bar{\gamma}^{I}$. It is clear from this figure that more the interference power, the worse the achieved performance, as expected. Also, a perfect fitting between the analytical and the asymptotic results is obvious in this figure. Finally, as the interference power is assumed to be fixed in each curve of this figure, the system continues in achieving performance gain as SNR keeps increasing and no noise floor appears in such case.

Figure 5 compares the outage performance of the MUSwiD and MUSwiDps selection schemes. The figure is generated for different values of SNR. It is obvious from this figure that as the outage threshold $\gamma_{\text {out }}$ increases, greater the outage probability, and hence, worse the system performance, as expected. Also, the figure shows that the MUSwiD and MUSwiDps selection schemes exactly behave the same and


Fig. 6 Outage probability versus $\gamma_{\top}$ for MUSwiD and MUSwiDps schemes with interference for different values of $L$
this is due to the effect of interference on the operation of these user selection schemes.

Figure 6 studies the effect of the switching threshold $\gamma_{\top}$ and the number of desired users $L$ on the outage performance of the system. In the MUSwiD selection scheme, increasing $L$ leads to a significant gain in system performance, especially in the medium SNR region. On the other hand, as $\gamma_{\top}$ becomes much smaller or much larger than the average SNR, the performance improvement decreases, as all curves asymptotically converge to the case of two desired users. This is due to the fact that, if the average SNR is very small compared to $\gamma_{\mathrm{T}}$, all the desired users will be unacceptable most of the time. On the other hand, if the average SNR is very high compared to $\gamma_{\mathrm{T}}$, all the desired users will be acceptable and one user will be used most of the time. Thus, in both cases, the additional desired users will not lead to any gain in the system behavior. On the other hand, the MUSwiDps selection scheme gives the same performance as the MUSwiD scheme in the region where $\gamma_{\top}$ is much smaller than the average SNR , as expected; whereas in the region where $\gamma_{\top}$ is much larger than the average SNR, the MUSwiDps gives better performance compared to the MUSwiD scheme. This is because in the MUSwiDps scheme, when the last desired user is reached and found unacceptable, the scheme selects the best user among all checked users in contrast to the MUSwiD scheme which in this case sticks to that last user. This explains the gain achieved by the MUSwiDps scheme over the MUSwiD scheme in this region of $\gamma_{\mathrm{T}}$.

Figure 7 studies the effect of interference on the error probability performance of the MUSwiD scheme. The figure is generated for different numbers of interferers $I_{d}$. It is obvious from this figure that as $I_{d}$ increases, worse the achieved behavior. Also, a perfect fitting between the asymptotic and analytical results of the symbol error probability is


Fig. 7 Symbol error probability versus SNR for MUSwiD scheme with interference for different values of $I_{d}$ when interference power is fixed


Fig. 8 Outage probability versus $L$ for MUSwiD scheme with interference for different values of $\bar{\gamma}{ }^{I}$
clear in this figure. Again, this figure shows that the number of interferers degrades the system performance via reducing the coding gain without affecting the diversity order of the system.

The outage performance of the MUSwiD scheme is portrayed versus number of desired users $L$ in Fig. 8 for different values of interference power $\bar{\gamma}^{I}$. It is clear from this figure that at the case where the interference power is not scaling with SNR, the system can still achieve performance gain as more desired users are available. This gain is more noticeable at the smaller values of $\bar{\gamma}^{I}$, as expected. Also, we can notice from this figure that the gain achieved in system performance becomes smaller as $L$ keeps increasing where a noise floor can be seen in the results.


Fig. 9 Average number of channel estimations versus $\gamma_{\top}$ for the MUSwiD and MUSwiDps user selection schemes in comparison with the opportunistic selection with $L=4$ and $\bar{\gamma}_{p}=10 \mathrm{~dB}$

The complexity of the MUSwiD and MUSwiDps user selection schemes in terms of average number of channel estimations and in comparison with the opportunistic or best user selection scheme is illustrated in Fig. 9. We can see from this figure that as the quality of desired users channels is required for its operation, opportunistic selection scheme is always of need for 4 channel estimations. On the other hand, the MUSwiD user selection scheme needs to estimate at most 3 channels because when these channels are found unacceptable, the last checked user will be used at the destination. Therefore, the MUSwiD scheme requires less path estimations than the MUSwiDps selection scheme. Also, we can notice from this figure that as $\gamma_{\top}$ increases, the average number of channel estimations of users increases since it is more difficult to find a user with an acceptable channel quality.

## 6 Conclusion

In this paper, exact and asymptotic e2e outage and symbol error probabilities were derived for multiuser switched diversity systems in the presence of interference. Two multiuser switching schemes were studied: the MUSwiD and MUSwiD with post-examine selection. Monte Carlo simulations proved the accuracy of the achieved analytical and asymptotic results. Findings illustrated that for fixed interference power, the system can still achieve performance gain when more desired users are available. This gain is noticeable in the range of SNR values that are comparable to the switching threshold. Also, results showed that the diversity order of the system with the two selection schemes is con-

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stant and equals 1 and that the two schemes have the same coding gain. Furthermore, findings illustrated that the gain achieved in system performance due to having more desired users becomes smaller as the interference power continues in increasing. Finally, results showed that the studied selection schemes are efficient for systems which operate at low SNR values and this makes them an attractive candidate to be implemented in the emerging mobile broadband communication systems.

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