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Performance of CDMA-Based Multi-hop Wireless Networks in Nakagami Fading

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Abstract In this paper, we derive the error probability of multi-hop networks employing code-division multiple access transmission in a multiuser environment. The standard Gaussian approximation is used to model the multiple access interference in the network. We consider multi-hop networks employing serial, parallel, and selection relaying using the decode-and-forward technique. For the selection relaying, we consider two schemes; namely, the threshold-based relay selection and the maximum relay selection. In addition, we investigate the effect of different system parameters on the error performance of the system, such as the spreading gain, the number of interfering users, and the path loss exponent. Results show that as the number of users increases, the improvement coming from the relaying decreases.

Keywords CDMA · Spread spectrum · Error probability · Rayleigh · Nakagami · Fading · Multiple access · Interference · Multi-hop · Relaying

الخلاصة

تم - في هذه الورقة العلمية - القيام بحساب احتمال وقوع خطأ في الإرسال الناتج عند استخدام شبكات اتصال متعددة القفزات مع تقنية إرسال (CDMA) في بيئة متعددة المستخدمين.

لقد تم استخدام نظام التقريب الجاوسي القياسي (SGA) لصناعة أنموذج للتداخل الناتج من تعددية الدخول (MAI) في شبكة الاتصال. وتم اختبار شبكات متعددة القفزات تستخدم طرق ترحيل على التوالي أو التوازي أو اختيار الترحيل مع استخدام المرحلين لتقنية الفك والإرسال. وعند استخدام اختيار الترحيل تم اختبار طريقتين مختلفتين، هما الاختيار عن طريق الحد (TRS) والاختيار عن طريق الحد الأقصى (MRS).

وقد تمت - إلى جانب ذلك - دراسة تأثير عدد من متغيرات النظام في احتمال وقوع خطأ الإرسال لشبكة الاتصال مثل عامل كسب الانتشار، وعدد المستخدمين المتداخلين ومعامل فقد المسار. وقد قد أظهرت النتائج أن الكسب الناتج من الترحيل يقل مع زيادة عدد المستخدمين.

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1 Introduction

Next generation wireless networks are expected to provide services that need high data rates with very high bandwidth efficiency compared to current systems. As the number of wireless terminals increases, higher system capacity is needed to provide the high data rate required by all these terminals. It is not practical to install more base stations to serve the large number of terminals, especially when these terminals are located in a pico-cell environment. In addition, the multipath fading arises from the reflection of the transmitted signal over many objects in the environment surrounding the transmitter and receiver. The multipath fading increases the number of errors in the transmission. The effects of multipath fading can be combated through the use of diversity.

In diversity, several replicas of the message are transmitted over orthogonal channels to reduce the multipath fading effect. While multiple-input multiple-output (MIMO) schemes improve the network capacity greatly [1] at large hardware complexity, a distributed MIMO scheme known as cooperative communication is introduced [2–6]. This technique allows single-antenna mobile units to get some of the benefits of MIMO systems in a distributed manner. The basic idea is that single-antenna mobile units in a multi-user scenario can share their antennas in a manner that creates a virtual MIMO system. Furthermore, terminals can relay signals for other terminals without the need for base stations. Such a network is referred to as a multi-hop network, which is the main theme for this paper [7,8].

Code division multiple access (CDMA) is a multiple access scheme utilized by various radio communication technologies. CDMA employs spread-spectrum (SS) technology in which each user is assigned a pseudo-random noise (PN) code so that multiple users are multiplexed over the same physical channel. In CDMA networks, multiple access interference (MAI) arises due to time jitter, random delays or the imperfect orthogonality of the PN codes used. At any given time, a subset of these users may transmit information simultaneously over a common channel to corresponding receivers.

Hasna et al. [9] focused on two-hop wireless networks and derived the end-to-end error performance over independent Rayleigh fading channels. Hasna [10] considered an N-hop series of non-regenerative relaying model. He derived an expression and an upper bound for the end-to-end signal-to-noise ratio (SNR) of the N-hop network. The derived expression was used to evaluate the average SNR, the amount of fading, and average bit error probability (BEP) of the system. Hasna et al. [11] studied the performance of a two-hop wireless networks with amplify-and-forward (AF) relays having fixed gains in terms of outage probability and average BEP. They proposed a relaying scheme in which the relays select their gains based on the first hop's average SNR. Yang et al. [12] presented a closed-form expression for the average outage duration of multi-hop networks employing decode-and-forward (DF) relays over different fading channel models, namely, "Rayleigh, Rician, and Nakagami". The average outage duration expression was used to compare between direct and relayed transmission schemes over a Rayleigh fading channel. Sunay et al. [13] showed how to calculate the error probabilities for various CDMA systems using the standard Gaussian approximation (SGA), improved Gaussian approximation (IGA), and Fourier series (FS) based schemes. Zhao et al. [14] investigated the performance of coding-spreading tradeoff in CDMA systems when using rate-compatible punctured turbo and rate-compatible punctured convolutional codes.

In [15], the authors investigated two different multi-hop bi-directional relay transmission schemes. These two schemes were based on AF relaying technique and analog network coding (ANC). In the first scheme, which they called the AF-ANC-central scheme, AF is employed at each of the intermediate nodes, while ANC is only utilized at the central relay. While in the second scheme, referred to as the AF-ANC-even scheme, they proposed that the even numbered relays perform ANC while the odd numbered relays only perform subtraction and AF. In addition, for reference purpose, they also considered the AF-No-ANC scheme, where the intermediate relays only perform AF and no ANC is employed anywhere. Lower bounds on BEP of these three schemes were obtained and verified by Monte Carlo simulations to be asymptotically tight ones at high SNRs. Li et al. [16] Proposed a new repetition-based multi-hop transmission system with non-coherent detection. They derived in the paper the symbol error probability (SEP) of the proposed system for both AF and DF relaying schemes. They showed that the repetition-based non-coherent AF relaying achieves a diversity order equal to the number of repetitions, while the repetition-based non-coherent DF relaying does not achieve any diversity order. In addition, they have evaluated the throughput of the proposed system. As a final conclusion, they showed that repetition does not enhance the throughput of the DF relaying systems, whereas it improves the throughput of the AF relaying systems for small and moderate values of SNR.

Rong et al. [17] considered multiuser multi-hop MIMO relay communication systems over correlated MIMO fading channels. They considered fast fading channel where the instantaneous channel state information (CSI) is only available at the destination node, but unknown at all users and all relay nodes. They derived the structure of the optimal user pre-coding matrices and relay amplifying matrices which maximize the users-destination ergodic sum mutual information. Their results can be considered more general than most of the other works, since they addressed multiuser scenarios, considered MIMO relays with a finite dimension, and taken into account the noise vector at each relay node. In a previous work of the authors [18], they derived the end-to-end error probability of multi-hop networks employing CDMA transmission in a multiuser environment over Rayleigh fading channels. The SGA is used to model the MAI in the system. They considered multi-hop networks employing serial, parallel, and selection relaying using DF technique. In addition, they investigated the effect of the number of users and the path loss exponent on the performance of the system. they have showed in the results that as the number of users increases, the improvement coming from the relaying decreases. However, relaying still gives significant improvement, especially in parallel relaying.

Up to the authors knowledge, there has been no research that investigates the relaying performance in CDMA networks under Nakagami fading in an analytical way. In this paper, we analyze the performance of a multi-hop network employing CDMA on the physical layer over Nakagami fading channels. In particular, we derive the error performance of multi-hop CDMA-based networks employing serial, parallel and selection relaying over different values of spreading gain and number of users. For simplicity, we use the SGA approach to model the MAI in the CDMA network. The rest of this paper is organized as follows. Section 2 describes the CDMA-based multi-hop network. The performance analysis of CDMA-based multi-hop networks is presented in Sect. 3. In Sect. 4, we discuss the numerical results obtained from simulation and analysis. Finally, Sects. 5 and 6 present complexity discussion and the main conclusions of this work.

2 System Model

2.1 CDMA System

In this paper, we consider multi-hop wireless networks employing CDMA transmission in the physical layer. The CDMA tranmitter is composed of a PN code generator and a binary phase-shift keying (BPSK) modulator, whereas the receiver is composed of a BPSK demodulator and a correlator that uses the same PN code of the transmsitter. Note that BPSK is being considered to make the analysis focused on the effect of multi-hop relaying. We assume that the information rate at the input to the modulator is R bits/s, the available channel bandwidth is W Hz, where R is much smaller than W. To utilize the entire available channel bandwidth, the phase of the carrier is shifted pseudorandomly according to the PN code at a rate W times/s. The reciprocal of W, denoted by T_{c}, defines the duration of a rectangular pulse, called a chip, which constitutes the basic element in the CDMA signal. Define $T_{\mathrm{b}} = \frac{1}{R}$ to be the transmission time of an information bit, the bandwidth expansion factor or the spreading gain V can be expressed as

$$V = \frac{W}{R} = \frac{T_{\mathrm{b}}}{T_{\mathrm{c}}}. \tag{1}$$

In practical systems, the spreading gain is an integer, which is the number of chips per information bit. At the transmitter side of each user, the information bits are multiplied by the PN code of the user at the modulator. The signal model of the transmitted signal for user k can be written as

$$s_k(t) = \sqrt{2P_k}\, b_k(t) c_k(t) \cos(\omega_{\mathrm{c}} t + \theta_k), \tag{2}$$

where P_k is the power of the transmitted signal of user k, $b_k(t)$ and $c_k(t)$ are the data and spreading signals of user k, respectively, ω_{c} is the signaling frequency and θ_k is the phase shift of user k.

Assuming that there are K active users in the network, the total received signal at any of the receivers can be expressed as

$$\begin{aligned} r(t) = &\sum_{k=1}^{K} \sqrt{2P_k d_k^{-n}}\, \alpha_k b_k(t-\tau_k) c_k(t-\tau_k) \\ &\times \cos(\omega_{\mathrm{c}} t + \phi_k) + \eta(t), \end{aligned} \tag{3}$$

where d_k is the distance from user k to the receiver, n is the path loss exponent, τ_k and ϕ_k are the phase and time delay introduced by the channel of user k, and $\eta(t)$ is the additive white Gaussian noise (AWGN) introduced at the receiver. Here, α_k is the fading component from user k to the receiver modeled as a Nakagami random variable (r.v.) [19]. At each user, a correlation receiver is typically used to filter the desired user's signal from all the other users' signals which share the same channel. Assuming the receiver has perfect estimate about the delay of the signal of user 1, the decision statistics for user 1 are given by [13]

$$\begin{aligned} Z_1 &= \int_0^{T_{\mathrm{b}}} r(t) c_1(t) \cos(\omega_{\mathrm{c}} t) \mathrm{d}t \\ &= \sqrt{\frac{1}{2} P_1 T_{\mathrm{b}}^2 d_1^{-n}}\, \alpha_1 b_1 \\ &\quad + \int_0^{T_{\mathrm{b}}} \sum_{k=2}^{K} \sqrt{2P_k d_k^{-n}}\, \alpha_k b_k(t) c_k(t) c_1(t) \\ &\quad \times \cos(\omega_{\mathrm{c}} t) \cos(\omega_{\mathrm{c}} t + \phi_k) \mathrm{d}t + \hat{\eta}, \end{aligned} \tag{4}$$

where b_1 is the information data of user 1, and $\hat{\eta}$ is an AWGN r.v. with variance equal to $N_0 T_{\mathrm{b}}/4$. The first term in (4) is the desired signal component, whereas the second term constitutes the MAI component, and the third part is the AWGN noise. In our analysis and for simplicity, we assume that all interfering users are located at the same distance from the destination, i.e., $d_k = d, k = 1, \ldots, K$.

2.2 Multi-hop Network

In multi-hop networks, ordinary mobile terminals are used to relay the data from the transmitter to the receiver without the need for a base station. As shown in Figs. 1 and 2, the source node (S) transmits information to the destination node (D) with the help of other users referred to as relays (R). In this paper, all relays employ DF relaying scheme, which is described below. It is assumed that the distance between the source and destination nodes is d, and relays are uniformly located at equal distances between the source and destination. It is worth noting that this assumption is only used to simplify the analysis, and should not be limiting by any way to the presented network model and the derived analysis. In the parallel network, the distance between the source and destination is assumed to be much larger than the distance between the relays.

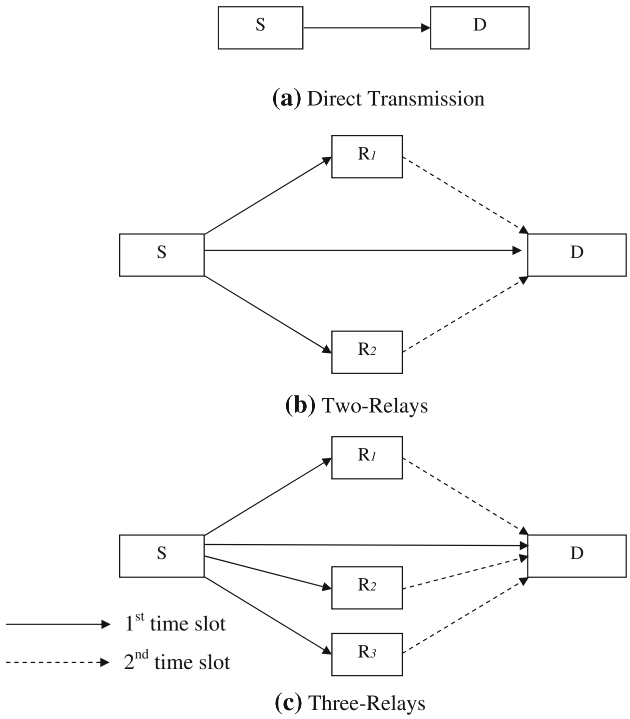

Fig. 1 Parallel multi-hop network. **a** Direct transmission, **b** Two-relay, and **c** Three-relay

Let us consider the serial network shown in Fig. 2. For fair comparison between the direct and multi-hop tranmission, we assume that transmission takes place according to the time slotting shown in Fig. 3. In the case of single-relay case, the source sends the spread signal to the relay in the first time slot, which despreads it, spreads it again and forwards it to the destination during the second time slot. To get the maximum of the system, the destination uses maximal-ratio combining (MRC) to combine the received signals from both the relay and the source i the two time slots. To be fair in comparing the relayed system with the direct system, we reduce the spreading gain in the relayed systems by a factor of 2. By doing so, we maintain the system throughput to be equal in all cases.

3 Performance Analysis

3.1 Direct Transmission

In this section, we review the derivation of BEP of the direct CDMA transmission with K users in the system. A method called the standard Gaussian approximation (SGA) [20] is used to derive the BEP of CDMA systems based on the argument that the decision statistic, Z_1 given in (4), may be modeled as a Gaussian r.v. The first component represents the desired signal, whereas the other two components

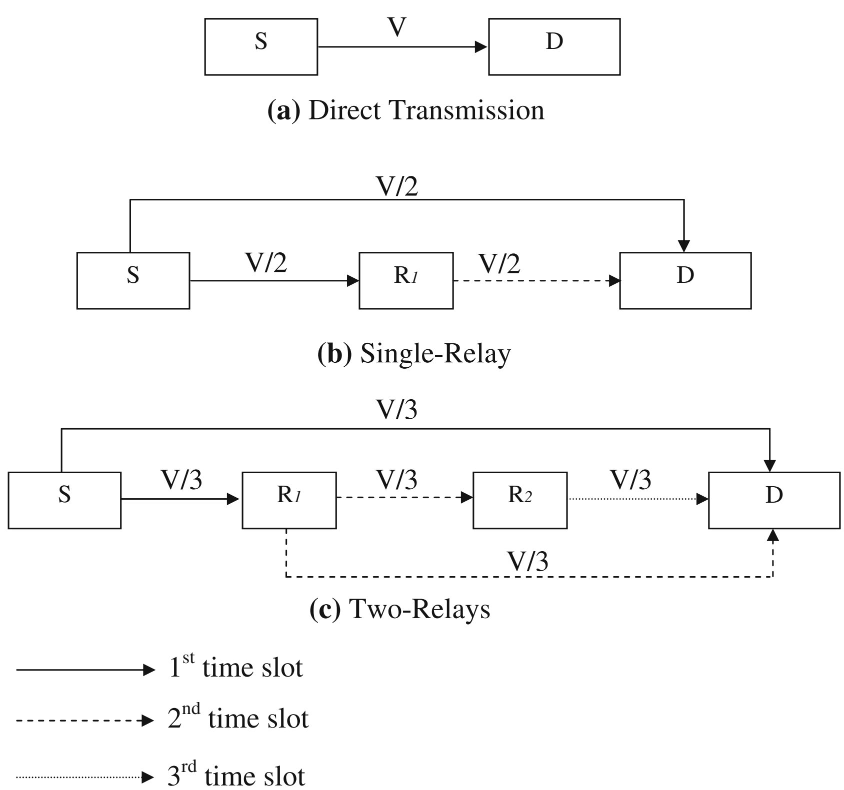

Fig. 2 Serial multi-hop network. **a** Direct transmission with spreading gain V, **b** single-relay network with spreading gain V/2, and **c** Two-relay network with spreading gain V/3

Source			Direct Transmission
Source	Relay1		Single-Relay
Source	Relay1	Relay2	Two-Relays

$\longrightarrow$ t

Fig. 3 Time slotting in the serial multi-hop network

are assumed to be zero-mean Gaussian r.v.'s [13]. Under the assumption that the PN codes are generated randomly, the SGA method results in an expression for the BEP based on the approximation of the MAI term by a Gaussian r.v. The SGA method results in the conditional BEP can be written as

$$P(E|\{\alpha_1\}) = Q\left(\sqrt{\frac{\frac{1}{2}P_1 T_\mathrm{b}^2 d^{-n}\alpha_1^2}{\frac{N_0 T_\mathrm{b}}{4}+\sigma_m^2}}\right), \tag{5}$$

where $Q(.)$ is the Q-function, the numerator represents the power of the desired signal, whereas the terms $\frac{N_0 T_\mathrm{b}}{4}$ and σ_m^2 are the respective variances of the AWGN and MAI terms under the SGA assumption. We have omitted the time subscript from the analysis for simplicity. The variance of the multiple interference was found in [13] to be

$$\sigma_m^2 = \frac{T_\mathrm{b}^2}{6V}\sum_{k=2}^{K} P_k d^{-n}\alpha_k^2. \tag{6}$$

By using (6) in (5), we get the conditional BEP for user 1 in a CDMA system using SGA as

$$P(E|\{\alpha_1\}) = Q\left(\sqrt{\left(\frac{1}{3V}\sum_{k=2}^{K}\frac{P_k}{P_1}\frac{\alpha_k^2}{\alpha_1^2}+\frac{N_0}{2T_\mathrm{b}P_1 d^{-n}\alpha_1^2}\right)^{-1}}\right). \tag{7}$$

Assume that all the users have equal power $P_1 = P_k$ for all k, and define $E_\mathrm{b} = P_1 T_\mathrm{b} d^{-n}$ as the average received energy per bit of User 1. Assuming the receiver uses MRC to combine the signals of the intended user from L diversity branches, the conditional BEP expression using SGA assumption [21] simplifies to

$$P(E|\{\alpha_1\}) = Q\left(\sqrt{\frac{\sum_{i=1}^{L} 2\alpha_i^2 E_\mathrm{b}/N_0}{\frac{2E_\mathrm{b}(K-1)}{3VN_0}+1}}\right). \tag{8}$$

Define the signal-to-interference-and-noise-ratio (SINR) as

$$\mathrm{SINR}(V,K) = \frac{2E_\mathrm{b}/N_0}{\frac{2(K-1)E_\mathrm{b}/N_0}{3V}+1}. \tag{9}$$

Using the integral form of the Q-function given by $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \mathrm{e}^{-x^2/2\sin^2(\theta)} d\theta$ in (8) results in

$$P(E|\{\alpha_1\}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L}\mathrm{e}^{-\frac{\mathrm{SINR}(V,K)\alpha_i^2}{2\sin^2(\theta)}} d\theta. \tag{10}$$

For generality, we assume that all α_i's are independent and identically distributed (i.i.d.) Nakagami-m r.v.'s. In this case, the probability density function (pdf) [19] of the r.v. $\gamma_i = \alpha_i^2$ is given by

$$f_{\gamma_i}(\gamma) = \frac{m^m}{\Gamma(m)}\gamma^{m-1}\mathrm{e}^{-m\gamma}, \quad \gamma \geq 0, \; m \geq 0.5, \tag{11}$$

where $\Gamma(\cdot)$ is the Gamma function and m is the fading parameter that indicates the fading severity. As m increases, the fading becomes less severe and reaches the non-fading case (i.e., AWGN channel) when $m \to \infty$. The Nakagami distribution covers a wide range of fading scenarios including Rayleigh fading when $m = 1$ and single-sided Gaussian distribution when $m = 0.5$. Averaging (10) over the pdf in (11) yields the unconditional BEP to be in the following form

$$P_\mathrm{b}(L,V,K) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{1}{1+\frac{\mathrm{SINR(V,K)}/m}{2\sin^2(\theta)}}\right)^{mL} d\theta. \tag{12}$$

Equation (12) gives the average BEP of a CDMA system over Nakagami-m fading channels with L-branch diversity, spreading gain V, and K users. The rest of the paper uses this result to derive the end-to-end error probability of multi-hop wireless networks employing different network topologies.

3.2 Serial Relaying

In serial relaying described in Sect. 2, the receiver combines signals from the source and the relays which have successfully detected the source data. To be fair in the comparison, the spreading gain is always $V/(J+1)$, and the energy of relays is divided by the number of the relays. Of course, this behavior will result in different scenarios, which need to be accounted for in the analysis. To simplify the analysis, we define the following events, and we will use them in the rest of the analysis.

- E = Detection error at the destination.
- $P_\mathrm{b}(L,V,K)$ = BEP in (12) given diversity order L, spreading gain V, and number of users K.

For the case of single-relay serial network as shown in Fig. 2b, the end-to-end error probability can be written as

$$\begin{aligned} P(E) = {} & P_\mathrm{b}(2,V/2,K)(1-P_\mathrm{b}(1,V/2,K)) \\ & + P_\mathrm{b}(1,V/2,K)P_\mathrm{b}(1,V/2,K), \end{aligned} \tag{13}$$

which can be easily evaluated using Eq. (12). A general formula for the end-to-end error probability for serial relaying employing J relays can be expressed as

$$P(E) = P_b\left(J+1, \frac{V}{J+1}, K\right)\left[1 - P_b\left(1, \frac{V}{J+1}, K\right)\right]^J + \sum_{i=0}^{J-1} P_b\left(i+1, \frac{V}{J+1}, K\right) \times \left[1 - P_b\left(1, \frac{V}{J+1}, K\right)\right]^i P_b\left(1, \frac{V}{J+1}, K\right). \quad (14)$$

3.3 Parallel Relaying

Figure 1 shows an example of parallel relaying, in which the source sends the data to all the relays, each of which despread and then spread it again and forward it to the destination. The destination uses MRC to combine the received signals from all the relays and the source node. The source sends the data to all the relays in the first time slot, and then all the relays send it in the next time slot to the destination node. To be fair in the comparison, the spreading gain is always $V/2$ but the energy in the second hop is divided by the number of the relays. The topology is shown in Fig. 1. Note that the spreading gain in the first time slot is $V/2$ regardless of the number of relays. In addition, the number of interfering users will depend on the number of relays which detected the source data correctly, and hence participated in relaying the data to the destination. For the case of two-relay, the end-to-end error probability is written as

$$P(E) = P_b(3, V/2, K+2)(1 - P_b(1, V/2, K))^2 + 2P_b(2, V/2, K+1)(1 - P_b(1, V/2, K)) \times P_b(1, V/2, K) + P_b(1, V/2, K)(P_b(1, V/2, K))^2. \quad (15)$$

A general formula for the end-to-end error probability for parallel relaying with J relays can be expressed as

$$P(E) = \sum_{i=0}^{J} \binom{J}{i} P_b(i+1, V/2, K+i) \times [1 - P_b(1, V/2, K)]^i [P_b(1, V/2, K)]^{J-i}, \quad (16)$$

which can be evaluated using Eq. (12).

3.4 Selection Relaying

In selection relaying, the topology is the same as the parallel topology in Fig. 1 but not all of the relays forward the data to the destination. Some negotiation must be done between the source and the relays to select which of the relays are going to cooperate in the transmission. In this paper, we consider the following two selection techniques:

- Threshold-based relay selection (TRS): In this scheme, the fading components of the links between the source and the relays are measured before each transmission. Only the relays with fading above a certain threshold α_{th} are allowed to join in the relaying. The spreading gain V is selected according to the number of the relays joining in the transmission. For example, when only one relay is joining in the transmission, the spreading gain becomes $V/2$, to be fair when comparing with the direct transmission case.
- Maximum relay selection (MRS): As in threshold-based relay selection, the fading of each link between the source and the relays is measured before each transmission. Only one relay which has the maximum fading will relay the source information. Here, the spreading gain is always $V/2$.

For the sake of analysis of the TRS scheme, we define the outage probability as the probability that the fading value α is less than the threshold α_{th} required for relaying, which is given by $P_{out} = \int_0^{\alpha_{th}} f(\alpha)d\alpha$. A general formula for the end-to-end error probability for TRS relaying employing J relays can be expressed as

$$P(E) = \sum_{j=0}^{J} \binom{J}{j} [1 - P_{out}]^j P_{out}^{J-j} \times \sum_{i=0}^{j} \binom{j}{i} P_b(i+1, V/2, K+i) \times [1 - P_b(1, V/2, K)]^i [P_b(1, V/2, K)]^{j-i}, \quad (17)$$

which can be evaluated using (12) and the outage probability for a Nakagami r.v. [19] given by

$$P_{out} = 1 - \frac{1}{\Gamma(m)} \Gamma(m, m\alpha_{th}^2), \quad (18)$$

where $\Gamma(a, x) = \int_0^x e^{-u} u^{a-1} du$ is the incomplete Gamma function [22].

For the MRS, the BEP derivation is the same as the single-relay serial but with a different direct probability of error than in (12). In general, the end-to-end error probability for the MRS scheme over Nakagami-m fading channels is given by the following modified version of (13) reflecting the selection of the best source-relay channel out of J relays

$$P(E) = P_b(2, V/2, K)(1 - P_b^s(J, V/2, K)) + P_b(1, V/2, K) P_b^s(J, V/2, K), \quad (19)$$

where $P_b^s(J, V, K))$ is the direct BEP for a CDMA system selecting the best channel out of J channels, and employing

BPSK with a spreading gain V and the number of users K. The term $P_b(.,.,.)$ in (19) is defined in (12). To calculate the new direct BEP, i.e., $P_b^s(J, V, K)$), we need to derive the distribution for the r.v. that is the maximum of J Nakagami r.v.'s for the J-relay network. Then, starting from the conditional BEP in (10), we can find the unconditional BEP expression. Define the r.v. $\beta = \max(\gamma_1, \gamma_2, \ldots, \gamma_J)$, where α_i is the fading coefficient from the source to the ith relay. The pdf of the r.v. β is given by

$$f_\beta(\beta) = J f_\gamma(\beta)[F_\gamma(\beta)]^{J-1}. \tag{20}$$

Upon substituting (11) in (20), the pdf of the max of J Nakagami r.v.'s [23] becomes

$$f_\beta(\beta) = \frac{Jm^m}{\Gamma(m)}\beta^{m-1}\mathrm{e}^{-m\beta}[\Gamma(m, m\beta)]^{J-1}, \quad \beta \geq 0,\ m \geq 0.5. \tag{21}$$

Now, to find an expression for the BEP similar to the one in (12), we need to average the conditional BEP in (10) over the pdf in (21) resulting in

$$P_b^s(J, V, K) = \frac{1}{\pi}\int_0^{\pi/2} \Psi_\beta(s(\theta))\mathrm{d}\theta, \tag{22}$$

where

$$s(\theta) = \frac{\mathrm{SINR}(V/2, K)}{2\sin^2(\theta)}. \tag{23}$$

and $\Psi_\beta(s)$ is the moment-generating function (MGF) of the r.v. β defined as $\Psi_\beta(s) = \mathrm{E}_\beta[\mathrm{e}^{s\beta}]$, where $\mathrm{E}_\beta[\cdot]$ is the expectation operator. Following the derivation in [23], the MGF of the r.v. β can be written finally as

$$\begin{aligned}\Psi_\beta(s) &= \frac{Jm^{J(m-1)}}{\Gamma(m)} \times \int_0^\infty \mathrm{e}^{-\beta(mJ-s)}\beta^{mJ-1}[{}_1F_1(1; 1+m; m\beta)]^{J-1}\mathrm{d}\beta \\ &= \frac{Jm^{J(m-1)}\Gamma(mJ)}{\Gamma(m)(mJ-s)^{mJ}} F_A \\ &\quad \times \Bigg(mJ; \underbrace{1, \ldots, 1}; \underbrace{1+m, \ldots, 1+m}; \\ &\qquad \underbrace{\frac{m}{mJ-s}, \ldots, \frac{m}{mJ-s}}\Bigg),\end{aligned} \tag{24}$$

where the underbraced terms are repeated for J times, and $F_A(\cdot; \cdots; \cdots; \cdots)$ is the Laurecella's hypergeometric function [22]. The final general expression of the BEP for the MRS scheme over Nakagami-m fading channels is finally obtained by substituting (24) in (22) and using (12) and (22) in (19).

The expression in (24) can be evaluated numerically using the Gauss–Laguerre Integration (GLI) rule from [24], the integral in (24) can be evaluated efficiently as

$$\int_0^\infty \mathrm{e}^{-y}y^{m-1}g(y)\mathrm{d}y \approx \sum_{p=1}^{P} w_m(p)g(y_m(p)), \tag{25}$$

where we used the function $g(y) = [{}_1F_1(1; 1+m; my)]^{J-1}$, $\{w_m(p)\}$ are the weights of the GLI rule for a specific m and $y_m(p)$ is the pth abscissa. Both $\{w_m(p)\}$ and $\{y_m(p)\}$ are computed according to the GLI rule as in [24]. It was found [24] that $P = 20$ is enough to get the required accuracy.

For the special case of Rayleigh fading channels, the performance analysis of MRS becomes easier. In this case, the pdf of the r.v. β, which is the maximum of J Rayleigh r.v.'s simplifies to

$$f_\beta(\beta) = J\sum_{j=1}^{J-1}\binom{J-1}{j}(-1)^j\mathrm{e}^{-(1+j)\beta}. \tag{26}$$

Averaging (10) over (26) results in the following expression for the BEP of the MRS scheme over Rayleigh fading channels

$$P_b^s(J, V, K) = \frac{J}{\pi}\sum_{j=1}^{J-1}\binom{J-1}{j}(-1)^j \times \int_0^{\pi/2}\left(\frac{1}{1+j+\frac{\mathrm{SINR(V,K)}/m}{2\sin^2(\theta)}}\right)\mathrm{d}\theta. \tag{27}$$

The final general expression of the BEP for the MRS scheme over Rayleigh fading channels is finally obtained by substituting (27) and (12) in (19).

4 Numeric Results

In this section, we illustrate some numerical results comparing the derived performance analysis with simulations. For the sake of simplicity, we assume that the relays are located at uniform distances between the source and the destination. To compare with the analytical results, we employ random PN sequences and BPSK transmission. The path loss exponent n is set to 3, except when we test the effect of the path loss on the network performance.

Figure 4 shows the analysis and simulation results of the error performance of the direct and serial topology employing single-relay and two relays over Rayleigh fading channels with number of users $K = 4$. The overall performance is poor due to MAI, but it is clear that relaying improves the performance significantly. Figure 5 shows the same results for the direct transmission, two-relay parallel, and three-relay parallel schemes. Note that the performance of the parallel

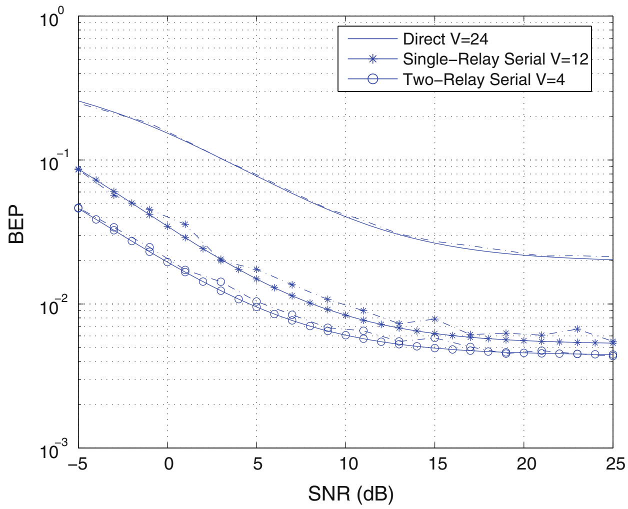

Fig. 4 Performance of the direct, single-relay and two-relay serial networks over Rayleigh fading channels with $K = 4$: *solid* analysis, *dashed* simulation

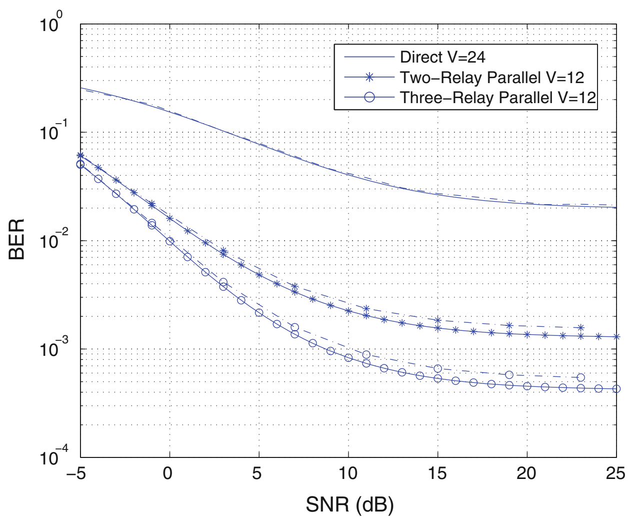

Fig. 5 Performance of the direct, two-relay and three-relay parallel networks over Rayleigh fading channels with $K = 4$: *solid* analysis, *dashed* simulation

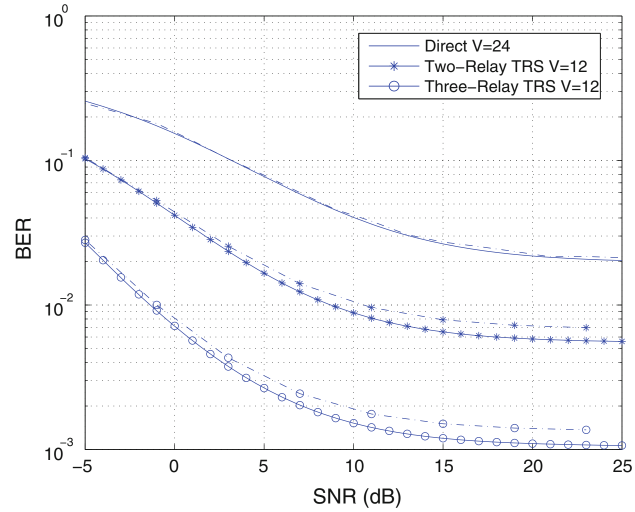

Fig. 6 Performance of the direct, two-relay and three-relay TRS networks over Rayleigh fading channels with $K = 4$ and $\alpha_{TH} = 0.5$: *solid* analysis, *dashed* simulation

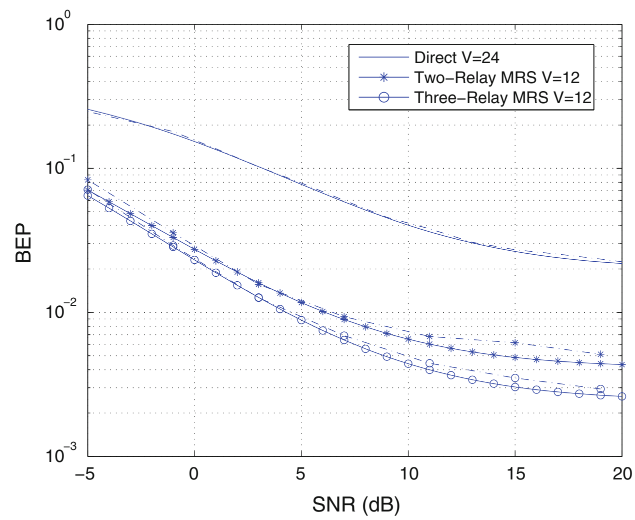

Fig. 7 Performance of the direct, two-relay and three-relay MRS networks over Rayleigh fading channels with $K = 4$: *solid* analysis, *dashed* simulation

relaying is much better than the serial scheme due to the added diversity in the parallel scheme. In addition, the relaying gain in the three-relay scheme is much much more than the two-relay scheme, even though it uses a smaller spreading gain. This is because the diversity gain has a greater effect than the loss due to MAI especially as SNR increases. The overall gain in the performance is poor due to the MAI, but there is still a significant improvement in the performance over the direct transmission because of the increased diversity order.

Figure 6 shows the analysis and simulation results for the direct, two-relay TRS, and three-relay TRS schemes over Rayleigh fading channels with four users and using a selection threshold of $\alpha_{\text{th}} = 0.5$, which is chosen so that the outage probability is set to 0.1175. It is observed that the performance of the TRS is generally better than the performance of the serial scheme but worse than the parallel scheme. This depends on the choice of the threshold α_{th}. As the threshold increases, fewer relays join the relaying, and so the performance approaches the serial scheme. On the other hand, as the threshold decreases, more relays join the relaying, and the performance approaches the parallel scheme. Figure 7 shows the same results for the MRS schemes with two and three relays. It can be noticed that the performance of MRS is worse than the TRS scheme. This is because MRS uses only one relay, whereas the TRS uses more than one relay with non-zero probability, resulting in more effective

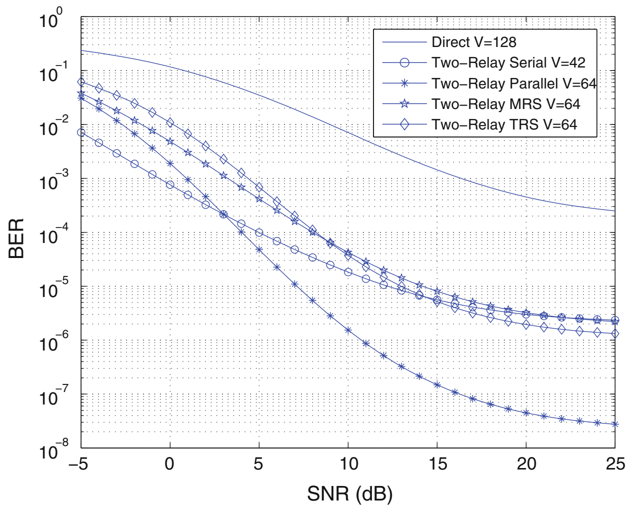

Fig. 8 Performance of the direct, two-relays serial, parallel, TRS and MRS relaying over a Nakagami-2 channel with $K = 4$

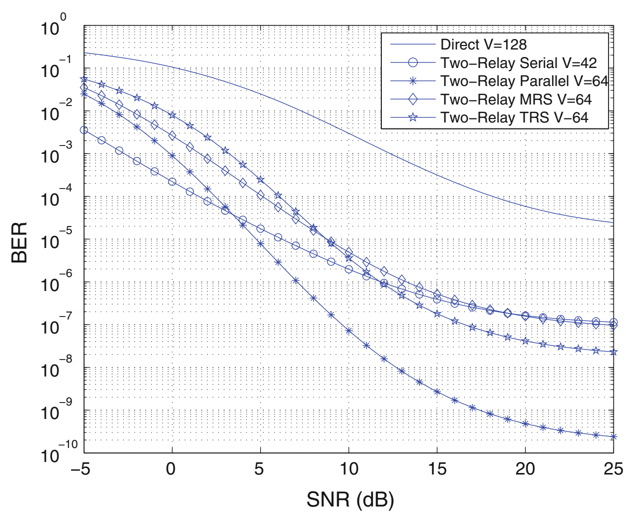

Fig. 9 Performance of the direct, two-relays serial, parallel, TRS and MRS relaying over a Nakagami-3 channel with $K = 4$

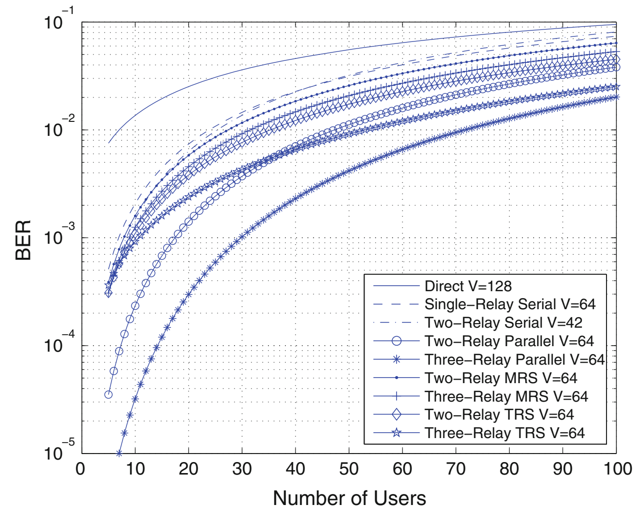

Fig. 10 Performance for the direct, serial, parallel, MRs and TRS networks in Rayleigh fading channels against K when SNR = 20 dB and $V = 128$

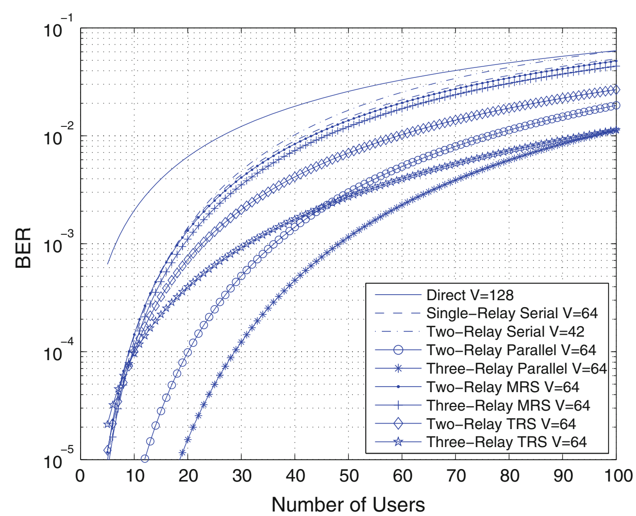

Fig. 11 Performance for the direct, serial, parallel, MRs and TRS networks in Nakagami-2 fading channels against K when SNR = 20 dB and $V = 128$

diversity gain in TRS. However, MRS performs better than the single-relay serial scheme because the probability of going into deep fading is reduced since the relay has the best channel among J relays. Note that the MRS scheme has a similar structure as the serial scheme, but with a better fading conditions due to the selection of the best relay.

In all the above figures, we have selected the spreading gain to be $V = 24$ for the sake of having reasonable simulation time. However, since we have shown that the analysis and the simulation results are close enough, the remaining results will show more practical system parameters using the analysis only without simulations. Figures 8 and 9 show the performance analysis of the direct and two-relay networks employing the serial, parallel, TRS, and MRS relaying over Nakagami-m fading channels with $m = 2$, $m = 3$, respectively, and the number of users set to $K = 4$. Because the diversity order is higher in the serial and the parallel schemes, they perform better than the MRS and the TRS schemes. Note that a two-relay serial has two relays between the source and destination, and hence it is not similar in structure to the two-relay MRS. In fact, the two-relay serial outperforms the two-relay MRS as it has more diversity gain and less attenuation through the path loss exponent.

Figures 10 and 11 show the performance of the direct, serial, parallel, MRS, and TRS relaying schemes against the number of users when the SNR = 20 dB and spreading gain $V = 128$, over Rayleigh and Nakagami-2 fading channels, respectively. We observe that both the error performance and

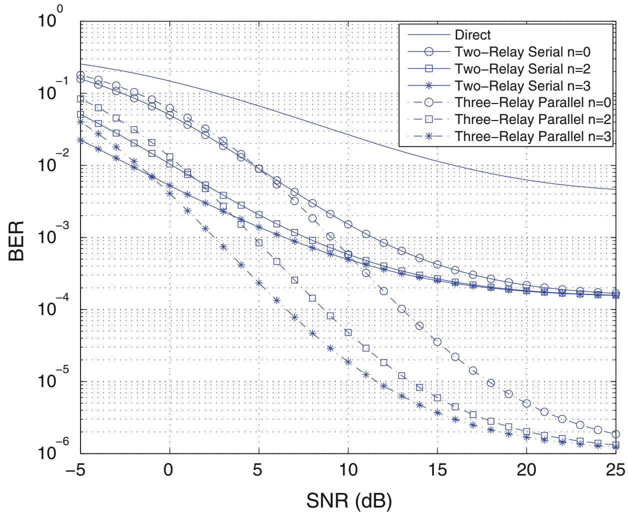

Fig. 12 Effect of path loss exponent of the performance of the serial two-relay and the parallel three-relay schemes over Rayleigh fading channel with $K = 4$, $V = 128$, and $n = 0, 2$, and 3

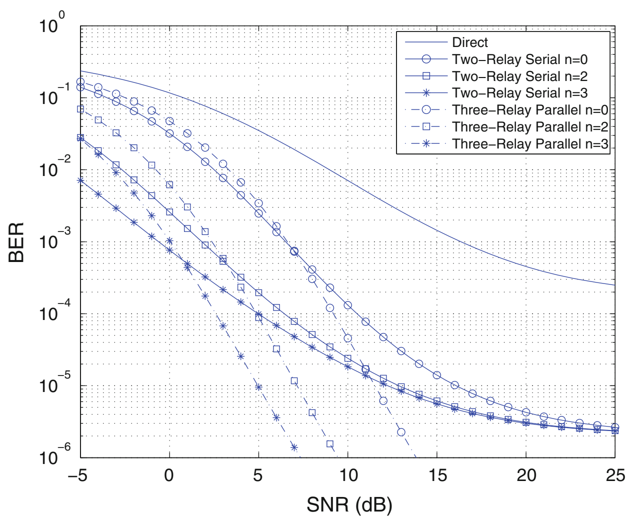

Fig. 13 Effect of path loss exponent of the performance of the serial two-relay and the parallel three-relay schemes over Nakagami-2 fading channel with $K = 4$, $V = 128$, and $n = 0, 2$, and 3

the relaying gain decrease as the number of users increases. Increasing the spreading gain reduces the effect of MAI, and hence the diversity effect becomes more visible. The improvement in error performance is clear when comparing it with the Rayleigh fading case in 10. In addition, the effect of diversity becomes less valuable as the value of m increases in a Nakagami fading channel, and hence the MAI effect becomes more dominant.

In Figs. 12 and 13, we show the effect of the path loss exponent on the performance of multi-hop networks employing two-relay serial and three-relay parallel schemes, over Rayleigh and Nakagami-2 fading channels, respectively, with four users and 128 spreading gain. We considered the cases

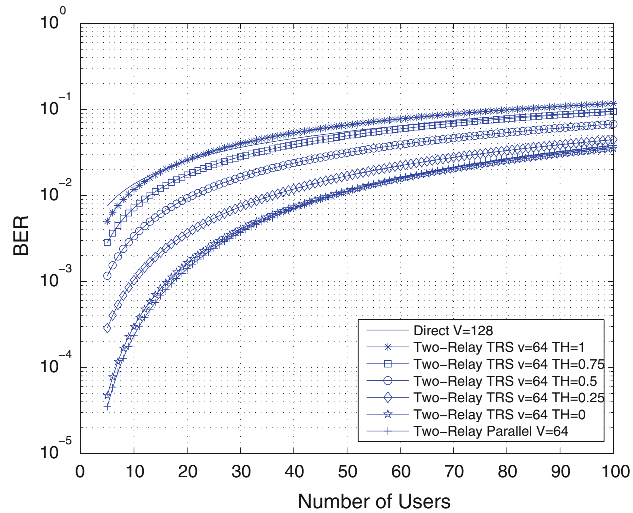

Fig. 14 Effect of the threshold on the performance of the TRS two-relay scheme over Rayleigh fading channels against K, with $V = 128$, and SNR = 20 dB

when $n = 0,2$, and 3, where $n = 0$ means that the distances between source, destination and relays were not taken into consideration. Results show that the higher the path loss exponent, the better the performance of the relayed system. In environments with large path loss exponent, sending the data over short distances saves more power than in environments with small path loss. So the relaying gain gets better as the path loss exponent increases, as the attenuation effect becomes less.

In Fig. 14, we investigated the effect of the threshold α_{th} on the performance of the two-relay TRS scheme against the number of users with SNR = 20 dB and $V = 128$. It is clear from the figure that as the threshold increases, fewer relays join the transmission and hence the performance approaches the direct scheme. However as the threshold decreases, more relays join the transmission and so the performance approaches the parallel scheme.

5 Complexity Comparison

As has been discussed in the previous section, the relaying clearly improves the performance of wireless networks. However, it would be very interesting to shed some light on the complexity incurred when using the relaying in wireless networks. First, the time slotting should be taken care on the network level. For example, in serial relaying with J relays the time slot for one packet shall be divided into J sub-slots. Of course, the spreading gain is adjusted to account for the loss in the bandwidth due to relaying. The second factor affecting the complexity is that all relays which will relay information will have to decode and forward. The decoding here refers to decoding symbol-by-symbol without any decoding at the error control layer. Symbol-by-symbol decoding is very simple and only needs matched filter at each relay, which exists by default if the relay is one normal wireless node.

To quantify the complexity, we need to know the number of relays which will participate in relaying the information. In serial and parallel relaying, J relays will need to decode information on symbol-by-symbol basis. In MRS and TRS relaying, only one relay needs to decode information. To compare MRS with TRS relaying, MRS needs to estimate the channel and chose the best relay for every packet, whereas in TRS the chosen relay continues to relay the information as long as his channel is above the pre-selected threshold. Hence, TRS incurres less complexity from the channel scanning point-of-view.

6 Conclusions

In this paper we investigated the performance of multi-hop wireless networks employing CDMA transmission based on the standard Gaussian approximation. In particular, we derived the probability of error of the network employing serial, parallel and two selection relaying schemes. In addition, the error performance of the system with serial, parallel, and selection relaying was investigated with different values of spreading gain, path loss exponent and number of users. Results show that the analysis and simulation results are close to each other, and that relaying greatly improves the system, especially in parallel and the TRS relaying schemes. Furthermore, when the spreading gain is increased, the effect of relaying on the overall performance increases, especially when the MAI is low. In addition, as the number of users increases, the improvement coming from relaying decreases. Finally, the error performance of the system improves as the path loss exponent increases. As a future research direction, we can investigate and derive the error performance of a CDMA-based multi-hop network employing AF technique. It is well-known that the AF technique consumes less energy than DF, which is a sensitive issue in multi-hop network design, and makes this as a promising future research.

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