Outage Analysis of N^{th}-Best DF Relay Systems in the Presence of CCI over Rayleigh Fading Channels

Anas M. Salhab, Fawaz Al-Qahtani, Salam A. Zummo, and Hussein Alnuweiri

***Abstract*—This letter investigates the outage behavior of a dual-hop N^{th}-best decode-and-forward (DF) relay system in which the relay and the destination undergo independent sources of co-channel interference (CCI). The fading envelopes associated with the desired and interfering users are assumed to follow independent but non-identically distributed (i.n.d.) Rayleigh fading models. Through the analysis, exact and approximate outage probability are derived based on which the diversity order and coding gain are obtained. Our findings suggest that the diversity order linearly increases with the number of relays and decreases with the order of the relay. Furthermore, results reveal that the system is still able to achieve full diversity gain in the presence of finite number of interferers with finite powers. The analytical results are supported and validated by Monte-Carlo simulations.**

***Index Terms*—Decode-and-forward, cooperative network, Rayleigh fading, co-channel interference, N^{th}-best relay.**

I. INTRODUCTION

COOPERATIVE networks are considered among efficient techniques to provide space diversity in wireless communication. Several relaying schemes were introduced in [1], among which are the amplify-and-forward (AF) and the decode-and-forward (DF). In the AF, the source signal is just scaled by a gain before being forwarded to the destination; whereas, in the DF, some signal processing and coding need to be performed by the relay before the source signal being forwarded.

Most of the work on cooperative systems has focused on noise-limited environments and ignored the interference effect on system behavior [2], [3]. Few papers have investigated the impact of interference in such cooperative systems. In [4], closed-form and asymptotic expressions for the outage and symbol error rate (SER) of an AF relay system were derived assuming Nakagami-m fading channels. The interference effect at the destination node over Nakagami-m fading channels was studied in [5]. In [6], the outage performance of an AF relay system was evaluated with interference at both the relay and the destination assuming Nakagami-m fading channels.

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Some key papers on multi-relay cooperative systems with interference are the ones presented in [7] and [8]. Particularly, in [7], the outage performance of an opportunistic DF relay system was evaluated with interference at the relays and destination assuming Rayleigh fading channels. In [9], the authors evaluated the outage performance of a conventional DF cooperative system over Nakagami-m fading channels. In some situations in relay networks, the best relay may not be available due to some scheduling or load balancing conditions. In this case, the decision may be made to use the second best relay or more generally the N^{th} best relay to cooperate. The performance of dual-hop AF and DF relay systems with such efficient relaying scheme was studied in [10] assuming Rayleigh fading channels.

In this letter, we derive an exact closed-form expression for the outage probability of a N^{th}-best DF relay system with interference at the relays and destination for the generic i.n.d. case of interferers' channels. We also evaluate the system performance at high SNR regime where an asymptotic expression for the outage probability is derived and analyzed in addition to deriving the diversity order and the coding gain. Due to its inherent effect on system behavior, the direct link is considered in all derivations in this paper.

II. SYSTEM MODEL

Consider a dual-hop relay system with one source, one destination, K relay nodes, and arbitrary number of interferers at both the relays and the destination with the N^{th}-best relaying scheme being used. The entire communication takes place in two phases. In the first phase, the source S transmits its signal to the destination D and the K relays. In the second phase, only the N^{th} best relay among all other relays who succeeded to decode the source signal is selected to forward it to D. We assume that the signal at the k^{th} relay is corrupted by interfering signals from I_k co-channel interferers $\{x_i\}_{i=1}^{I_k}$.

The received signal at the k^{th} relay can be expressed as

$$y_{\mathsf{r}_k} = h_{\mathsf{s},k}x_0 + \sum_{i_k=1}^{I_k} h_{i_k,k}^I x_{i_k,k}^I + n_{\mathsf{s},k}, \tag{1}$$

where $h_{\mathsf{s},k}$ is the channel coefficient between S and the k^{th} relay, x_0 is the transmitted symbol with $\mathbb{E}\{|x_0|^2\} = P_0$, $h_{i_k,k}^I$ is the channel coefficient between the i_k^{th} interferer and k^{th} relay, $x_{i_k,k}^I$ is the transmitted symbol from the i_k^{th} interferer with $\mathbb{E}\{|x_{i_k,k}^I|^2\} = P_{i_k,k}^I$, $n_{\mathsf{s},k} \sim \mathcal{CN}(0, N_0)$ is an additive white Gaussian noise (AWGN), and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. Let us define $h_{\mathsf{s},\mathsf{d}}$, $h_{k,\mathsf{d}}$, and $h_{i_d,\mathsf{d}}^I$ as the channel coefficients between S and D, the k^{th} relay and D, the i_d^{th} interferer and D, respectively. All the channel gains are assumed to follow the Rayleigh distribution. That is, the channel powers denoted by $|h_{\mathsf{s},\mathsf{d}}|^2$, $|h_{\mathsf{s},k}|^2$, $|h_{k,\mathsf{d}}|^2$, $|h^I_{i_k,k}|^2$, and $|h^I_{i_d,\mathsf{d}}|^2$ are exponential distributed random variables (RVs) with parameters $\sigma^2_{\mathsf{s},\mathsf{d}}$, $\sigma^2_{\mathsf{s},k}$, $\sigma^2_{k,\mathsf{d}}$, $\sigma^2_{I,i_k,k}$, and $\sigma^2_{I,i_d,\mathsf{d}}$, respectively. Using (1), the signal-to-interference plus noise ratio (SINR) at the k^{th} relay can be written as $\gamma_{\mathsf{s},k} = \frac{P_0}{N_0}|h_{\mathsf{s},k}|^2 \Big/ \left(\sum_{i_k=1}^{I_k} \frac{P^I_{i_k,k}}{N_0}|h^I_{i_k,k}|^2 + 1\right)$.

Let B_L denote a decoding set defined by the set of relays who successfully decoded the source message at the first phase. It is defined as

$$B_L \triangleq \left\{k \in \mathcal{S}_r : \gamma_{\mathsf{s},k} \geq 2^{2R} - 1\right\}, \tag{2}$$

where $\mathcal{S}_r$ is the set of all relays and R denotes a fixed spectral efficiency threshold.

In the second phase and after decoding the received signal, only the N^{th} best relay in B_L forwards the re-encoded signal to the destination. The N^{th} best relay is the relay with the N^{th} maximum $\gamma_{l,\mathsf{d}}$, where $\gamma_{l,\mathsf{d}}$ is the SINR at the destination resulting from the l^{th} relay being the relay which forwarded the source information. It can be written as $\gamma_{l,\mathsf{d}} = \frac{P_l}{N_0}|h_{l,\mathsf{d}}|^2 \Big/ \left(\sum_{i_d=1}^{I_d} \frac{P^I_{i_d,\mathsf{d}}}{N_0}\left|h^I_{i_d,\mathsf{d}}\right|^2 + 1\right)$, where P_l, $P^I_{i_d,\mathsf{d}}$, and N_0 are the transmit power of the l^{th} active relay, the transmit power of the i_d^{th} interferer, and the AWGN power at the destination, respectively, and I_d is the number of interferers at the destination node. Since the denominator is common to the SINRs from all relays belonging to B_L, the N^{th} best relay is the relay with the N^{th} maximum $\left\{\frac{P_l}{N_0}|h_{l,\mathsf{d}}|^2\right\}$.

In the analysis of this paper, the destination is assumed to be located at the same point during the two phases. This means the same interference is affecting the destination node in both phases. As the maximal-ratio combining (MRC) is being used at the destination, the signals on the direct link and the N^{th} best relay are added. The end-to-end (e2e) SINR at the destination output can be written as $\gamma_{\mathsf{d}} \triangleq \gamma_{\mathsf{s},\mathsf{d}} + \gamma_{N_b^{\text{th}},\mathsf{d}} = \left(\frac{P_0}{N_0}|h_{\mathsf{s},\mathsf{d}}|^2 + \frac{P_{N_b^{\text{th}}}}{N_0}|h_{N_b^{\text{th}},\mathsf{d}}|^2\right) \Big/ \left(\sum_{i_d=1}^{I_d} \frac{P^I_{i_d,\mathsf{d}}}{N_0}\left|h^I_{i_d,\mathsf{d}}\right|^2 + 1\right)$.

III. Outage Performance Analysis

In this section, we derive a closed-from expression for the system outage probability. Before getting in details, we find it is appropriate to first present some preliminary studies and hence the new results of the considered system can be revealed.

A. Preliminary Study

The probability of the decoding set defined in (2) can be written as

$$\mathrm{P_r}\left[B_L\right] = \prod_{l \in B_L} \mathrm{P_r}\left[\gamma_{\mathsf{s},l} \geq u\right] \prod_{m \notin B_L} \mathrm{P_r}\left[\gamma_{\mathsf{s},m} < u\right], \tag{3}$$

where $u = \left(2^{2R} - 1\right)$. The outage probability of the system can be achieved by averaging over all possible decoding sets as follows [7]

$$\begin{aligned} P_{\mathsf{out}} &\triangleq \mathrm{P_r}\left[\frac{1}{2}\log_2\left(1 + \gamma_{\mathsf{d}}\right) < R\right] \\ &= \sum_{L=0}^{K}\sum_{B_L} \mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right] \mathrm{P_r}\left[B_L\right], \end{aligned} \tag{4}$$

where the internal summation is taken over all of $\binom{K}{L}$ possible subsets of size L from the set with K relays. In order to evaluate (4), we need first to derive $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u|B_L\right]$ and $\mathrm{P_r}\left[B_L\right]$ which are presented in the following section.

Throughout the analysis below, it assumed that $\rho \triangleq P_0/N_0 = P_l/N_0$ and $\rho_I \triangleq P^I_{i_k,k}/N_0 = P^I_{i_d,\mathsf{d}}/N_0$. The terms $\rho|h_{\mathsf{s},\mathsf{d}}|^2$, $\rho|h_{\mathsf{s},k}|^2$, $\rho_I|h^I_{i_k,k}|^2$, $\rho|h_{l,\mathsf{d}}|^2$, and $\rho_I|h^I_{i_d,\mathsf{d}}|^2$ are exponential distributed with parameters $\lambda_{\mathsf{s},\mathsf{d}} = 1/\rho\sigma^2_{\mathsf{s},\mathsf{d}}$, $\lambda_{\mathsf{s},k} = 1/\rho\sigma^2_{\mathsf{s},k}$, $\lambda^I_{i_k,k} = 1/\rho_I\sigma^2_{I,i_k,k}$, $\lambda_{l,\mathsf{d}} = 1/\rho\sigma^2_{l,\mathsf{d}}$, and $\lambda^I_{i_d,\mathsf{d}} = 1/\rho_I\sigma^2_{I,i_d,\mathsf{d}}$. For the i.n.d. case, we have $\lambda^I_{i_n,n} \neq \lambda^I_{j_n,n}$, when $i_n \neq j_n$, $n \in \mathcal{S}_r \bigcup \{d\}$.

B. Exact Outage Probability

The outage probability of the considered system is summarized in following key result:

Theorem 1: The outage probability of the N^{th}-best DF relay systems with i.n.d interferers' powers can be obtained in a closed-form expression by using (4), after evaluating the terms $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u|B_L\right]$ and $\mathrm{P_r}\left[\gamma_{\mathsf{s},k} < u\right]$ as follows

$$\begin{aligned} \mathrm{P_r}\left[\gamma_{\mathsf{d}} < u|B_L\right] &= \prod_{i_d=1}^{I_d} \lambda^I_{i_d,\mathsf{d}} e^{\lambda^I_{i_d,\mathsf{d}}} \sum_{g=1}^{I_d} \frac{\sum_{l=1}^{L} \lambda_{l,\mathsf{d}}\lambda_{\mathsf{s},\mathsf{d}}}{\prod_{\substack{m=1\\m\neq g}}^{I_d}\left(\lambda^I_{m,\mathsf{d}} - \lambda^I_{g,\mathsf{d}}\right)} \\ &\times \sum_{\mathcal{P}}\left[\frac{\left(\frac{\Xi_1}{\lambda_{\mathsf{s},\mathsf{d}}} - \frac{\Xi_2}{C_a}\right)}{C_a - \lambda_{\mathsf{s},\mathsf{d}}} + \sum_{q=1}^{L-N}(-1)^q \sum_{s_1<\ldots<s_q} \frac{\left(\frac{\Xi_1}{\lambda_{\mathsf{s},\mathsf{d}}} - \frac{\Xi_2}{C_c}\right)}{C_c - \lambda_{\mathsf{s},\mathsf{d}}}\right]. \end{aligned} \tag{5}$$

$$\mathrm{P_r}\left[\gamma_{\mathsf{s},k} < u\right] = \prod_{i_k=1}^{I_k} \lambda^I_{i_k,k} e^{\lambda^I_{i_k,k}} \sum_{g=1}^{I_k} \frac{\Xi_3}{\prod_{\substack{m=1\\m\neq g}}^{I_k}\left(\lambda^I_{m,k} - \lambda^I_{g,k}\right)}, \tag{6}$$

where $C_c = \sum_{w=L-N+1}^{L-1} \lambda_{i_w,\mathsf{d}} + \sum_{n=1}^{q} \lambda_{s_n,\mathsf{d}} + \lambda_{l,\mathsf{d}}$, $\Xi_1 = \Gamma(1, \lambda^I_{i_d,\mathsf{d}})/\lambda^I_{i_d,\mathsf{d}} - \Gamma(1, \lambda^I_{i_d,\mathsf{d}} + \lambda_{\mathsf{s},\mathsf{d}}u)/(\lambda^I_{i_d,\mathsf{d}} + \lambda_{\mathsf{s},\mathsf{d}}u)$, $\Xi_2 = \Gamma(1, \lambda^I_{i_d,\mathsf{d}})/\lambda^I_{i_d,\mathsf{d}} - \Gamma(1, \lambda^I_{i_d,\mathsf{d}} + C_a u)/(\lambda^I_{i_d,\mathsf{d}} + C_a u)$, $\Xi_3 = \Gamma(1, \lambda^I_{i_k,k})/\lambda^I_{i_k,k} - \Gamma(1, \lambda^I_{i_k,k} + \lambda_{\mathsf{s},k}u)/(\lambda^I_{i_k,k} + \lambda_{\mathsf{s},k}u)$, and $\Gamma(.,.)$ denotes the incomplete gamma function [11, Eq. (8.352.2)].

Proof: See Appendix A. ■

IV. Asymptotic Outage Behavior

In this section, we evaluate the system performance at high SNR values in which the outage probability can be expressed as $P_{\mathsf{out}} \approx (G_{\mathsf{c}}\rho)^{-G_{\mathsf{d}}}$, where G_{d} is referred to as the achieved diversity order of the system while G_{c} is referred to as the coding gain of the system.

Theorem 2: At high SNR regime and with finite number of interferers of finite powers, the outage probability of the

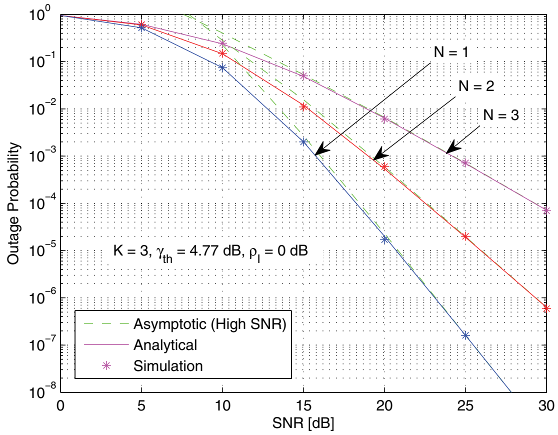

Fig. 1. Outage probability versus average SNR for $N^{\rm th}$-best DF relay system with interference at the relays and destination when interference power does not scale with SNR. We have set $\sigma^2_{\mathsf{s,d}} = 1$, $\sigma^2_{\mathsf{s},1} = 0.2$, $\sigma^2_{\mathsf{s},2} = 0.6$, $\sigma^2_{\mathsf{s},3} = 0.8$, $\sigma^2_{k,\mathsf{d}} = 0.4$ and $(\sigma^I_k)^2 = 0.01$ and $I_k = 1$ for $k = 1, \ldots, 3$, $(\sigma^I_{\mathsf{d}})^2 = 0.01$, and $I_d = 1$.

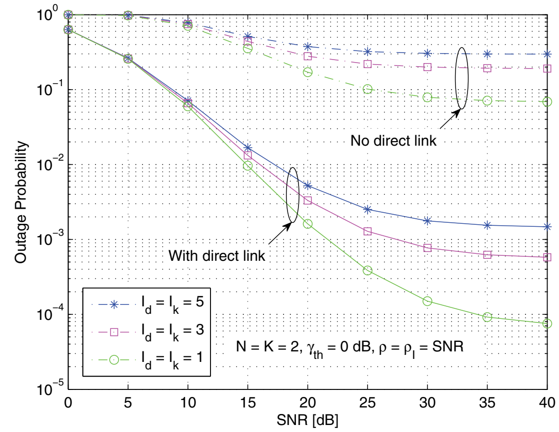

Fig. 2. Outage probability versus average SNR for $N^{\rm th}$-best DF relay system with interference at the relays and destination when interference power scales with SNR. We have set $\sigma^2_{\mathsf{s,d}} = 1$, $\sigma^2_{\mathsf{s},1} = 0.2$, $\sigma^2_{\mathsf{s},2} = 0.6$, $\sigma^2_{k,\mathsf{d}} = 0.4$ and $(\sigma^I_k)^2 = 0.01$ for $k = 1, 2$, and $(\sigma^I_{\mathsf{d}})^2 = 0.001$.

$N^{\rm th}$-best DF relay systems with interference under Rayleigh fading channels can be approximated as

$$P_{\mathsf{out}} \approx \frac{\Gamma\left(L-N+3, \lambda^I_{\mathsf{d}}\right)(\lambda^I_{\mathsf{d}})^{-(L-N-I_d+3)}(-1)^{I_d}}{(L-N+1)(L-N+2)\,(I_d-1)!}\exp\left(\lambda^I_{\mathsf{d}}\right) \times -L\binom{L-1}{N-1}\left(1+\frac{L}{\lambda^I_{\mathsf{d}}}\right)\rho^{-(L-N+2)}u^{L-N+3}. \quad (7)$$

Proof: See Appendix B. ■

Based on (7), the diversity order G_{d} and coding gain G_{c} can be characterized as follows.

Corollary 1: The diversity order of the system is given by $G_{\mathsf{d}} = L - N + 2$; while the coding gain is given by

$$G_{\mathsf{c}} = \left\{\frac{\Gamma\left(L-N+3, \lambda^I_{\mathsf{d}}\right)(\lambda^I_{\mathsf{d}})^{-(L-N-I_d+3)}(-1)^{I_d}}{(L-N+1)(L-N+2)\,(I_d-1)!}\exp\left(\lambda^I_{\mathsf{d}}\right) \times -L\binom{L-1}{N-1}\left(1+\frac{L}{\lambda^I_{\mathsf{d}}}\right)u^{L-N+3}\right\}^{L-N+2}, \quad (8)$$

Note that increasing the number of relays provides extra diversity order. Also, it helps to reduce the outage probability via improving the coding gain achieved by the system.

V. Numerical Results

Fig. 1 validates the achieved analytical results. It can be seen that the analytical results as well as the asymptotic curves perfectly fit with the simulation ones. We can also notice that the outage probability increases as N increases. Furthermore, it is obvious that the diversity order linearly increases with the number of active relays L although we use only one relay. Also, the diversity order linearly decreases as we move from the best relay ($N = 1$) to the second best relay and generally the $N^{\rm th}$ best relay case. This figure also shows that for the case when interference power does not scale with SNR, the system still can achieve full diversity gain.

Fig. 2 illustrates the interference effect on the system performance when $I_k = I_d$. It is obvious that when the number of interferers increases and hence, the total interference power, the outage probability performance deteriorated further, as expected. In addition, as the number of interferers decreases, the amount of achievement in system performance increases. Furthermore, the gain achieved in system performance due to the existence of the direct link compared to the case where it is not existed is obvious in this figure. Finally, as the interference power scales with SNR, the system diversity gain reaches zero and a noise floor appears in all cases in this figure.

In this letter, exact and asymptotic analysis of the outage performance of a dual-hop $N^{\rm th}$-best DF relay system was evaluated in the presence of interference at the relays and destination assuming Rayleigh fading channels. Results showed that the diversity order linearly increases with the number of relays and linearly decreases with the order of the relay. Findings illustrated that under the condition of finite number of interferers with finite powers, the system can still achieve full diversity. Finally, for the case where the interference power scales with SNR, zero diversity gain is achieved and a noise floor appears in the results due to the effect of interference.

Appendix A
Proof of Theorem 1

In this Appendix, we evaluate the first term in (4) $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u|B_L\right]$. The e2e SINR can be written as $\gamma_{\mathsf{d}} = Y_1/Z_1$. The CDF of γ_{d} given a decoding set B_L is given by $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u|B_L\right] = \int_1^\infty f_Z(z)\int_0^{uz} f_Y(y)dydz$. First, we evaluate the PDF of $Z_1 = \sum_{i_d=1}^{I_d}\rho_I|h^I_{i_d,\mathsf{d}}|^2 + 1 = X_1 + 1$, where the PDF of X_1 is given by $f_{X_1}(x) = \prod_{i_d=1}^{I_d}\lambda^I_{i_d,\mathsf{d}}\sum_{g=1}^{I_d}\frac{\exp\left(-\lambda^I_{i_d,\mathsf{d}}x\right)}{\prod_{m=1, m\neq g}^{I_d}\left(\lambda^I_{m,\mathsf{d}}-\lambda^I_{g,\mathsf{d}}\right)}$.

Using the transformation of RVs, we get

$$f_{Z_1}(z) = \prod_{i_d=1}^{I_d}\lambda^I_{i_d,\mathsf{d}}e^{\lambda^I_{i_d,\mathsf{d}}}\sum_{g=1}^{I_d}\frac{\exp\left(-\lambda^I_{i_d,\mathsf{d}}z\right)}{\prod_{m=1, m\neq g}^{I_d}\left(\lambda^I_{m,\mathsf{d}}-\lambda^I_{g,\mathsf{d}}\right)}. \quad (9)$$

Now, we evaluate the PDF of $Y_1 = \rho|h_{\mathsf{s,d}}|^2 + \rho|h_{N_b^{\rm th},\mathsf{d}}|^2$ given $\left|B_L\right| = L$, $L \geq 1$. It is given by $f_Y(y) = \int_0^y f_{\rho|h_{\mathsf{s,d}}|^2}(y -$

$\tau) f_{\rho|h_{N_b^{\text{th}},\mathsf{d}}|^2}(\tau) d\tau$. The PDF of $\rho|h_{N_b^{\text{th}},\mathsf{d}}|^2$ given B_L can be written as [10]

$$f_{\rho|h_{N_b^{\text{th}},\mathsf{d}}|^2}(\tau) = \sum_{l=1}^{L} f_{\rho|h_{l,\mathsf{d}}|^2}(\tau) \sum_{\mathcal{P}} \prod_{j=1}^{L-N} \bar{\chi}_{i_j} \prod_{w=L-N+1}^{L-1} \chi_{i_w}, \tag{10}$$

where $\sum_{\mathcal{P}}$ denotes the summation over all $n!$ permutations $(i_1, i_2, \ldots, i_L)$ of $(1, 2, \ldots, L)$, $\bar{\chi}_{i_j} = F_{\rho|h_{i_j,\mathsf{d}}|^2}(\tau)$, and $\chi_{i_w} = \left(1 - F_{\rho|h_{i_w,\mathsf{d}}|^2}(\tau)\right)$.

For Rayleigh fading, the PDF $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau)$ and the CDF $F_{\rho|h_{i_j,\mathsf{d}}|^2}(\tau)$ are respectively given by $\lambda_{l,\mathsf{d}} \exp\left(-\lambda_{l,\mathsf{d}}\tau\right)$ and $1 - \exp\left(-\lambda_{i_j,\mathsf{d}}\tau\right)$. Upon substituting these statistics in (10) and after some algebraic manipulations, we get

$$f_{\rho|h_{N_b^{\text{th}},\mathsf{d}}|^2}(\tau) = \sum_{l=1}^{L} \lambda_{l,\mathsf{d}} \sum_{\mathcal{P}} \exp\left(-\frac{m_{l,\mathsf{d}}}{\Omega_{l,\mathsf{d}}}\tau\right) \Bigg[\exp\left(-C_a\tau\right) + \sum_{q=1}^{L-N} (-1)^q \sum_{s_1 < \ldots < s_q} \exp\left(-\left(C_a + C_b\right)\tau\right)\Bigg], \tag{11}$$

where $\sum_{s_1<\ldots<s_q} = \sum_{s_1=1}^{L-N-q+1} \sum_{s_2=s_1+1}^{L-N-q+2} \cdots \sum_{s_q=s_{q-1}+1}^{L-N}$, $C_a = \sum_{w=L-N+1}^{L-1} \lambda_{i_w,\mathsf{d}}$, and $C_b = \sum_{n=1}^{q} \lambda_{s_n,\mathsf{d}} + \lambda_{l,\mathsf{d}}$.

Having $f_{Z_1}(z)$ and $f_{Y_1}(y)$ being evaluated, the term $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right]$ can be obtained as in (5).

Now, in evaluating the second term in (4) $\mathrm{P_r}\left[B_L\right]$, the CDF of $\gamma_{\mathsf{s},k}$ is required to be found first. This SINR can be written as Y_a/Z_b. For Rayleigh fading channels, the PDF of Y_a is $\lambda_{\mathsf{s},k} \exp\left(-\lambda_{s,k}\tau\right)$ and the PDF of Z_b is similar to that found in (9) with replacing i_d by i_k and d by k. Having the CDF of $\gamma_{\mathsf{s},k}$ obtained as in (6), the term $\mathrm{P_r}\left[B_L\right]$ can be evaluated.

APPENDIX B
PROOF OF THEOREM 2

For simplicity we consider the second hops of relays have identical CDFs ($\lambda_{1,\mathsf{d}} = \lambda_{2,\mathsf{d}} = \ldots = \lambda_{K,\mathsf{d}} = \lambda_{R,\mathsf{d}}$). To find the approximate outage probability in (7), we first need to obtain $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right]$. As $\rho \to \infty$ and with finite values of ρ_I, I_k, and I_d, the CDF and the PDF of the exponential distribution can be respectively approximated as $F_{\rho|h_{m,n}|^2}(\tau) \approx \lambda_{m,n}\tau$ and $f_{\rho|h_{m,n}|^2}(\tau) \approx \lambda_{m,n}$. Based on that, the PDF in (10) can be evaluated in a closed-form expression as

$$f_{\rho|h_{N_b^{\text{th}},\mathsf{d}}|^2}(\tau) \approx L \binom{L-1}{N-1} \left(\lambda_{R,\mathsf{d}}\right)^{L-N+1} \sum_{k=0}^{N-1} \binom{N-1}{k} \times (-1)^k \left(\lambda_{R,\mathsf{d}}\right)^k (\tau)^{k+L-N}. \tag{12}$$

Now, following the same procedure as in Appendix A, the term $\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right]$ in (4) can be evaluated at high SNR as

$$\begin{aligned}\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right] \approx & -L \binom{L-1}{N-1} \frac{(\lambda_{\mathsf{d}}^I)^{I_d} (-1)^{I_d} \left(\lambda_{R,\mathsf{d}}\right)^{(L-N+1)}}{(I_d-1)!} \\ & \times \frac{e^{\lambda_{\mathsf{d}}^I}}{\lambda_{\mathsf{s},\mathsf{d}}^{-1}} \sum_{g=0}^{I_d-1} \binom{I_d-1}{g} (-1)^g \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{(-1)^k}{(k+L-N+1)} \\ & \times \frac{\Gamma\left(g+k+L-N+3, \lambda_{\mathsf{d}}^I\right)}{(k+L-N+2)} (\lambda_{\mathsf{d}}^I)^{-(g+k+L-N+2)-1} \left(\lambda_{R,\mathsf{d}}\right)^k \\ & \times u^{k+L-N+2}. \end{aligned} \tag{13}$$

This expression can be further simplified due to the fact that it is still dominant when $k = 0$ and $g = 0$. Therefore, the result in (13) can be simplified as

$$\begin{aligned}\mathrm{P_r}\left[\gamma_{\mathsf{d}} < u | B_L\right] \approx & -L \binom{L-1}{N-1} \frac{(\lambda_{\mathsf{d}}^I)^{I_d} (-1)^{I_d} \left(\lambda_{R,\mathsf{d}}\right)^{(L-N+1)}}{(I_d-1)!} \\ & \times \frac{e^{\lambda_{\mathsf{d}}^I}}{\lambda_{\mathsf{s},\mathsf{d}}^{-1}} (\lambda_{\mathsf{d}}^I)^{-(+L-N+2)-1} \frac{\Gamma\left(L-N+3, \lambda_{\mathsf{d}}^I\right)}{(L-N+1)(L-N+2)} u^{L-N+2}. \end{aligned} \tag{14}$$

Now, the second term in (4) $\mathrm{P_r}\left[B_L\right]$ can be obtained after evaluating the CDF of $\gamma_{\mathsf{s},k}$ which can be approximated as

$$\mathrm{P_r}\left[\gamma_{\mathsf{s},k} < u\right] \approx \lambda_{\mathsf{s},k} \left(1 + \frac{I_k}{\lambda_k^I}\right) u. \tag{15}$$

Having (14) and (15) being evaluated, the asymptotic outage probability can be obtained in a closed-from as in (7).

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