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Exact Outage Probability of Opportunistic DF Relay Systems With Interference at Both the Relay and the Destination Over Nakagami-m Fading Channels

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***Abstract*—In this paper, we investigate the outage behavior of a dual-hop opportunistic decode-and-forward (DF) relay system with cochannel interference (CCI) at both the relay and the destination. The source–relay and relay–destination channels and the interferers' channels at both the relay and the destination nodes are assumed to follow Nakagami-m distribution. Exact closed-form expressions for the outage probability for both independent nonidentically distributed (i.n.d.) and independent identically distributed (i.i.d.) cases of interferers' channels are derived in this paper. Furthermore, the system behavior at high SNR values is studied via deriving the asymptotic outage probability, and hence, the diversity order and the coding gain are characterized. Our results show that the cochannel interferers do not reduce the diversity gain of the system; instead, they degrade the outage performance by affecting the coding gain of the system. The accuracy of the analytical results is supported by Monte Carlo simulations.**

***Index Terms*—Cochannel interference (CCI), decode-and-forward (DF), dual-hop networks, Nakagami-m fading, opportunistic relaying.**

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I. Introduction

Cooperative or relay networks are considered among efficient techniques to provide space diversity in wireless communication. In relay networks, single or multiple relays are used in helping a user to forward his signal to the desired destination. Several relaying schemes have been introduced in [1], among which are the amplify-and-forward (AF) and the decode-and-forward (DF). In the AF scheme, the source signal is just scaled by a gain before being forwarded to the destination, whereas in the DF scheme, some signal processing and coding need to be performed by the relay before the source signal is forwarded.

Most of the work on cooperative systems has focused on noise-limited environments and ignored the interference effect on system behavior [2]–[4]. Few papers have investigated the impact of interference in such cooperative systems [5]–[7]. Particularly, in [7], Al-Qahtani *et al.* derived closed-form expressions for the outage and symbol error rate of a dual-hop AF relay system over Nakagami-m fading channels, in addition to studying the system behavior at the high-SNR regime. The interference effect at the destination node over Rayleigh and Nakagami-m fading channels was studied in [8] and [9], respectively. Recently, several papers have studied the interference impact at both the relay and the destination [10], [11]. The outage performance of a dual-hop AF relay system over Rayleigh and Nakagami-m fading channels was presented in [10] and [11], respectively.

Some key papers on cooperative systems with multiple relays are the ones presented in [12] and [13]. Particularly, in [12], Kim and Kim evaluated the outage performance of an opportunistic DF cooperative system with interference at both the relay and the destination nodes. All channels were assumed to follow Rayleigh distribution with unequal-power interferers. In [14], Yu *et al.* evaluated the outage performance of a conventional DF cooperative system over Nakagami-m fading channels by using the moment-generating function (MGF) approach. Unequal-power interferers were assumed, and the direct link was considered in the analysis. The work that includes the impact of cochannel interference (CCI) looks important since the interference can be one of the major limiting factors of the cooperative performance. Most of the aforementioned papers considered the effect of CCI in single-relay systems with interference at the relay or the destination or at both. Few studies considered the interference impact in multirelay systems with a single interferer at the relay or the destination node. The lack of strong studies that evaluate the performance of multirelay cooperative systems with interference at both the relay and the destination nodes motivates us to contribute in this area of research. Another contribution of this paper is that it is a development to the work presented in [14], where the conventional DF relaying is extended to an opportunistic DF relaying by using a different approach rather than the MGF approach. Using the best relay scheme reduces the amount of cooperation overheads and enhances the system spectral efficiency. An important contribution of this work is that it evaluates the system behavior at the high-SNR regime, where more insights about the system performance such as the diversity order and coding gain are provided.

To the best of our knowledge, the interference effect at both the relay and the destination nodes in opportunistic DF cooperative system over Nakagami-m fading channels has not been studied yet. In this paper, we consider the system in [12], where the best DF relay is selected to relay the source information with interference at both the relays and the destination. We derive exact closed-form expressions for the outage probability for both nonidentically distributed (i.n.d.) and independent identically distributed (i.i.d.) cases of interferers' channels. Furthermore, we evaluate the system performance at the high-SNR regime, where an asymptotic expression for the outage probability is derived and analyzed, and the diversity order and coding gain are provided. Due to its inherent effect on system behavior, the direct link is considered in all derivations in this paper. The situation where the direct link is not existed becomes a special case of the considered system.

The remainder of this paper is organized as follows: Section II introduces the system and channel models. The analysis of system performance is conducted in Section III. Section IV proposes the asymptotic outage behavior of the considered system. In Section V, some simulation and numerical results are presented and discussed. Finally, conclusions are given in Section VI.

II. SYSTEM AND CHANNEL MODELS

Fig. 1 shows the cooperative communication system under consideration. It consists of one source, one destination, K relay nodes, and arbitrary number of interferers at both the relays and the destination. The entire communication takes place in two phases. In the first phase, the source S transmits its signal to the destination D and the K relays. In the second phase, only the best relay among all other relays that succeeded to decode the source signal is selected to forward it to D. We assume that the signal at the kth relay is corrupted by interfering signals from I_k cochannel interferers $\{x_i\}_{i=1}^{I_k}$. The received signal at the kth relay can be expressed as

$$y_{\mathsf{r}_k} = h_{\mathsf{s},k} x_0 + \sum_{i_k=1}^{I_k} h_{i_k,k}^I x_{i_k,k}^I + n_{\mathsf{s},k} \tag{1}$$

where $h_{\mathsf{s},k}$ is the channel coefficient between S and the kth relay, x_0 is the transmitted symbol with $\mathbb{E}\{|x_0|^2\} = P_0$, $h_{i_k,k}^I$ is the channel coefficient between the i_k^{th} interferer and the kth relay, $x_{i_k,k}^I$ is the transmitted symbol from the i_k^{th} interferer with $\mathbb{E}\{|x_{i_k,k}^I|^2\} = P_{i_k,k}^I$, $n_{\mathsf{s},k} \sim \mathcal{CN}(0, N_0)$ is an additive white Gaussian noise (AWGN), and $\mathbb{E}\{\cdot\}$ denotes the expectation operation. Let us define $h_{\mathsf{s,d}}$, $h_{k,\mathsf{d}}$, and $h_{i_d,\mathsf{d}}^I$ as the channel coefficients between S and D, the kth relay and D, and the i_d^{th} interferer and D, respectively. All the channel gains are assumed to follow the Nakagami-m distribution. That is, the channel powers denoted by $|h_{\mathsf{s,d}}|^2$, $|h_{\mathsf{s},k}|^2$, $|h_{k,\mathsf{d}}|^2$, $|h_{i_k,k}^I|^2$, and $|h_{i_d,\mathsf{d}}^I|^2$ are gamma distributed random variables (RVs) with parameters $(m_{\mathsf{s,d}}, \sigma_{\mathsf{s,d}}^2/m_{\mathsf{s,d}})$, $(m_{\mathsf{s},k}, \sigma_{\mathsf{s},k}^2/m_{\mathsf{s},k})$, $(m_{k,\mathsf{d}}, \sigma_{k,\mathsf{d}}^2/m_{k,\mathsf{d}})$, $(m_{i_k,k}^I, \sigma_{I,i_k,k}^2/m_{i_k,k}^I)$, and $(m_{i_d,\mathsf{d}}^I, \sigma_{I,i_d,\mathsf{d}}^2/m_{i_d,\mathsf{d}}^I)$, respectively. In this paper, $F_X(.)$ and $f_X(.)$ denote the cumulative distribution function (cdf) and the probability density function (pdf) of an RV X. Using (1), the signal-to-interference plus noise ratio (SINR) at the kth relay can be written as $\gamma_{\mathsf{s},k} = (P_0/N_0|h_{\mathsf{s},k}|^2)/(\sum_{i_k=1}^{I_k} P_{i_k,k}^I/N_0|h_{i_k,k}^I|^2 + 1)$.

Let C_L denote a decoding set defined by the set of active relays that could have correctly decoded the message sent from the source in the first phase. It is defined as [12]

$$\begin{aligned} C_L &\triangleq \left\{k \in \mathcal{S}_r : \frac{1}{2}\ \log_2(1+\gamma_{\mathsf{s},k}) \geq R\right\} \\ &= \{k \in \mathcal{S}_r : \gamma_{\mathsf{s},k} \geq 2^{2R} - 1\} \end{aligned} \tag{2}$$

where $\mathcal{S}_r$ is a set of L relays, and R denotes a fixed spectral efficiency threshold.

In the second phase after decoding the received signal, only the best relay in C_L forwards the reencoded signal to the destination. The received signal at the destination can be expressed as

$$y_{\mathsf{d}} = h_{\mathsf{s,d}} x_0 + h_{b,\mathsf{d}} y_{\mathsf{r}_b} + \sum_{i_d=1}^{I_d} h_{i_d,\mathsf{d}}^I x_{i_d,\mathsf{d}}^I + n_{b,\mathsf{d}} \tag{3}$$

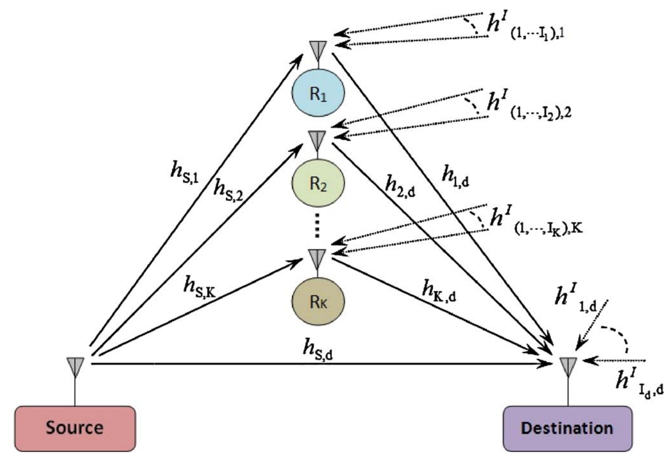

Fig. 1. Dual-hop opportunistic DF relaying system with CCI at both the relay and the destination.

where $h_{b,\mathsf{d}}$ is the channel coefficient between the best relay and the destination, y_{r_b} is the received signal at the best relay, and $n_{b,\mathsf{d}} \sim \mathcal{CN}(0, N_0)$ is the AWGN. The best relay is selected such that

$$b = \arg\ \max_{l \in C_L} \{\gamma_{l,\mathsf{d}}\} \tag{4}$$

where $\gamma_{l,\mathsf{d}}$ is the SINR at the destination resulting from the l^{th} relay being the relay that forwarded the source information. It can be written as $\gamma_{l,\mathsf{d}} = (P_l/N_0|h_{l,\mathsf{d}}|^2)/(\sum_{i_d=1}^{I_d} P_{i_d,\mathsf{d}}^I/N_0|h_{i_d,\mathsf{d}}^I|^2 + 1)$, where P_l, $P_{i_d,\mathsf{d}}^I$, and N_0 are the transmit power of the lth active relay, the transmit power of the i_d^{th} interferer, and the AWGN power at the destination, respectively, and I_d is the number of interferers at the destination node. Equivalently

$$b = \arg\ \max_{l \in C_L} \left\{\frac{P_l}{N_0}|h_{l,\mathsf{d}}|^2\right\} \tag{5}$$

since the denominator is common to the SINRs from all relays belonging to C_L.

The destination finally combines the signals from the source and the best relay using maximal-ratio combining. The end-to-end (e2e) SINR at the destination output can be written as $\gamma_{\mathsf{d}} \triangleq \gamma_{\mathsf{s,d}} + \gamma_{b,\mathsf{d}} = (P_0/N_0|h_{\mathsf{s,d}}|^2 + P_b/N_0|h_{b,\mathsf{d}}|^2)/(\sum_{i_d=1}^{I_d} P_{i_d,\mathsf{d}}^I/N_0|h_{i_d,\mathsf{d}}^I|^2 + 1)$.

III. PERFORMANCE ANALYSIS

Here, we evaluate the system outage for both i.n.d. and i.i.d. interferers' fading channels. The probability of the decoding set defined in (2) can be written as

$$\mathrm{P_r}[C_L] = \prod_{l \in C_L} \mathrm{P_r}[\gamma_{\mathsf{s},l} \geq u] \prod_{m \notin C_L} \mathrm{P_r}[\gamma_{\mathsf{s},m} < u] \tag{6}$$

where $u = (2^{2R} - 1)$. The outage probability for the considered system can be achieved by averaging over all possible decoding sets as follows [12]:

$$\begin{aligned} P_{\mathsf{out}} &\triangleq \mathrm{P_r}\left[\frac{1}{2}\ \log_2(1+\gamma_{\mathsf{d}}) < R\right] \\ &= \sum_{L=0}^{K} \sum_{C_L} \mathrm{P_r}[\gamma_{\mathsf{d}} < u | C_L] \mathrm{P_r}[C_L] \end{aligned} \tag{7}$$

where the internal summation is taken over all $\binom{K}{L}$ possible decoding sets C_L of size L from the original set with K relays. To evaluate (7), we first need to derive $\mathrm{P_r}[\gamma_{\mathsf{d}} < u | C_L]$ and then $\mathrm{P_r}[C_L]$ as follows.

A. Nonidentical Interferers

Let $\rho \triangleq P_0/N_0 = P_l/N_0$ and $\rho_I \triangleq P^I_{i_k,k}/N_0 = P^I_{i_d,\mathsf{d}}/N_0$. Then, $\rho|h_{\mathsf{s,d}}|^2$, $\rho|h_{\mathsf{s},k}|^2$, $\rho_I|h^I_{i_k,k}|^2$, $\rho|h_{l,\mathsf{d}}|^2$, and $\rho_I|h^I_{i_d,\mathsf{d}}|^2$ are gamma distributed with parameters $(m_{\mathsf{s,d}}, \rho\sigma^2_{\mathsf{s,d}}/m_{\mathsf{s,d}} = 1/\alpha_{\mathsf{s,d}} m_{\mathsf{s,d}})$, $(m_{\mathsf{s},k}, \rho\sigma^2_{\mathsf{s},k}/m_{\mathsf{s},k} = 1/\alpha_{\mathsf{s},k} m_{\mathsf{s},k})$, $(m^I_{i_k,k},\ \ \rho_I\sigma^2_{I,i_k,k}/m^I_{i_k,k} = 1/\alpha^I_{i_k,k} m^I_{i_k,k})$, $(m_{l,\mathsf{d}},\ \rho\sigma^2_{l,\mathsf{d}}/m_{l,\mathsf{d}} = 1/\alpha_{l,\mathsf{d}} m_{l,\mathsf{d}})$, and $(m^I_{i_k,k},\ \rho_I\sigma^2_{I,i_d,\mathsf{d}}/m^I_{i_k,k} = 1/\alpha^I_{i_d,\mathsf{d}} m^I_{i_k,k})$, respectively. We can then calculate P_{out} in (7) using the following result.

Lemma 1: For the case of unequal-power interferers, that is, $\alpha^I_{i_n,n} \neq \alpha^I_{j_n,n}$, when $i_n \neq j_n$, $n \in \mathcal{S}_r \bigcup \{d\}$, the term $\mathrm{P_r}[\gamma_{\mathsf{d}} < u|C_L]$ for $L \geq 1$ is given by (8), shown at the bottom of the page. where $C_a = m^{m_{\mathsf{s,d}}}_{\mathsf{s,d}}/\Omega^{m_{\mathsf{s,d}}}_{\mathsf{s,d}}\Gamma(m_{\mathsf{s,d}})$, $C_b(l) = m^{m_{l,\mathsf{d}}}_{l,\mathsf{d}}/\Omega^{m_{l,\mathsf{d}}}_{l,\mathsf{d}}\Gamma(m_{l,\mathsf{d}})$, $C_1(r,l) = m_{l,\mathsf{d}} + r - 1$, $C_2(l) = m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}} - m_{\mathsf{s,d}}/\Omega_{\mathsf{s,d}}$, $C_3(r) = m_{\mathsf{s,d}} - r - 1$, $C_4(i_d,g) = (\alpha^I_{i_d,\mathsf{d}})^{-g-1}\Gamma(g+1, \alpha^I_{i_d,\mathsf{d}})$, $C_5(r,k_1) = m_{\mathsf{s,d}} - r + k_1 - 1$, $C_6(r,l,q_i) = \sum_{i=1}^n q_i + m_{l,\mathsf{d}} + r - 1$, $C_7(l,j_s) = \sum_{s=1}^n m_{j_s,\mathsf{d}}/\Omega_{j_s,\mathsf{d}} + m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}} - m_{\mathsf{s,d}}/\Omega_{\mathsf{s,d}}$, $C_8(r,k_2) = m_{\mathsf{s,d}} - r + k_2 - 1$, $C_9(l,j_s) = \sum_{s=1}^n m_{j_s,\mathsf{d}}/\Omega_{j_s,\mathsf{d}} + m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}}$, $\Lambda_1(i_d,u) = m_{\mathsf{s,d}}/(\Omega_{\mathsf{s,d}})u + \alpha^I_{i_d,\mathsf{d}}$, $\Lambda_2(i_d,l,u) = m_{l,\mathsf{d}}/(\Omega_{l,\mathsf{d}})u + \alpha^I_{i_d,\mathsf{d}}$, and $\Lambda_3(i_d,l,j_s,u) = (\sum_{s=1}^n m_{j_s,\mathsf{d}}/\Omega_{j_s,\mathsf{d}} + m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}})u + \alpha^I_{i_d,\mathsf{d}}$.

Proof: See Appendix A. ■

For $L = 0$, the term $\mathrm{P_r}[\gamma_{\mathsf{d}} < u|C_L]$ can be simply found by replacing k for relay by d for destination and i_k by i_d in the cdf of $\gamma_{\mathsf{s},k}$ in Lemma 2.

Now, we find the second term in (7), which is $\Pr[C_L]$.

Lemma 2: The cdf of $\gamma_{\mathsf{s},k}$, which is a part of $\Pr[C_L]$ in (6) for the nonidentical interferers' case, is given by

$$\begin{aligned}
&\mathrm{P_r}[\gamma_{\mathsf{s},k} < u]_{\mathrm{NI}} \\
&\quad = -\sum_{i_k=1}^{I_k}\sum_{i=1}^{m^I_{i_k,k}} \frac{(-1)^{m^I_{i_k,k}}\beta^{i-1}_{i_k}\exp\left(\alpha^I_{i_k,k}\right)}{(i-1)!} \\
&\qquad \times \sum_{g=0}^{m^I_{i_k,k}-1}\binom{m^I_{i_k,k}-1}{g}(-1)^g \frac{m^{m_{\mathsf{s},k}}_{\mathsf{s},k}}{\Omega^{m_{\mathsf{s},k}}_{\mathsf{s},k}\Gamma(m_{\mathsf{s},k})} \\
&\qquad \times \left(\frac{(m_{\mathsf{s},k}-1)!}{\left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}}}\Xi_1 - \sum_{j=0}^{m_{\mathsf{s},k}-1}\frac{(m_{\mathsf{s},k}-1)!u^j}{j!\left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}-j}}\Xi_2\right)
\end{aligned} \tag{9}$$

where $\Xi_1 = \Gamma(g+1, \alpha^I_{i_k,k})/(\alpha^I_{i_k,k})^{g+1}$, $\Xi_2 = \Gamma(g+j+1, m_{\mathsf{s},k}/\Omega_{\mathsf{s},k}u + \alpha^I_{i_k,k})/(m_{\mathsf{s},k}/\Omega_{\mathsf{s},k}u + \alpha^I_{i_k,k})^{g+j+1}$, and $\Gamma(.,.)$ denotes the incomplete gamma function [17, eq. (8.352.2)].

Proof: See Appendix B. ■

B. Identical Interferers

This section considers the case of identical interferers, i.e., $(m^I_{i_k,k} = \cdots = m^I_k,\ \ \alpha^I_{i_k,k} = \cdots = \alpha^I_k)$ and $(m^I_{i_d,\mathsf{d}} = \cdots = m^I_{\mathsf{d}},$

$$\begin{aligned}
\mathrm{P_r}[\gamma_{\mathsf{d}} < u|C_L] = &-\sum_{i_d=1}^{I_d}(-1)^{m^I_{i_d,\mathsf{d}}}\exp\left(\alpha^I_{i_d,\mathsf{d}}\right)\sum_{i=1}^{m^I_{i_d,\mathsf{d}}}\frac{\beta^{i-1}_{i_d}}{(i-1)!}\sum_{g=0}^{m^I_{i_d,\mathsf{d}}-1}\binom{m^I_{i_d,\mathsf{d}}-1}{g}(-1)^g C_a\sum_{r=0}^{m_{\mathsf{s,d}}-1}\binom{m_{\mathsf{s,d}}-1}{r}(-1)^r\sum_{l=1}^{L}C_b(l) \\
&\times\Bigg[\frac{C_1(r,l)!}{C_2(l)^{m_{l,\mathsf{d}}+r}}\left(\frac{C_3(r)!C_4(i_d,g)}{\left(\frac{m_{\mathsf{s,d}}}{\Omega_{\mathsf{s,d}}}\right)^{m_{\mathsf{s,d}}-r}} - \sum_{k_3=0}^{C_3(r)}\frac{C_3(r)!\Lambda_1(i_d,u)^{-g-k_3-1}u^{k_3}}{k_3!\left(\frac{m_{\mathsf{s,d}}}{\Omega_{\mathsf{s,d}}}\right)^{m_{\mathsf{s,d}}-r-k_3}}\Gamma\left(g+k_3+1, \Lambda_1(i_d,u)\right)\right) \\
&-\sum_{k_1=0}^{C_1(r,l)}\frac{1}{k_1!}\frac{C_1(r,l)!}{C_2(l)^{m_{l,\mathsf{d}}+r-k_1}} \\
&\times\left(\frac{C_5(r,k_1)!C_4(i_d,g)}{\left(\frac{m_{l,\mathsf{d}}}{\Omega_{l,\mathsf{d}}}\right)^{m_{\mathsf{s,d}}-r+k_1}} - \sum_{k_4=0}^{C_5(r,k_1)}\frac{C_5(r,k_1)!\Lambda_2(i_d,l,u)^{-g-k_4-1}u^{k_4}}{k_4!\left(\frac{m_{l,\mathsf{d}}}{\Omega_{l,\mathsf{d}}}\right)^{m_{\mathsf{s,d}}-r+k_1-k_4}}\Gamma\left(g+k_4+1, \Lambda_2(i_d,l,u)\right)\right) \\
&+\sum_{n=1}^{L}(-1)^n\sum_{j_1<\cdots<j_n, j_{(.)}\neq l}\ \sum_{q_1=\cdots=q_n=0}\frac{\prod_{w=1}^n\left(\frac{m_{j_w,\mathsf{d}}}{\Omega_{j_w,\mathsf{d}}}\right)^{q_w}}{\prod_{p=1}^n q_p!} \\
&\times\Bigg\{\frac{C_6(r,l,q_i)!}{C_7(l,j_s)^{\sum_{i=1}^n q_i+m_{l,\mathsf{d}}+r}}\left(\frac{C_3(r)!C_4(i_d,g)}{\left(\frac{m_{\mathsf{s,d}}}{\Omega_{\mathsf{s,d}}}\right)^{m_{\mathsf{s,d}}-r}} - \sum_{k_5=0}^{C_3(r)}\frac{C_3(r)!}{k_5!}\frac{\Lambda_1(i_d,u)^{-g-k_5-1}u^{k_5}}{\left(\frac{m_{\mathsf{s,d}}}{\Omega_{\mathsf{s,d}}}\right)^{m_{\mathsf{s,d}}-r-k_5}}\Gamma(g+k_5+1, \Lambda_1(i_d,u))\right) \\
&-\sum_{k_2=0}^{C_6(r,l,q_i)}\frac{C_6(r,l,q_i)!}{k_2!C_7(l,j_s)^{\sum_{i=1}^n q_i+m_{l,\mathsf{d}}+r-k_2}} \\
&\times\left(\frac{C_8(r,k_2)!C_4(i_d,g)}{C_9(l,j_s)^{m_{\mathsf{s,d}}-r+k_2}} - \sum_{k_6=0}^{C_8(r,k_2)}\frac{C_8(r,k_2)!u^{k_6}}{k_6!}\sum_{k_6=0}^{C_8(r,k_2)}\frac{\Lambda_3(i_d,l,j_s,u)^{-g-k_6-1}}{C_9(l,j_s)^{m_{\mathsf{s,d}}-r+k_2-k_6}}\Gamma\left(g+k_6+1, \Lambda_3(i_d,l,j_s,u)\right)\right)\Bigg\}\Bigg]
\end{aligned} \tag{8}$$

$\alpha^I_{i_d,\mathsf{d}} = \cdots = \alpha^I_{\mathsf{d}}$). Now, the denominator of γ_{d} becomes $Z_2 = \sum_{i_d=1}^{I_d} \rho_I |h^I_{i_d,\mathsf{d}}|^2 + 1 = X_2 + 1$, where X_2 is a summation of gamma distributed RVs with the same parameter and the same average power whose pdf is given by $f_{X_2}(x) = (\alpha^I_{\mathsf{d}})^{I_{\mathsf{d}} m^I_{\mathsf{d}}} / \Gamma(I_{\mathsf{d}} m^I_{\mathsf{d}})\, x^{I_{\mathsf{d}} m^I_{\mathsf{d}}-1} \exp(-\alpha^I_{\mathsf{d}} x)$. Using the transformation of RVs and then the binomial formula, we get

$$f_{Z_2}(z) = -\frac{\left(\alpha^I_{\mathsf{d}}\right)^{I_d m^I_{\mathsf{d}}} (-1)^{I_d m^I_{\mathsf{d}}} \exp\left(\alpha^I_{\mathsf{d}}\right)}{\Gamma\left(I_{\mathsf{d}} m^I_{\mathsf{d}}\right)} \times \sum_{g=0}^{I_d m^I_{\mathsf{d}}-1} \binom{I_{\mathsf{d}} m^I_{\mathsf{d}} - 1}{g} (-1)^g z^g \exp\left(-\alpha^I_{\mathsf{d}} z\right). \tag{10}$$

Finally, using the relation (19) in Appendix A, the term $\mathrm{P_r}[\gamma_{\mathsf{d}} < u | C_L]$ in (7) can be evaluated. The expression is omitted due to space limitations.

In evaluating the term $\mathrm{P_r}[C_L]$ for this case, the SINR $\gamma_{\mathsf{s},k}$ can be written as $\gamma_{\mathsf{s},k} = Y_1/Z_3$. The pdf of Y_1 is similar to $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau)$ in Appendix A with replacing l by s and d by k. The pdf of Z_3 is similar to that found in (10) with replacing d by k.

Now, following the same procedure as in Appendix B, the cdf of $\gamma_{\mathsf{s},k}$ can be evaluated as

$$\begin{aligned}
\mathrm{P_r}\left[\gamma_{\mathsf{s},k} < u\right]_{\mathrm{I}} &= -\frac{\left(\alpha^I_k\right)^{I_k m^I_k} (-1)^{I_k m^I_k} \exp\left(\alpha^I_k\right)}{\Gamma\left(I_k m^I_k\right)} \times \sum_{g=0}^{I_k m^I_k - 1} \binom{I_k m^I_k - 1}{g} (-1)^g \frac{m_{\mathsf{s},k}^{m_{\mathsf{s},k}}}{\Omega_{\mathsf{s},k}^{m_{\mathsf{s},k}} \Gamma(m_{\mathsf{s},k})} \\
&\times \left(\frac{(m_{\mathsf{s},k} - 1)!}{\left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}}} \frac{\Gamma\left(g+1, \alpha^I_k\right)}{(\alpha^I_k)^{g+1}} - \sum_{j=0}^{m_{\mathsf{s},k}-1} \frac{(m_{\mathsf{s},k}-1)! u^j}{j! \left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}-j}} \times \frac{\Gamma\left(g+j+1, \frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}} u + \alpha^I_k\right)}{\left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}} u + \alpha^I_k\right)^{g+j+1}} \right).
\end{aligned} \tag{11}$$

IV. Asymptotic Outage Behavior

Since the achieved results are too complex to give any insight about the key performance measures, it is of great interest to look into the high-SNR regime, where simple expressions can be obtained. Note that in the high-SNR regime, we can express the outage probability as $P_{\mathsf{out}} \approx (G_{\mathsf{c}}\rho)^{-G_{\mathsf{d}}}$, where G_{d} is the achieved diversity order of the system, whereas G_{c} is the coding gain of the system. When $\rho \to \infty$ with finite values of ρ_I, I_k, and I_d, the outage probability is approximated as follows:

$$P_{\mathsf{out}} \approx \begin{cases} \mathbb{L}_1 \frac{\left(\frac{m_{\mathsf{s},k}}{\alpha^I_k \Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}}}{\Gamma(m_{\mathsf{s},k}+1)} \left(\frac{u}{\rho}\right)^{m_{\mathsf{s},k}}, & m_{\mathsf{s},k} < m_{l,\mathsf{d}} \\ \mathbb{L}_2 \frac{\left(\frac{m_{\mathsf{s},k}}{\alpha^I_k \Omega_{\mathsf{s},k}}\right)\left(\frac{m_{l,\mathsf{d}}}{\alpha^I_k \Omega_{l,\mathsf{d}}}\right)^{\sum_{l=1}^L m_{l,\mathsf{d}}}}{\Gamma\left(\sum_{l=1}^L m_{\mathsf{s},k}+1\right)\Gamma(m_{\mathsf{s},d})} \times \left(\frac{u}{\rho}\right)^{\sum_{l=1}^L m_{l,\mathsf{d}} + m_{\mathsf{s},d}}, & m_{\mathsf{s},k} > m_{l,\mathsf{d}} \end{cases} \tag{12}$$

where $\mathbb{L}_1$ and $\mathbb{L}_2$ are constants given by $\mathbb{L}_1 = (-1)^{I_k m^I_k} \exp(\alpha^I_k) / m_{\mathsf{s},k} (\alpha^I_k)^{-1} \Gamma(I_k m^I_k) \sum_{g=0}^{I_k m^I_k - 1} \binom{I_k m^I_k - 1}{g} \Gamma(m_{\mathsf{s},k}+g+1, \alpha^I_k)(-1)^g (\alpha^I_k)^{-g}$ and $\mathbb{L}_2 = (-1)^{I_d m^I_{\mathsf{d}}} B(a_1, m_{\mathsf{s},d}) (\alpha^I_{\mathsf{d}})^{-I_d m^I_{\mathsf{d}}+1} \exp(\alpha^I_{\mathsf{d}}) / \Gamma(I_d m^I_{\mathsf{d}}) \sum_{g=0}^{I_d m^I_{\mathsf{d}}-1} \binom{I_d m^I_{\mathsf{d}} - 1}{g} (-1)^g (\alpha^I_{\mathsf{d}})^{-g} \Gamma(a_2, \alpha^I_{\mathsf{d}})$, $B(\lambda, \mu)$ is the beta function defined as $B(\lambda, \mu) = \Gamma(\lambda)\Gamma(\mu)/\Gamma(\lambda+\mu)$ [17, eq. (8.38)], $a_1 = \sum_{l=1}^L m_{l,\mathsf{d}} + 1$, and $a_2 = \sum_{l=1}^L m_{l,\mathsf{d}} + m_{\mathsf{s},d} + g + 1$.

Proof: See Appendix C. ■

We observe that the diversity order of the system is given by $\min(m_{\mathsf{s},k}, m_{l,d})L + m_{\mathsf{s},d}$, whereas the coding gain of the system is given by (13), shown at the bottom of the page. We assume that the fading parameters over all links are equal, i.e., $(m_{\mathsf{s},r} = m_{l,k} = m_{\mathsf{s},d} = m)$. This assumption does not affect the system asymptotic behavior. Therefore, it is clear that this system model achieves a full diversity order that is equal to $m(L+1)$, despite the presence of the interference. This is expected since the interferers' powers α^I_k and α^I_{d} are assumed to be finite while deriving the diversity order of the system. In addition, we can observe that the coding gain is affected by the interferers' powers. More importantly, our findings show that the diversity order is determined by the most severely faded link while the coding gain is affected by the total interferers' powers.

V. Numerical Results

Here, we illustrate the validity of the expressions derived in Sections III and IV. We also provide some numerical examples to show the effect of the interference and the fading parameter on the system performance.

In Fig. 2, we illustrate the validity of our analysis via providing the analytical, simulation, and asymptotic curves for the considered system. It is shown that the analytical results and the asymptotic curves perfectly fit with the simulation results. Moreover, the enhancement on system performance due to use of more relays is obvious in this figure. It is also clear that the system can still achieve full diversity gain in the presence of a finite number of interferers with finite power.

$$G_{\mathsf{c}} = \begin{cases} \left(\mathbb{L}_1 \frac{\left(\frac{m_{\mathsf{s},k}}{\alpha^I_k \Omega_{\mathsf{s},k}}\right)^{m_{\mathsf{s},k}}}{\Gamma(m_{\mathsf{s},k}+1)}\right)^{-\frac{1}{m_{\mathsf{s},k}}}, & m_{\mathsf{s},k} < m_{l,\mathsf{d}} \\ \left(\mathbb{L}_2 \frac{\left(\frac{m_{\mathsf{s},k}}{\Omega_{\mathsf{s},k}}\right)\left(\frac{m_{l,\mathsf{d}}}{\alpha^I_k \Omega_{l,\mathsf{d}}}\right)^{\sum_{l=1}^L m_{\mathsf{s},k}}}{\Gamma\left(\sum_{l=1}^L m_{l,\mathsf{d}}+1\right)\Gamma(m_{\mathsf{s},d})}\right)^{-\frac{1}{\sum_{l=1}^L m_{l,\mathsf{d}}+m_{\mathsf{s},d}}}, & m_{\mathsf{s},k} > m_{l,\mathsf{d}} \end{cases} \tag{13}$$

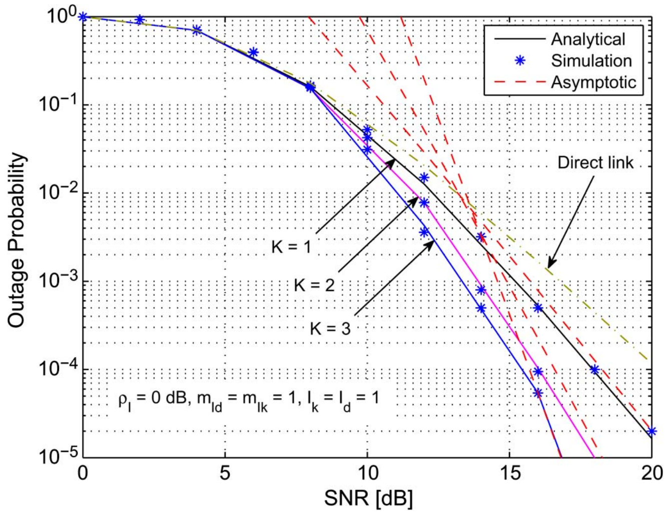

Fig. 2. Outage probability versus average SNR of the opportunistic DF relaying system with a single interferer at the relays and the destination for different number of relays. We have set $\gamma_{\text{th}} = 4.77$ dB, $m_{\text{s},d} = 3$, $\sigma^2_{\text{s},d} = 1$, $m_{\text{s},1} = m_{1,\text{d}} = 1$, $\sigma^2_{\text{s},1} = 0.2$, $\sigma^2_{1,\text{d}} = 0.4$, $m_{\text{s},2} = m_{2,\text{d}} = 2$, $\sigma^2_{\text{s},2} = 0.2$, $\sigma^2_{2,\text{d}} = 0.6$, $m_{\text{s},3} = m_{3,\text{d}} = 3$, $\sigma^2_{\text{s},3} = 0.2$, $\sigma^2_{3,\text{d}} = 0.8$, and $(\sigma^I_k)^2 = (\sigma^I_{\text{d}})^2 = 0.01$.

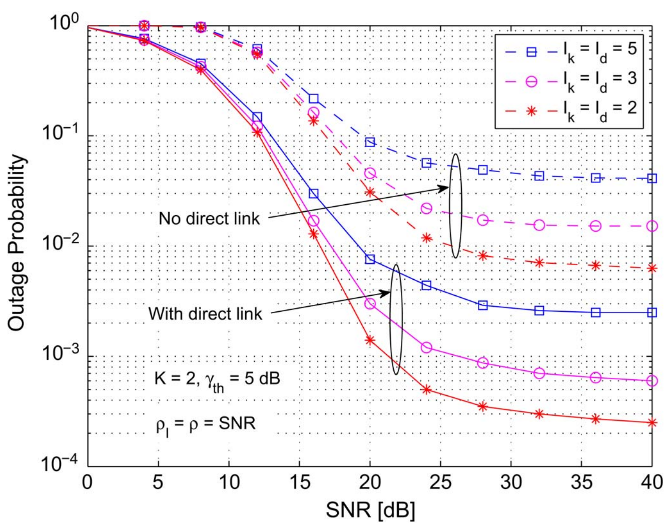

Fig. 3. Outage probability versus average SNR of the opportunistic DF relaying system with two relays for different number of interferers at the relays and the destination. We have set $m_{\text{s},d} = 1$, $\sigma^2_{\text{s},d} = 1$, $m_{\text{s},k}$ and $m_{k,\text{d}} = k$ for $k = 1, 2$, $\sigma^2_{\text{s},1} = \sigma^2_{\text{s},2} = 0.2$, $\sigma^2_{1,\text{d}} = 0.4$, $\sigma^2_{2,\text{d}} = 0.6$, $m^I_1 = m^I_2 = 1$, $(\sigma^I_k)^2 = (\sigma^I_{\text{d}})^2 = 0.01$, and $m^I_{\text{d}} = 1$.

Figs. 3 and 4 show the effect of interference on the system behavior. In Fig. 3, the number of interferers at both the relays and the destination nodes is assumed to be equal. It is clearly shown that when the number of interferers increases, hence the total interference power, the outage probability performance deteriorates further, as expected. In addition, the improvement achieved in the system performance due to the direct link is obvious in this figure. Fig. 4 provides a comparison of the interference effect at the relay and the destination nodes on the system behavior. As shown, the interference at the destination node affects the system performance more severely than the interference at the relay. This is because the impact of the interference at the relay node is reduced by the decoding process performed by the relay itself. Finally, a floor appears in these two figures due to the effect of interference on the system performance. This is expected since the transmit SNR of both the desired user ρ and the interferers ρ_I and, thus, their powers are assumed to be increasing. This explains why the diversity order of this case is equal to zero.

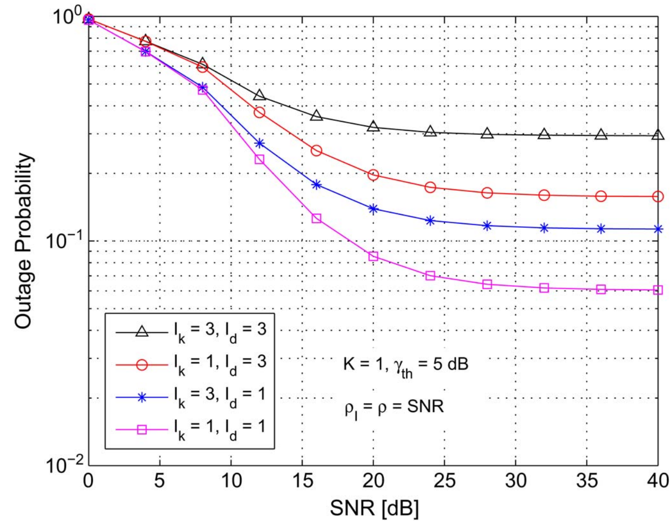

Fig. 4. Outage probability versus average SNR of the opportunistic DF relaying system with one relay for different number of interferers at the relay and the destination. We have set $m_{\text{s},d} = 1$, $\sigma^2_{\text{s},d} = 1$, $m_{\text{s},1} = 1$, $m_{1,\text{d}} = 2$, $\sigma^2_{\text{s},1} = 0.2$, $\sigma^2_{1,\text{d}} = 0.4$, $m^I_1 = 1$, $(\sigma^I_1)^2 = (\sigma^I_{\text{d}})^2 = 0.05$, and $m^I_{\text{d}} = 1$.

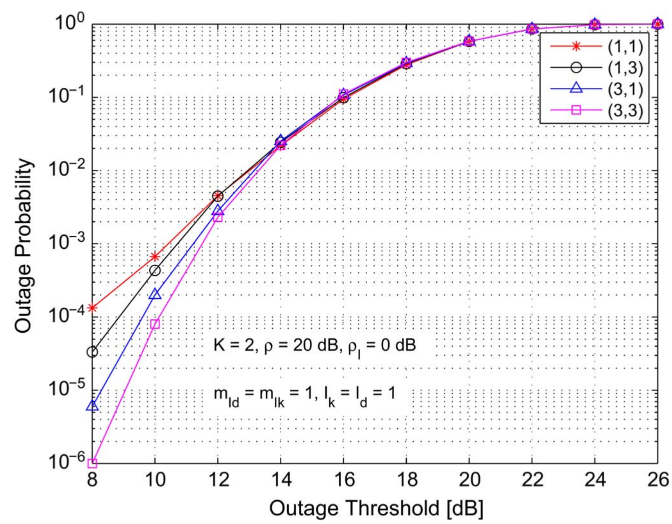

Fig. 5. Outage probability versus outage threshold of the opportunistic DF relaying system with two relays for different values of fading parameters $(m_{\text{s},k}, m_{k,\text{d}})$. We have set $m_{\text{s},d} = 3$, $\sigma^2_{\text{s},d} = 1$, $\sigma^2_{\text{s},1} = 0.2$, $\sigma^2_{1,\text{d}} = 0.4$, $\sigma^2_{\text{s},2} = 0.2$, $\sigma^2_{2,\text{d}} = 0.6$, and $(\sigma^I_k)^2 = (\sigma^I_{\text{d}})^2 = 0.01$.

Fig. 5 describes the effect of fading parameter m on the outage performance. In this figure, we use m_1 and m_2 to represent the fading parameters of the first hop and second hop of both relays, respectively. It is shown in this figure that the worst behavior is achieved when the fading parameters of the two hops of both relays equal unity, which is the Rayleigh case, as expected. Moreover, it is shown that having m_1 larger than m_2 gives better performance compared with the case where m_2 is larger than m_1. This is because having m_1 being smaller may cause the relays unable to decode the source signal, which will affect the transmission on the second hop and, thus, the overall system behavior. Finally, the best behavior can be achieved by enhancing the channels of both hops at the same time, as expected.

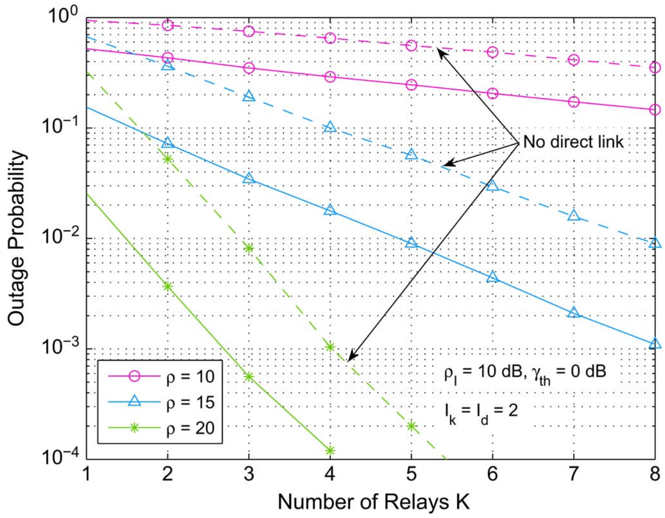

Fig. 6. Outage probability versus number of relays of the opportunistic DF relaying system for different values of ρ. We have set $m_{\mathrm{s},d} = 1$, $\sigma^2_{\mathrm{s},d} = 1$, $m_{\mathrm{s},k} = m_{k,\mathrm{d}} = k$ for $k = 1,\ldots,8$, $\sigma^2_{\mathrm{s},1} = \cdots = \sigma^2_{\mathrm{s},8} = 0.2$, $\sigma^2_{1,\mathrm{d}} = 0.3$, $\sigma^2_{2,\mathrm{d}} = 0.4$, $\sigma^2_{3,\mathrm{d}} = 0.5$, $\sigma^2_{4,\mathrm{d}} = 0.6$, $\sigma^2_{5,\mathrm{d}} = 0.7$, $\sigma^2_{6,\mathrm{d}} = 0.8$, $\sigma^2_{7,\mathrm{d}} = 0.9$, $\sigma^2_{8,\mathrm{d}} = 1$, $m^I_k = 1$ and $(\sigma^I_k)^2 = 0.5$ for $k = 1,\ldots,8$, $m^I_{\mathrm{d}} = 1$, and $(\sigma^I_{\mathrm{d}})^2 = 0.5$.

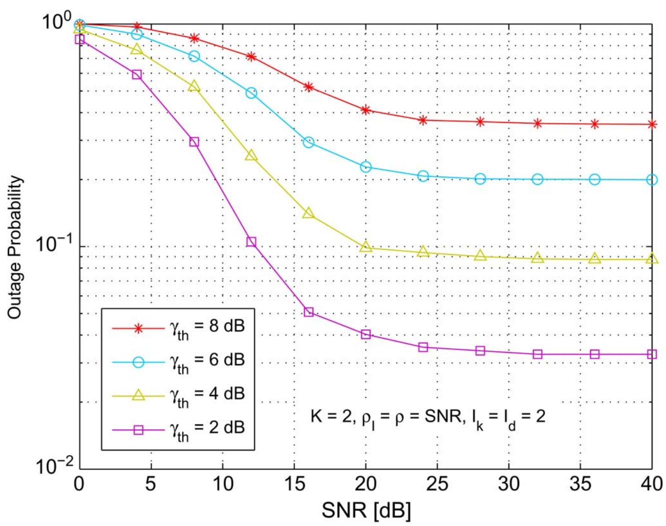

Fig. 7. Outage probability versus average SNR of the opportunistic DF relaying system with two relays for different values of outage threshold. We have set $m_{\mathrm{s},d} = 1$, $\sigma^2_{\mathrm{s},d} = 1$, $m_{\mathrm{s},k} = m_{k,\mathrm{d}} = k$ for $k = 1,2$, $\sigma^2_{\mathrm{s},1} = \sigma^2_{\mathrm{s},2} = 0.2$, $\sigma^2_{1,\mathrm{d}} = 0.4$, $\sigma^2_{2,\mathrm{d}} = 0.6$, $m^I_1 = m^I_2 = 2$, $(\sigma^I_k)^2 = (\sigma^I_{\mathrm{d}})^2 = 0.1$, and $m^I_{\mathrm{d}} = 1$.

Fig. 6 shows that the considered relay system still achieves performance gain, and the outage probability decreases when the number of relays K increases, but the slope depends on the SNR values. In addition, the achieved gain in system behavior due to the existence of the direct link is clear in the figure.

In Fig. 7, we investigate the effect of outage threshold on the system performance. Clearly and as expected, as the value of the outage threshold increases, the worse the achieved behavior becomes.

VI. Conclusion

In this paper, we have evaluated the outage performance of a dual-hop opportunistic DF relay system in the presence of interference at both the relay and the destination nodes. We derived exact closed-form expressions for the outage probability with the desired user channels and the interferers' channels being Nakagami-m distributed. Furthermore, the outage performance of the proposed system was studied at the high-SNR regime via deriving the asymptotic outage probability. The achieved results were validated via the Monte Carlo simulations, which show an accurate fitting with the analytical and asymptotic results. In addition, the findings of this paper reveal that for the case where the interferers' powers do not scale with the SNR, the presence of interference does not reduce the diversity order of the system; however, it affects the system performance through the coding gain. On the other hand, having the interferers' powers increase with the SNR results in a noise floor in the outage performance and, hence, a zero diversity gain. Finally, the results show that having the fading parameter of the first hop better than that of the second hop gives better performance compared with the opposite case.

Appendix A
Proof of Lemma 1

Here, we evaluate the first term $\mathrm{P_r}[\gamma_{\mathrm{d}} < u|C_L]$ in (7). The e2e SINR γ_{d} can be written as a ratio of two RVs $\gamma_{\mathrm{d}} = Y/Z$. The proof is carried out through the following series of results.

Proposition 1: The pdf of $Z = \sum_{i_d=1}^{I_d} \rho_I |h^I_{i_d,\mathrm{d}}|^2 + 1 = X + 1$ is given by

$$f_Z(z) = -\sum_{i_d=1}^{I_d} (-1)^{m^I_{i_d,\mathrm{d}}} \exp\left(\alpha^I_{i_d,\mathrm{d}}\right) \sum_{i=1}^{m^I_{i_d,\mathrm{d}}} \frac{\beta^{i-1}_{i_d}}{(i-1)!} \times \sum_{g=0}^{m^I_{i_d,\mathrm{d}}-1} \binom{m^I_{i_d,\mathrm{d}}-1}{g} (-1)^g z^g \exp\left(-\alpha^I_{i_d,\mathrm{d}} z\right). \quad (14)$$

Proof: The pdf of X is given by

$$f_X(x) = \sum_{i_d=1}^{I_d} \sum_{i=1}^{m^I_{i_d,\mathrm{d}}} \frac{\beta^{i-1}_{i_d} x^{m^I_{i_d,\mathrm{d}}-1}}{(i-1)!} \exp\left(-\alpha^I_{i_d,\mathrm{d}} x\right) \quad (15)$$

where $\beta^{i-1}_{i_d} = \prod_{l=1}^{I_d} (\alpha^I_{l,\mathrm{d}})^{m^I_{l,\mathrm{d}}} / (m^I_{i_d,\mathrm{d}} - i)! d^{i-1}/ds^{i-1} [\prod_{\substack{n=1\\ n\neq i_d}}^{N} (\alpha^I_{n,\mathrm{d}} + s)^{-m^I_{n,\mathrm{d}}}]$ evaluated at $s = \alpha^I_{i_d,\mathrm{d}}$.
Using the transformation of RVs for $Z = X + 1$, then the binomial rule, we can get the pdf of Z as (14). ■

Proposition 2: Let $Y = \rho|h_{\mathrm{s,d}}|^2 + \rho|h_{b,\mathrm{d}}|^2$ with $|C_L| = L$, $L \geq 1$. The pdf of Y is given by

$$f_Y(y) = C_a \sum_{r=0}^{m_{\mathrm{s,d}}-1} \binom{m_{\mathrm{s,d}}-1}{r} (-1)^r \sum_{l=1}^{L} C_b(l) \times \left[\frac{C_1(r,l)!}{C_2(l)^{m_{l,\mathrm{d}}+r}} y^{m_{\mathrm{s,d}}-r-1} \exp\left(-\frac{m_{\mathrm{s,d}}}{\Omega_{\mathrm{s,d}}} y\right) - \sum_{k_1=0}^{C_1(r,l)} \frac{C_1(r,l)!}{k_1! C_2(l)^{m_{l,\mathrm{d}}+r-k_1}} y^{m_{\mathrm{s,d}}-r+k_1-1} \times \exp\left(-\frac{m_{l,\mathrm{d}}}{\Omega_{l,\mathrm{d}}} y\right) + \sum_{n=1}^{L} (-1)^n \right.$$

$$\times \sum_{j_1<\cdots<j_n, j_{(.)}\neq l} \sum_{q_1=\cdots=q_n=0} \frac{\prod_{w=1}^{n}\left(\frac{m_{j_w,\mathsf{d}}}{\Omega_{j_w,\mathsf{d}}}\right)^{q_w}}{\prod_{p=1}^{n} q_p!}$$
$$\times \left\{ \frac{C_6(r,l,q_i)!\, y^{m_{\mathsf{s},\mathsf{d}}-r-1}}{C_7(l,j_s)^{\sum_{i=1}^{n} q_i + m_{l,\mathsf{d}}+r}} \exp\left(-\frac{m_{\mathsf{s},\mathsf{d}}}{\Omega_{\mathsf{s},\mathsf{d}}}y\right) \right.$$
$$- \sum_{k_2=0}^{C_6(r,l,q_i)} \frac{C_6(r,l,q_i)!}{k_1! C_7(l,j_s)^{\sum_{i=1}^{n} q_i + m_{l,\mathsf{d}}+r-k_2}}$$
$$\left.\left. \times y^{m_{\mathsf{s},\mathsf{d}}-r+k_2-1} \exp\left(-\left(\sum_{s=1}^{n}\frac{m_{j_s,\mathsf{d}}}{\Omega_{j_s,\mathsf{d}}} + \frac{m_{l,\mathsf{d}}}{\Omega_{l,\mathsf{d}}}\right)y\right)\right\}\right] \tag{16}$$

where C_a, $C_b(l)$, $C_1(r,l)$, $C_2(l)$, $C_6(r,l,q_i)$, and $C_7(l,j_s)$ are as defined before.

Proof: The pdf of $\rho|h_{b,\mathsf{d}}|^2$ given C_L can be written as

$$f_{\rho|h_{b,\mathsf{d}}|^2}(\tau) = \sum_{l=1}^{L} f_{\rho|h_{l,\mathsf{d}}|^2}(\tau) \prod_{\substack{i=1\\ i\neq l}}^{L} F_{\rho|h_{i,\mathsf{d}}|^2}(\tau). \tag{17}$$

For Nakagami-m fading, the pdf of $\rho|h_{l,\mathsf{d}}|^2$ and the cdf of $\rho|h_{m,\mathsf{d}}|^2$ are given by $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau) = m_{l,\mathsf{d}}^{m_{l,\mathsf{d}}} \tau^{m_{l,\mathsf{d}}-1} \Omega_{l,\mathsf{d}}^{m_{l,\mathsf{d}}} \Gamma(m_{l,\mathsf{d}}) \exp(-m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}}\tau)$ and $F_{\rho|h_{i,\mathsf{d}}|^2}(\tau) = 1 - \sum_{q=0}^{m_{i,\mathsf{d}}-1} 1/q!(m_{i,\mathsf{d}}/\Omega_{i,\mathsf{d}})^q \tau^q \exp(-m_{i,\mathsf{d}}/\Omega_{i,\mathsf{d}}\tau)$, respectively. Upon substituting these cdfs in (17) and after some algebraic manipulations, the final result becomes

$$f_{\rho|h_{b,\mathsf{d}}|^2}(\tau)$$
$$= \sum_{l=1}^{L} \frac{m_{l,\mathsf{d}}^{m_{l,\mathsf{d}}} \tau^{m_{l,\mathsf{d}}-1}}{\Omega_{l,\mathsf{d}}^{m_{l,\mathsf{d}}} \Gamma(m_{l,\mathsf{d}})} \exp\left(-\frac{m_{l,\mathsf{d}}}{\Omega_{l,\mathsf{d}}}\tau\right)$$
$$\times \left[1 + \sum_{n=1}^{L} (-1)^n \sum_{\substack{j_1=1\\ j_1\neq l}}^{L-n+1} \sum_{\substack{j_2=j_1+1\\ j_2\neq l}}^{L-n+2} \cdots \sum_{\substack{j_n=j_{n-1}+1\\ j_n\neq l}}^{L} \right.$$
$$\times \sum_{q_1=0}^{m_{j_1,\mathsf{d}}-1} \sum_{q_2=0}^{m_{j_2,\mathsf{d}}-1} \cdots \sum_{q_n=0}^{m_{j_n,\mathsf{d}}-1} \frac{\prod_{w=1}^{n}\left(\frac{m_{j_w,\mathsf{d}}}{\Omega_{j_w,\mathsf{d}}}\right)^{q_w}}{\prod_{p=1}^{n} q_p!}$$
$$\left. \times \tau^{\sum_{i=1}^{n} q_i} \exp\left(-\sum_{s=1}^{n} \frac{m_{j_s,\mathsf{d}}}{\Omega_{j_s,\mathsf{d}}}\tau\right)\right]. \tag{18}$$

We then can get the pdf of Y from $f_Y(y) = \int_0^y f_{\rho|h_{\mathsf{s},\mathsf{d}}|^2}(y-\tau) f_{\rho|h_{b,\mathsf{d}}|^2}(\tau)d\tau$ as (16). ■

Using the results of Proposition 1 and Proposition 2, we can get the cdf of $\gamma_{\mathsf{d}} = Y/Z$ as (8) from

$$\mathrm{P_r}[\gamma_{\mathsf{d}} < u|C_L] = \int_1^{\infty} f_Z(z) \int_0^{uz} f_Y(y)dy\,dz. \tag{19}$$

By integrating (19), we get (8).

APPENDIX B
PROOF OF LEMMA 2

Here, we evaluate the second term $\mathrm{P_r}[C_L]$ in (7). The SINR $\gamma_{\mathsf{s},k}$ can be written as Y_1/Z_1, where the pdf of Y_1 is similar to $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau)$ by replacing l with s and d with k, and the pdf of Z_1 is similar to that found in (14) by replacing i_d with i_k and d with k.

Now, using the integral in (19) and with replacing Y by Y_1 and Z by Z_1, and after some algebraic manipulations, the cdf of $\gamma_{\mathsf{s},k}$ can be evaluated as (9). Having $\mathrm{P_r}[\gamma_{\mathsf{d}} < u|C_L]$ and $\mathrm{P_r}[C_L]$ evaluated, the outage probability in (7) can be calculated.

APPENDIX C
PROOF OF (12)

Here, we evaluate the system outage at high SNR values. When $\rho \to \infty$ and with finite values of ρ_I, I_k, and I_d, the Gamma density and distribution functions can be approximated by $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau) \approx (m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}})^{m_{l,\mathsf{d}}} \tau^{m_{l,\mathsf{d}}-1}/\Gamma(m_{l,\mathsf{d}})$ and $F_{\rho|h_{l,\mathsf{d}}|^2}(\tau) \approx (m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}})^{m_{l,\mathsf{d}}} \tau^{m_{l,\mathsf{d}}}/\Gamma(m_{l,\mathsf{d}}+1)$, respectively. Based on that, the pdf of the best relay can be written as $f_{\rho|h_{l,\mathsf{d}}|^2}(\tau) \approx \prod_{l=1}^{L} (m_{l,\mathsf{d}}/\Omega_{l,\mathsf{d}})^{m_{l,\mathsf{d}}} \tau^{m_{l,\mathsf{d}}-1}/\Gamma(m_{l,\mathsf{d}})$. Following the same procedure as in Appendix A, the pdf $f_Y(y)$ can be evaluated. Upon substituting $f_{Z_2}(z)$ given by (10) and $f_Y(y)$ in (19), the first term in (7) can be found. Following the same procedure as in Appendix B, the second term in (7) can be determined. After some mathematical manipulations, we get (12).

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The Effects of Limited Channel Knowledge on Cognitive Radio System Capacity

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***Abstract*—We examine the impact of limited channel knowledge on the secondary user (SU) in a cognitive radio system. Under a minimum signal-to-interference noise ratio (SINR) constraint for the primary user (PU) receiver, we determine the SU capacity under five channel knowledge scenarios. We derive analytical expressions for the capacity cumulative distribution functions and the probability of SU blocking as a function of allowable interference. We show that imperfect knowledge of the PU–PU link gain by the SU-Tx often prohibits SU transmission or necessitates a high interference level at the PU. We also show that errored knowledge of the PU–PU channel is more beneficial than statistical channel knowledge and that imperfect knowledge of the SU-Tx to PU-Rx link has limited impact on SU capacity.**

***Index Terms*—Channel capacity, cognitive radio (CR), partial channel-state information (CSI).**

I. Introduction

A large body of work is now available on various aspects of cognitive radio (CR) systems, including fundamental information-theoretic capacity limits and performance analysis, which often assumes perfect secondary user (SU)-Tx to primary user (PU)-Rx channel state information (CSI) [1]–[7]. In practice, there is expected to be limited (or no) collaboration between PU and SU systems. Hence, an important question is the impact of the nature of channel knowledge on CR capacity. Several recent contributions have considered imperfect CSI [8]–[14]. In [8], mean and outage capacities along with optimum power allocation policies have been investigated for a CR system in a fading environment with imperfect CSI. Here, probabilistic constraints were employed to maintain an acceptably low probability that interference exceeded some target. In our work, we also use probabilistic constraints but apply them to a signal-to-interference noise ratio (SINR) target.

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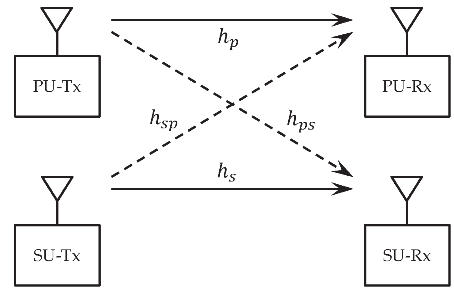

Fig. 1. System diagram.

This paper differs from the existing literature in several ways. There are four link gains in a two-user PU/SU channel to consider, and each of them may or may not be perfectly known at the SU transmitter. Previous studies [8]–[10], [12], [14] have only assumed imperfect knowledge of the SU-Tx to PU-Rx link. Additionally, in previous work, the effect of the interference from the PU-Tx on SU capacity is ignored. Moreover, we employ the SINR at the PU-Rx to impose probabilistic constraints to protect the PU-Rx, whereas prior works, with the exception of [10], have considered an interference outage constraint. Finally, we consider several cases where the imperfect CSI manifests itself in the form of statistical channel knowledge (i.e., knowledge of the mean link gains). Such a form of imperfect CSI is attractive from a practical standpoint since obtaining accurate knowledge is almost impossible for some links, such as the PU-Tx to PU-Rx link. Moreover, the mean value does not impose a large system burden as it only requires infrequent updates. Note that the inclusion of PU-Tx to SU-Rx interference and probabilistic constraints enables a rigorous evaluation of the benefits of various types of CSI. In this paper, we establish the following key observations and results:

1) In four of the five scenarios considered, we derive analytical expressions for the cumulative distribution function (cdf) of the SU SINR and use it to evaluate the SU capacity cdf.
2) For all scenarios, we derive the probability of SU blocking as a function of the permissible interference at the PU-Rx.
3) By evaluating our results for a range of system parameters, we demonstrate the importance of accurate knowledge of the PU-Tx to PU-Rx link at the SU-Tx.
4) We demonstrate the very high sensitivity of SU performance to the error in the estimation of the PU-Tx to PU-Rx and SU-Tx to PU-Rx links.
5) We show that errored knowledge of the PU-Tx to PU-Rx link and SU-Tx to PU-Rx link (if available) is better for SU capacity than knowledge of the mean link gains.
6) By considering a single probabilistic SINR constraint, a unified framework is presented that enables fair comparisons between different types of channel knowledge.

II. System Model

Consider a CR system (see Fig. 1) with the SU-Tx and PU-Tx transmitting simultaneously to their respective receivers. Independent point-to-point flat Rayleigh fading channels are assumed