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Optimized Power Allocation for Layered-Steered Space-Time Codes

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Abstract Layered Steered Space-Time Codes (LSSTC) is a recently proposed multiple-input multiple-output system that combines the benefits of vertical Bell Labs space-time (VBLAST) scheme, space-time block codes and beamforming. We suggest a new downlink scheme employing LSSTC with asymmetric power allocation, by assuming that the user feeds the BS with the average signal-to-noise ratio per VBLAST layer through the uplink feedback channel. The motivation behind proposing such a system is to enhance the error performance by assigning power to the layers in an optimal manner. We refer to the system proposed as the optimal power allocation LSSTC (OPA-LSSTC). Our analysis is general such that it includes asymmetric layered systems in which each layer may have different number of antennas and also the power can be assigned to layers asymmetrically.

Keywords Layered-steered space-time codes · LSSTC · Performance analysis · Optimal power allocation

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1 Introduction

Various techniques have been proposed to counter the problem of propagation conditions, and to achieve data rates that are very close to the Shannon limit. One of these techniques is using multiple-input multiple-output (MIMO) systems which employ antenna arrays at both the transmitter and the receiver. Wolniansky et al. has proposed in [1] the well-known MIMO scheme, known as vertical Bell Labs space-time (VBLAST). In VBLAST architecture, parallel data streams are sent via the transmit antennas at the same carrier frequency. Given that the number of receive antennas is greater than or equal to the number of transmit antennas, the receiver employs a low complexity method based on successive interference cancellation (SIC) to detect the transmitted data streams. In this manner, VBLAST can achieve high spectral efficiencies without any need for increasing the system's bandwidth or transmitted power.

Alamouti has presented in [2] a new scheme called STBC with two transmit and one receive antennas that provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit and two receive antennas. This scheme can be generalized to two transmit antennas and M receive antennas to provide a diversity order of $2M$. Similar work was considered in [3] where space time trellis codes (STTC) were used as the component codes. With the tempting advantages of VBLAST and STBC, many researchers have attempted to combine these two schemes to result in a multilayered architecture called MLSTBC [4] with each layer being composed of antennas that corresponds to a specific STBC. This combined scheme arises as a solution to jointly achieve spatial multiplexing and diversity gains simultaneously. With MLSTBC scheme, it is possible to increase the data rate while keeping a satisfactory link quality in terms of symbol error rate (SER) [5].

In [6], beamforming was combined with MLSTBC to produce a hybrid system called the layered steered space time codes (LSSTC). The addition of beamforming to MLSTBC further improves the performance of the system by focusing the energy towards one direction, where the antenna gain is increased in the direction of the desired user, while reducing the gain towards the interfering users.

The main contribution of this paper is deriving the optimal power allocation at the transmitter side for an LSSTC system in order to minimize the probability of error. In this paper, we optimally design a power allocation algorithm to antenna arrays comprising an LSSTC scheme. The power allocation algorithm utilizes the channel state information feedback from the mobile station on the uplink to the BS, and finds the best power allocation to the channel conditions encountered by the BS. The resulting system is an LSSTC scheme with optimal power allocation that enhances the SER performance without any further cost, except the feedback of the channel state information and simple calculations at the BS to find the best power allocation scheme. We claim that optimal power allocation is first designed for LSSTC in this paper. Some of the advantages of this design is that can be added to the existing and evolving wireless communication systems, that employ MIMO such as the long-term evolution (LTE) or WiMAX, seamlessly and with quite low cost as it only requires the running of the OPA processing algorithm without any additional hardware.

The paper is organized as follows. Section 2 presents a brief literature review on the power allocation in layered systems. Section 3 gives a description of the system model for the proposed scheme. Section 4 presents the notation used for the power allocation scheme. Section 5 shows the performance analysis of PA-LSSTC, in which we derive a formula for the probability of error of the individual layers employing different modulation schemes and the average SER of the LSSTC system. Furthermore, the optimal PA scheme for LSSTC is derived so that the probability of error is minimized. Section 6 presents the simulation results conducted to evaluate the PA-LSSTC system. Section 7 discusses the complexity of the proposed system. Finally, Section 8 presents our conclusions.

2 Related Work

In this section we briefly present some of the research literature pertaining to power allocation in layered systems. Many techniques have been proposed to improve the performance of layered systems such as VBLAST, one of which is to decrease error propagation from earlier layers by assigning more power to those layers. An analytical derivation for the optimum power allocation was presented in [7] for unordered VBLAST. The procedure proposed in [7] uses approximations for the total bit errorr rate (TBER) and block error rate (BLER) to derive closed forms of the optimized error rates at high SNR. According to [7], both BLER-based and and TBER-based optimization result in the same performance. A transmit power allocation scheme for VBLAST systems is proposed in [8], this scheme uses the Lagrange multiplier method to minimize the overall bit error rate (BER). In [9], a different approach is followed, where the authors find recursive expressions for the error rate of the individual layers and then use Newton's method to find the optimum power allocation for an unordered VBLAST system. In [10–12], the performance of optimized and non-optimized VBLAST systems was compared through numerical simulations. It was noted that applying optimum power allocation results in a few dBs gain in the BER curve. In [13], an algorithm is derived to find the optimum power allocation for a VBLAST system equipped with 2 transmit antennas. The proposed scheme in [13] numerically minimizes the probability of vector error and yields an SNR gain up to 3 dB for a 2×2 system at a probability of vector error of 10^{-3}. The drawback of the derivation in [13] is that it is limited to the case of two transmit antennas.

Our work explores the extent to which optimal power allocation is able to minimize the probability of error in LSSTC systems. Our analysis is a generalization of that for VBLAST in [9] to the LSSTC case, where beamforming and STBC are involved. We also investigate the performance of the power allocation scheme for LSSTC (PA-LSSTC) employing multi-level quadrature amplitude modulation ($M_q -$QAM). Unlike the analysis presented in [9], our analysis is more general such that it includes asymmetric layered systems in which each layer may have different number of antenna elements. Our study also investigates the effect of varying different LSSTC parameters on the power allocation gain. To the best of our knowledge, no analysis on power allocation has been derived for LSSTC systems before. It would be interesting and novel to investigate the effect of asymmetric power allocation in LSSTC systems.

3 System Model

Figure 1 shows the block diagram of a PA-LSSTC system, the system has N_T total transmitting antennas and N_R receiving antennas and is denoted by an (N_T, N_R) system. The antenna architecture employed in Fig. 1 has M transmit adaptive antenna arrays (AAs) spaced sufficiently far apart in order to experience independent fading and hence achieve transmit diversity. Each of the AAs consists of L elements that are spaced at a distance of $\lambda/2$ to ensure achieving beamforming.

A block of B input information bits is sent to the vector encoder of LSSTC and serial-to-parallel converted to produce K streams (layers) of length $B_1, B_2, \ldots, B_K$, where $B_1 + B_2 + \cdots + B_K = B$. Each group of B_k bits, $k \in [1, K]$, is then encoded by a component

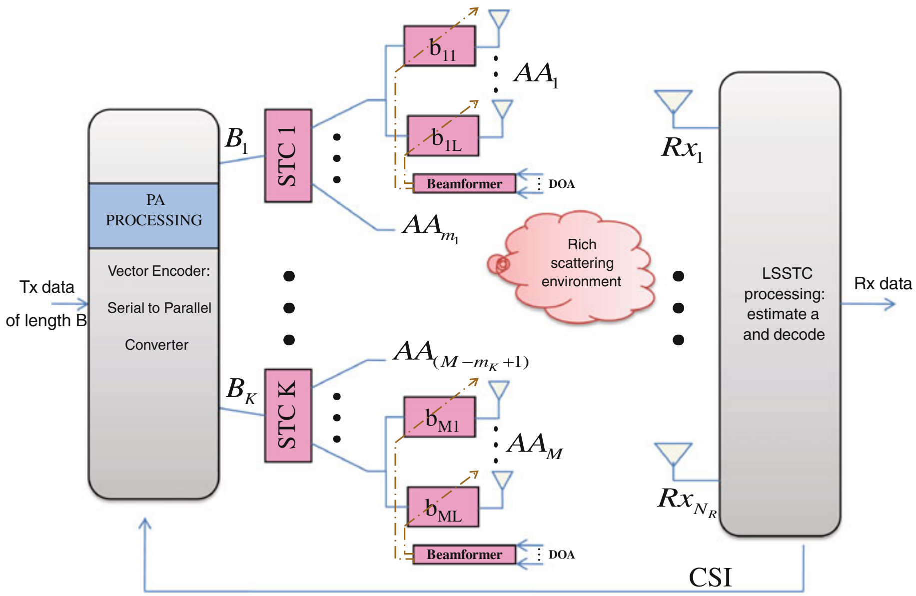

Fig. 1 Block diagram of a single user LSSTC system with power allocation

space-time code STC_k associated with m_k transmit AAs, where $m_1 + m_2 + \cdots + m_K = M$. The output of the kth STC encoder is a $m_K \times l$ codeword, $\mathbf{c}_i$, that is sent over l time intervals. The space-time coded symbols from all layers can be written as $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]^T$, where $\mathbf{C}$ is an $M \times l$ matrix.

The coded symbols from $\mathbf{C}$ are then processed by the corresponding beamformers, and then transmitted simultaneously over the wireless channels. The transmit antennas of all groups are synchronized and allocated equal power, moreover, the total transmission power is fixed, where the transmitted symbols have an average power of $P_T = 1$, where the average is taken across all codewords over both spatial and temporal components. For proper operation, N_R should be at least equal to K. The BS of PA-LSSTC prompts the user to feedback the CSI per layer via the feedback channel along with the direction of arrival (DOA) data. Also the transmitter is capable of performing PA processing.

The signal model can be described in matrix notation, where the received baseband data matrix can be written as

$$\mathbf{Y} = \mathbf{HWC} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is the received signal over l time intervals and has a dimension of $N_R \times l$, $\mathbf{H}$ is a matrix of vectors of dimension $N_R \times M$ whose entries are $\mathbf{h}_{n,m}$, where $\mathbf{h}_{n,m}$ is the L-dimensional channel impulse response (CIR) vector spanning the mth AA, $m \in [1, \ldots, M]$ and the nth receiver antenna, $n \in [1, \ldots, N_R]$ as $\mathbf{h}_{n,m}(t)$. The $N_R \times l$ noise matrix, $\mathbf{N}$, characterizes the additive white Gaussian noise (AWGN). The nth row of $\mathbf{N}$ denoted as $\mathbf{z}_n$, where $n \in [1, \ldots, N_R]$, is a row vector of l columns, the ith entry of $\mathbf{z}_n$ is a spatially uncorrelated circular-complex normal random variable, and can be written as $z_n^i = z_{I,n}^i + jz_{Q,n}^i$, where $z_{I,n}^i$ and $z_{Q,n}^i$ are two independent zero-mean Gaussian random variables having a variance of $N_0/2$. We will represent z_n^i as $\mathcal{CN}(0, N_0)$. Furthermore, $\mathbf{W}$ is a diagonal weight matrix of vectors of dimension $M \times M$ whose diagonal entry $\mathbf{w}_{m,m}$ is the L-dimensional beamforming weight vector for the mth beamformer AA and the nth receive antenna, and can be written as $\mathbf{w}_{m,m} = [b_{m1}, \ldots, b_{mL}]$, where b_{mi}, $i \in [1, \ldots, L]$, is the ith weighting gain of the mth AA.

Throughout this paper, the phrase "sub-stream" is used to refer to the data stream of each AA, whereas, the term "layer" represents the data stream to be encoded by STBC. The transmitted symbols can be written in vector form as $\mathbf{x} = [x_1, \ldots, x_M]^T$, where x_i is the ith sub-stream sent by the ith AA.

After multiplying the channel matrix ($\mathbf{H}$) by the weighting matrix ($\mathbf{W}$) and performing some matrix manipulations, the received signal can be written as

$$\mathbf{Y} = L\tilde{\mathbf{H}}\mathbf{C} + \mathbf{N}, \tag{2}$$

where $\tilde{\mathbf{H}}$ is an $(N_R \times M)$ matrix whose entries are $\alpha_{n,m}$, which is the Rayleigh faded channel coefficient coupling the mth AA to the nth receiver antenna. Further, $\tilde{\mathbf{H}}$ can be Partitioned into groups corresponding to each layer as in [4]

$$\tilde{\mathbf{H}} = \left[\tilde{\mathbf{h}}_1, \ldots, \tilde{\mathbf{h}}_K\right], \tag{3}$$

where $\tilde{\mathbf{h}}_k$ is the channel matrix of the kth layer. Looking at (2), the effect of beamforming can be clearly seen as a direct gain in the signal-to-noise ratio (SNR). Expressing $\tilde{\mathbf{H}}\mathbf{C}$ in terms of the layer components we get

$$\mathbf{Y} = L\sum_{k=1}^{K}\tilde{\mathbf{h}}_k\mathbf{c}_k + \mathbf{N}, \tag{4}$$

where $\mathbf{c}_k$ represents the component STBC used at layer k, where $k \in [1, \ldots, K]$.

4 The Power Allocation Scheme

The power allocation pattern for the PA-LSSTC system is characterized by the vector $\mathbf{K} = [K_1, K_2, \ldots, K_{M-1}]$, where K_i is defined as the transmit power ratio of the ith sub-stream to the sum of power of sub-streams $i+1, \ldots, M$. Hence, the parameter K_i is defined by

$$K_i = \frac{P_i}{\sum_{j=i+1}^{M} P_j}, \quad i = 1, 2, \ldots, M-1, \tag{5}$$

where P_i denotes the transmit power of the ith sub-stream. Similarly, we define the layer PA pattern as $\mathbf{K}_L = [K_{L,1}, K_{L,2}, \ldots, K_{L,K-1}]$ where K is the number of layers, and $K_{L,i}$ is defined as the transmit power ratio of the ith layer to the sum of power of layers $i+1, \ldots, K$. $K_{L,i}$ is defined by

$$K_{L,i} = \frac{P_{L,i}}{\sum_{j=i+1}^{K} P_{L,j}}, \quad i = 1, 2, \ldots, K-1, \tag{6}$$

where $P_{L,i}$ denotes the transmit power of the ith layer. For fair comparison among different PA patterns, the PA pattern must satisfy the total power constraint defined as $P_T = \sum_{i=1}^{M} P_i$, where P_T is assumed to to be equal to the average transmit power per modulation symbol.

5 Performance Analysis

In this section, the performance of LSSTC systems employing PA scheme is analyzed. The receiver is assumed to have a fixed detection ordering and uses serial group interference cancellation (SGIC) for detection [4]. The analysis is carried out for slow Rayleigh fading channels, in which we assume that the channel remains constant for many STBC blocks. Thus, the transmitter obtains the estimates of the average SNR per layer from the receiver, finds the optimal power allocation pattern, and uses the same power allocation pattern till the channel changes. Using this assumption minimizes the feedback load by a significant amount.

The sub-stream error will depend on the number of errors that occurred in the sub-stream itself and on the errors propagating from the previous layers, and will not depend on the errors occurring in the other sub-streams of the same layer. Therefore, we will calculate the layer probability of error, which will be equal to the probability of sub-stream error of the sub-streams sent from that layer. Therefore throughout this paper we will express the layer performance in terms of that of one of its substreams. For the ith layer the latter will be denoted as s_i. Similar to the analysis in [9], we first denote the SER of the ith layer under PA pattern $\mathbf{K}_L$ and noise of variance N_0 as $P_{e_i|(\mathbf{K}_L,N_0)} = P\{\hat{s}_i \neq s_i \mid \mathbf{K}_L, N_0\}$, where $\hat{s}_i$ represents the estimate of s_i. The SER of the ith layer has the form

$$P_{e_i|(\mathbf{K}_L,N_0)} = \sum_{l=0}^{i-1} P\{s_i \neq \hat{s}_i, A_{i-1}^l \mid \mathbf{K}_L, N_0\}, \tag{7}$$

where A_{i-1}^l defines the event of having l errors in the symbols $\hat{s}_1 \sim \hat{s}_{i-1}$. Let V_m denote one of the $\binom{i-1}{l}$ events which has detection errors at certain l layers among the $i-1$ processed layers at each time slot. Thus V_m is a set that contains the layer indices for one of the $\binom{i-1}{l}$ combinations of choosing l error symbols among the $i-1$ layers, where $m = 1, 2, \ldots, \binom{i-1}{l}$. We can express V_m as a set $V_m = \{v_{m,1}, v_{m,2}, \ldots, v_{m,l}\}$ where $v_{m,k}$ denotes the index of the layer in which the kth error has occurred. For instance, if $V_m = \{1, 3, 4\}$ then the first error was in the first layer, while the second was in the third layer, and the third was in the fourth layer. Also, we assume that $v_{m,1} < v_{m,2} < \cdots < v_{m,l}$, $v_{m,k} \in \{1, 2, \ldots, i-1\}$. Further more, the complement set of V_m is defined as

$$W_m = \{1, 2, \ldots, i-1\} - V_m = \{w_{m,1}, w_{m,2}, \ldots, w_{m,i-1-l}\},$$

where $w_{m,1} < w_{m,2} < \cdots < w_{m,i-1-l}$, $w_{m,k} \in \{1, 2, \ldots, i-1\}$. We define $e_{V_m}^i$ as the event of having the ith layer in error and having l erroneous layers indicated by V_m given a specific PA pattern $\mathbf{K}_L$ and a noise of variance N_0 as follows

$$\begin{aligned} e_{V_m}^i = \Big\{ s_i \neq \hat{s}_i & \bigcap_{\forall v_{m,k} \in V_m} \{s_{v_{m,k}} \neq \hat{s}_{v_{m,k}}\} \\ & \bigcap_{\forall w_{m,k} \in W_m} \{s_{w_{m,k}} = \hat{s}_{w_{m,k}}\} \,\Big|\, \mathbf{K}_L, N_0 \Big\}. \end{aligned} \tag{8}$$

Then, then the probability that the ith layer along with l proceeding layers defined by V_m given $\mathbf{K}_L$ and N_0 can be found by simply summing the probability of all the possible combinations of V_m. Mathematically, we can write this as

$$P\{s_i \neq \hat{s}_i, A_{i-1}^{l} \mid \mathbf{K}_L, N_0\} = \sum_{m=1}^{\binom{i-1}{l}} P(e_{V_m}^{i}). \tag{9}$$

Moreover, $P(e_{V_m}^{i})$ can be decomposed into a product of i components as follows

$$P(e_{V_m}^{i}) = P(e_{V_m}^{i,i}) \cdot P(e_{V_m}^{i,i-1}) \cdots P(e_{V_m}^{i,1}) = \prod_{t=1}^{i} P(e_{V_m}^{i,t}), \tag{10}$$

where $e_{V_m}^{i,t}$ is defined similar to $e_{V_m}^{i}$ except that it corresponds to the tth layer (whether erroneous or correct). $e_{V_m}^{i,t}$ is defined as

$$e_{V_m}^{i,t} = \begin{cases} \Big\{ s_t \neq \hat{s}_t \;\Big|\; \bigcap\limits_{\forall v_{m,k}<t} \{s_{v_{m,k}} \neq \hat{s}_{v_{m,k}}\} \\ \qquad \bigcap\limits_{\forall w_{m,k}<t} \{s_{w_{m,k}} = \hat{s}_{w_{m,k}}\}, \mathbf{K}_L, N_0 \Big\}, \quad t \in V_m. \\ \Big\{ s_t = \hat{s}_t \;\Big|\; \bigcap\limits_{\forall v_{m,k}<t} \{s_{v_{m,k}} \neq \hat{s}_{v_{m,k}}\} \\ \qquad \bigcap\limits_{\forall w_{m,k}<t} \{s_{w_{m,k}} = \hat{s}_{w_{m,k}}\}, \mathbf{K}_L, N_0 \Big\}, \quad t \in W_m. \end{cases} \tag{11}$$

Thus, substituting (10) in (9) results in

$$P\{s_i \neq \hat{s}_i, A_{i-1}^{l} \mid \mathbf{K}_L, N_0\} = \sum_{m=1}^{\binom{i-1}{l}} \prod_{t=1}^{i} P(e_{V_m}^{i,t}). \tag{12}$$

The exact SER of the tth layer without error propagation given the diversity order and the SNR have been derived in [14] for different modulation schemes. We denote the SER of the tth layer as $P_e(D_t, \rho_t)$, where D_t is the diversity order of the tth layer, defined as $D_t = m_t(N_R - K + t)$, and ρ_t is the SNR of the tth layer, which can be written in terms of the layer power and the effective noise variance as $\rho_t = P_t/\sigma_t^2$. The effective noise variance, σ_t^2, is composed of two parts; the noise from the the currently detected symbol and the potential error propagation signal from the earlier detected symbols. Noting the expression for the diversity order of LSSTC above, the effect of using STBC with VBLAST can be clearly seen, where the diversity order of each layer compared to VBLAST is increased by a factor equal to the STBC size of that specific layer. To find the probability of the event $e_{V_m}^{i,t}$ defined in (11), we express it in terms of $P_e(D_t, \rho_t)$ for both correct and erroneous layers as follows

$$P(e_{V_m}^{i,t}) = \begin{cases} P_e(m_t(N_R - K + t), P_t/\sigma_t^2), & t \in V_m \\ 1 - P_e(m_t(N_R - K + t), P_t/\sigma_t^2), & t \in W_m, \end{cases} \tag{13}$$

where the value of σ_t^2 depends on the modulation scheme and the symbol energy used in previous layers.

In this paper we consider QAM modulation. The reader may refer to our results in [15] for other modulation schemes such as phase shift keying (PSK). Under a Rayleigh faded channel with diversity order D_i, the SER of a square M_q − QAM [16] can be written as

$$P_e(D_i, \rho_i) = 4\left(1 - \frac{1}{\sqrt{M_q}}\right) I_1 - 4\left(1 - \frac{1}{\sqrt{M_q}}\right)^2 I_2, \tag{14}$$

where the terms I_1 and I_2 are defined as

$$I_1 = \left[\frac{1}{2}(1 - \mu_i)\right]^{D_i} \cdot \sum_{k=0}^{D_i - 1} \binom{D_i - 1 + k}{k} \left[\frac{1}{2}(1 + \mu_i)\right]^k, \tag{15}$$

$$\begin{aligned} I_2 &= \frac{1}{4} - \mu_i \cdot \left(\frac{1}{2} - \frac{1}{\pi} \cdot \tan^{-1}(\mu_i)\right) \cdot \sum_{k=0}^{D_i - 1} \binom{2k}{k} \cdot (4\tau_i)^{-k} \\ &+ \frac{\mu_i}{\pi} \sin\left(\tan^{-1}(\mu_i)\right) \sum_{k=1}^{D_i - 1} \sum_{i=1}^{k} \tau_i^{-k} \cdot T_{ik} \cdot \left(\cos\left(\tan^{-1}(\mu_i)\right)\right)^{2(k-i)+1}, \end{aligned} \tag{16}$$

where

$$\mu_i \triangleq \sqrt{\frac{\rho_i}{\frac{2}{3}(M_q - 1) + \rho_i}}, \tag{17}$$

$$\tau_i \triangleq \left(\frac{3\rho_i}{2(M_q - 1)} + 1\right), \tag{18}$$

$$T_{ik} \triangleq \frac{\binom{2k}{k}}{\binom{2(k-i)}{k-i} 4^i \cdot (2(k-i) + 1)}. \tag{19}$$

The variance of the effective noise affecting the tth layer is approximated by

$$\begin{aligned} \sigma_t^2 &= N_0 + \sum_{\forall v_{m,k} < t} \mathrm{E}\left[\|\tilde{\mathbf{h}}_{v_{m,k}}\|^2\right] \cdot \mathrm{Var}\left[e_{v_{m,k}} \mid x_{v_{m,k}} \neq \hat{x}_{v_{m,k}}\right] \\ &= N_0 + \sum_{\forall v_{m,k} < t} L^2 \cdot \frac{6}{M_q - 1} P_{L, v_{m,k}} \\ &= N_0 + \frac{6L^2}{M_q - 1} \cdot \sum_{\forall v_{m,k} < t} P_{L, v_{m,k}}, \end{aligned} \tag{20}$$

where $\tilde{\mathbf{h}}_{v_{m,k}}$, $P_{L,v_{m,k}}$, and $e_{v_{m,k}}$ denote the channel matrix, transmit power, and the error event of layer $v_{m,k}$, respectively. The notation $E[.]$ is the expectation operator, $Var[.]$ is the variance operator, and $\|.\|^2$ is the squared Frobenious norm.

After finding the expressions of σ_t^2 and $P_e(D_i, \rho_i)$, they can be substituted into (13). The SER of the ith layer, $P_{e_i|(\mathbf{K}_L, N_0)}$ can be evaluated by combining (7), (12), and (13), and from that we can find the probability of error of the ith sub-stream by

$$P_{e_{x_i}|(\mathbf{K}, N_0)} = \mathrm{Prob}\{x_i \neq \hat{x}_i \mid \mathbf{K}, N_0)\} \tag{21}$$

$$= P_{e_{\Gamma(i)}|(\mathbf{K}_L, N_0)}, \tag{22}$$

where $\Gamma(i)$ is the layer from which the ith sub-stream is sent. The average probability of symbol error over all M sub-streams is simply written as

$$P_{av|(\mathbf{K}, N_0)} = \frac{1}{M} \cdot \sum_{i=1}^{M} P_{e_{x_i}|(\mathbf{K}, N_0)}. \tag{23}$$

In the sequel, we aim to find the optimal PA pattern $\mathbf{K}$ that would result in optimizing the performance by minimizing the probability of error for the LSSTC system. To achieve this we need to differentiate the formula of the average SER $P_{av|(\mathbf{K},N_0)}$ with respect to $\mathbf{K}$ to find the minimum value of the SER. Clearly such analytical differentiation is very difficult, therefore we use a numerical approach applying Newton's method [9]. To minimize $P_{av|(\mathbf{K},N_0)}$, we need to find the value of $\mathbf{K} = [K_1, K_2, \ldots, K_{M-1}]$ that satisfies the following set of equations

$$\frac{\partial P_{av|(\mathbf{K},N_0)}}{\partial K_i} = 0, \quad i = 1, 2, \ldots, M-1. \tag{24}$$

To solve the set of equations in (24) by Newton's method, we start with an initial guess and the optimal PA pattern $\mathbf{K}_{opt}$ is obtained by iterating until the solution converges, which depends on the initial guess and the step size.

6 Numerical Results

In this section we present some simulation results of the proposed PA scheme for LSSTC systems with different modulation schemes and transmitter configurations. Throughout this paper, EPA-LSSTC will be used to denote equal power allocation LSSTC system in which all the layers are assigned the same amount of power. On the other hand, OPA-LSSTC will be used to denote optimal power allocation LSSTC system in which the layers are assigned different amounts of power according to $\mathbf{K}_{opt}$.

The detection process can be classified into two types [17]. The first type is the non-ordered detection, in which choosing the layer to be detected does not depend on the power of the layer, and the detection order is predetermined before the signal is received. The second type is the post-ordered detection, where the detection order is not known until the channel realization is perfectly estimated at the receiver.

In the Figs. 2, 3, 4, we verify our analysis by comparing it to the simulation results. Figure 2 shows the SER of the individual layers of an 8×2 LSSTC using non-ordered SGIC detector employing BPSK modulation with $K = 2$ and $L = 2$ obtained from both the simulation and the analysis. The Figure compares the analytical results of the SER to those obtained from the simulation with equal power allocation. It is clear that the simulation makes a nearly perfect match to the analysis results, which demonstrates the validity of the proposed analysis method.

Figure 3 compares the simulation results to those obtained from the analysis for SER of an LSSTC system employing non-ordered SGIC and BPSK modulation with $K = 4$ and $N_R = 2$ and different number of beamforming elements. It can be seen that the simulation and analysis results match quite well, which proves the validity of the analysis.

Figure 4 shows a fair comparison between different transmitter configurations of the LSSTC system in terms of the SER, obtained from both the EPA-LSSTC analysis and simulation. The three configurations use a total number of transmit antennas, $N_T = 8$, and the receiver is equipped with 4 antennas. In this comparison a different modulation scheme is used such that the spectral efficiency would be the same for all of them, which is set to 4 bps/Hz.

In the Figs. 5, 6, 7, 8, 9, 10, we show some numerical results for the OPA-LSSTC. A 16×4 OPA-LSSTC employing 16-QAM modulation with $K = 4$ and $L = 2$ is considered. The optimal PA for each layer versus E_s/N_0 is plotted in Fig. 5, where it can be seen that at high SNR, the impact of error propagation is more dominant than the noise. It can be seen

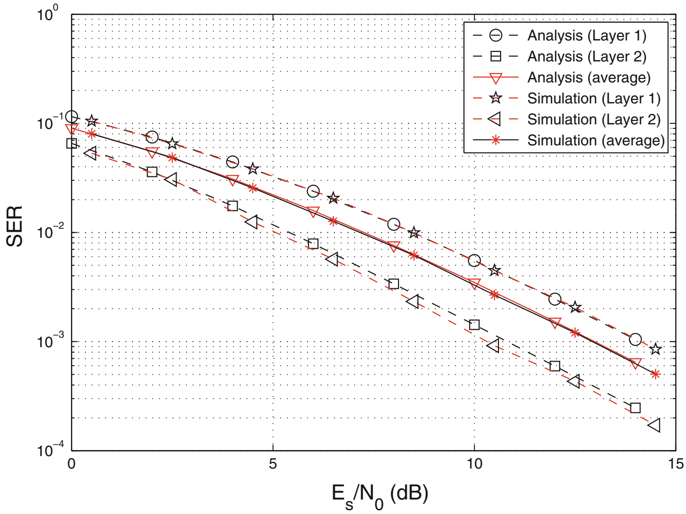

Fig. 2 SER of the individual sub-streams of an 8×2 LSSTC employing SGIC without ordering and BPSK modulation with $K = 2$ and $L = 2$ (comparing analysis to simulation results)

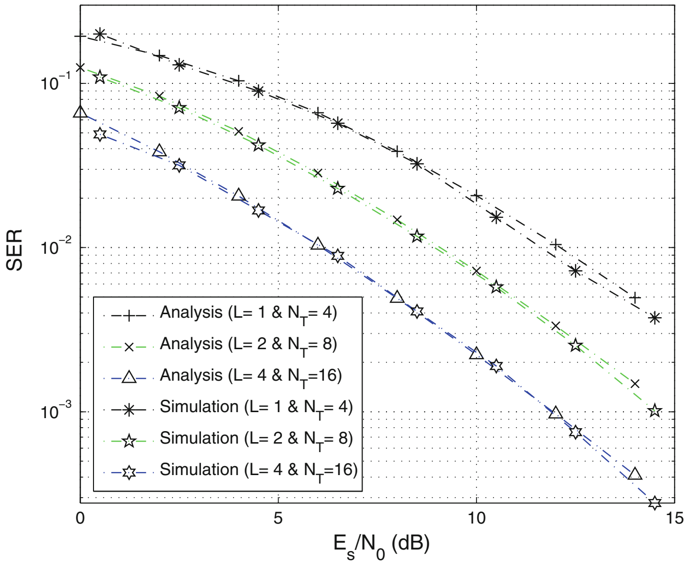

Fig. 3 SER of LSSTC employing non-ordered SGIC and BPSK modulation with $K = 4$ and $N_R = 4$ (comparing analysis to simulation results)

from Fig. 5 that the SER is dominated by the first layer, and that the detection errors in the first layer would cause severe detection errors to the following layers, therefore, the optimal PA scheme suggests assigning the earlier layers higher power than the later ones as the SNR increases. Note that the first layer gets most of the transmit power at high SNR since it is the weakest layer that has the lowest diversity order among all layers. In Fig. 6, we plot the SER of a 16×4 LSSTC system employing 16-QAM modulation with $K = 4$ and $L = 2$. We compare two cases, PA-LSSTC with equal power allocation (EPA-LSSTC), and the

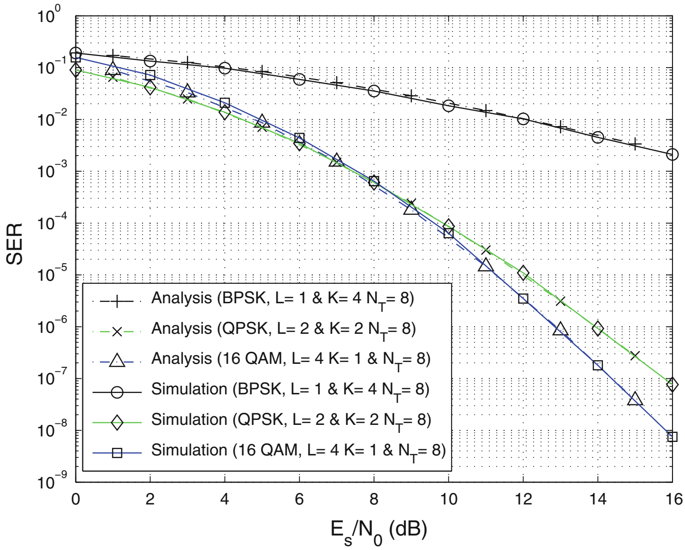

Fig. 4 SER of LSSTC employing non-ordered SGIC at 4 bps/Hz and different modulation schemes with $N_T = 8$ and $N_R = 4$ (comparing analysis to simulation results)

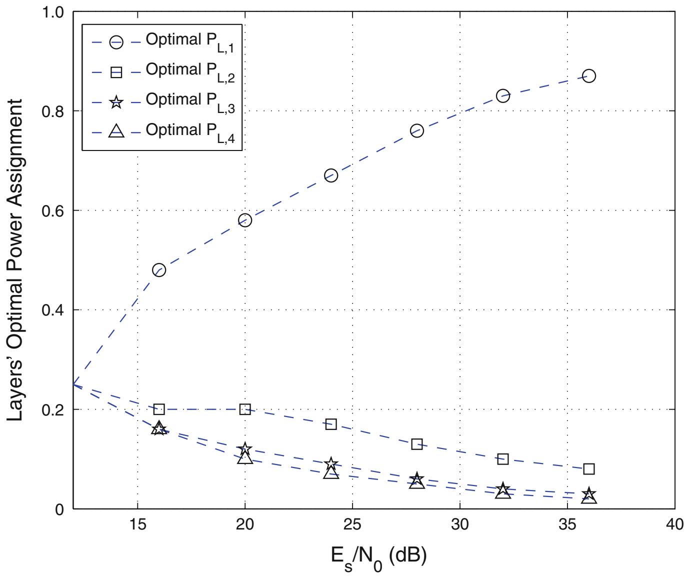

Fig. 5 Optimal PA for each layer for a 16 × 4 OPA-LSSTC scheme employing 16-QAM modulation with $K = 4$ and $L = 2$

PA-LSSTC with optimal power allocation (OPA-LSSTC). Our SER analysis is shown to be very accurate as compared to simulation results. It is observed that the proposed OPA-LSSTC has about 2.8 dB gain at a SER of 10^{-4} compared to EPA-LSSTC. This shows the superior performance of the proposed scheme.

Now, we want to study the effect of changing the parameters of the LSSTC system on the PA gain, which we define as the difference between the SNR levels of the EPA-LSSTC

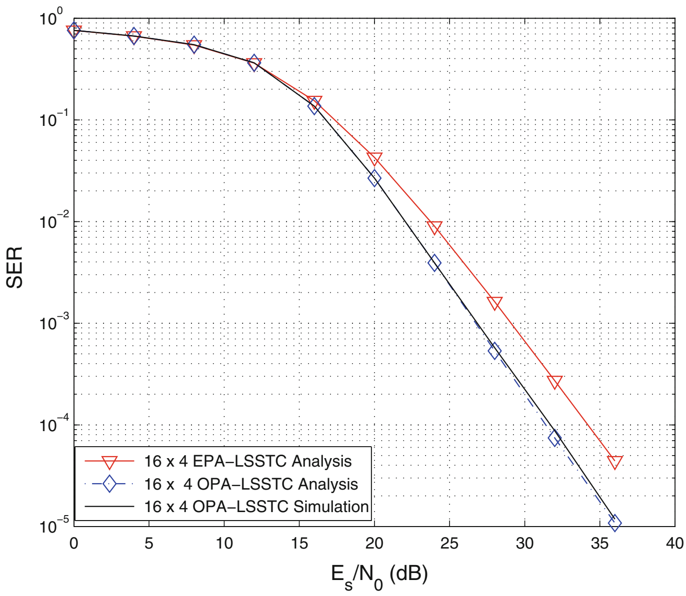

Fig. 6 SER of 16×4 LSSTC system using OPA-LSSTC scheme employing 16-QAM modulation with $K = 4$ and $L = 2$

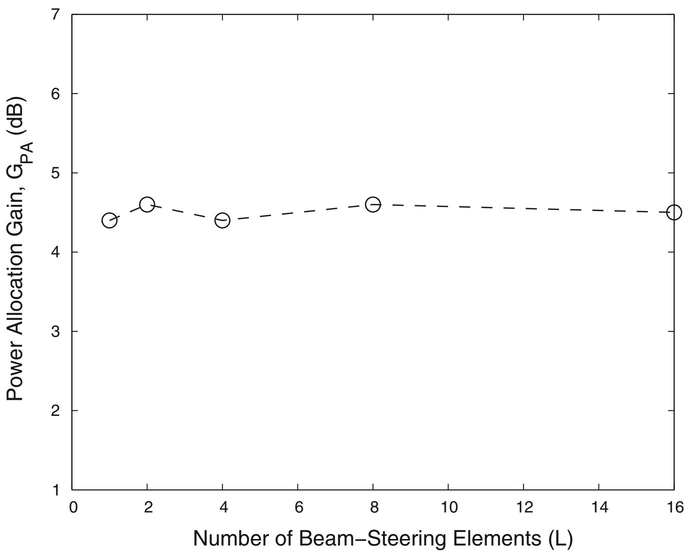

Fig. 7 PA gain, G_{PA} versus the number of beam-steering elements (L) at a SER of 10^{-6} using SGIC employing BPSK modulation with $K = 4$, $m_k = 2$ and $N_R = 4$

and the OPA-LSSTC required to reach the same SER. Figure 7 shows the PA gain, G_{PA} (dB) versus the number of beam-steering elements (L) at a SER of 10^{-6} using SGIC and BPSK modulation with $K = 4$, $m_k = 2$, and $N_R = 4$. It is observed that the gain remains approximately constant and does not depend on L, which is actually expected since L is not

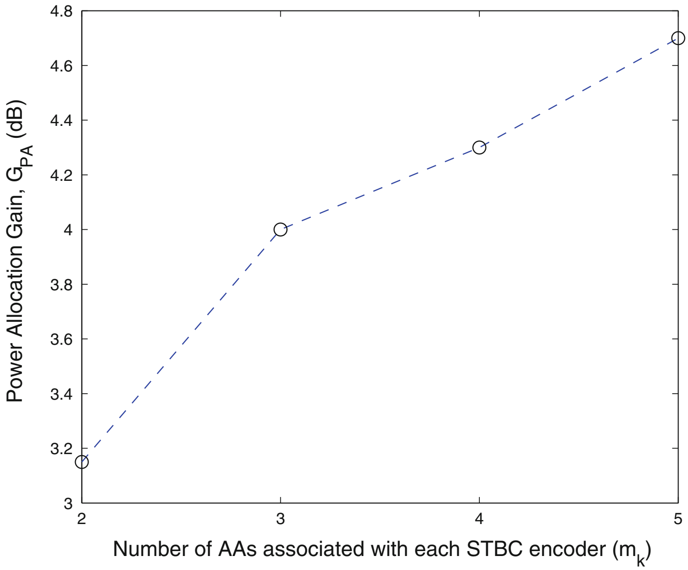

Fig. 8 PA gain, G_{PA} versus the number of AAs associated with each STBC encoder (m_k) at a SER of 10^{-6} using SGIC employing BPSK modulation with $K = 4$, $L = 2$ and $N_R = 4$

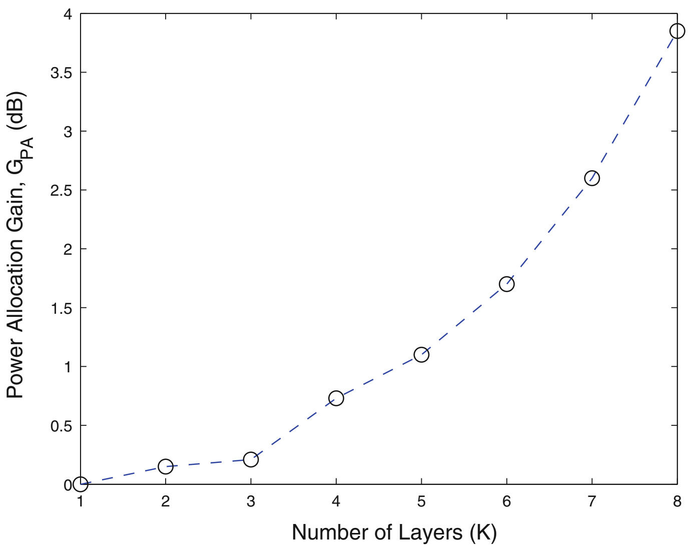

Fig. 9 PA gain, G_{PA} versus the number of Layers (K) at a SER of 10^{-6} using SGIC employing BPSK modulation with $m_k = 2$, $L = 2$ and $N_R = 8$

related directly to the distribution of the power among the layers, i.e. if L increase or decrease the layer will still get the same amount of power.

Figure 8 shows the PA gain, G_{PA} (dB) versus the number of AAs associated with each STBC encoder (m_k) at a SER of 10^{-6} using SGIC and BPSK modulation with $K = 4$, $L = 2$, and $N_R = 4$. It can be seen that G_{PA} does not increase significantly with increasing m_k. The small increase can be related to the diversity order.

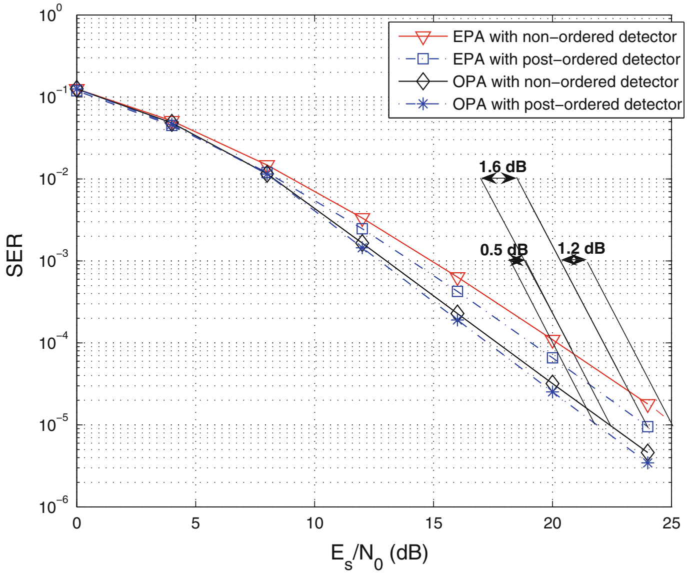

Fig. 10 SER of 16 × 4 LSSTC employing SGIC and BPSK modulation with $K = 4$ and $L = 2$ (simulation results)

Table 1 Possible transmitter-receiver configurations for LSSTC

Transmitter	Receiver	
	Non-ordered detector	Post-ordered detector
Equal power allocation	EPA-LSSTC with non-ordered detector	EPA-LSSTC with post-ordered detector
Optimal power allocation	OPA-LSSTC with non-ordered detector	OPA-LSSTC with post-ordered detector

In Fig. 9 we plot the power allocation gain, G_{PA} (dB) versus number of Layers (K) at a SER of 10^{-6} using SGIC employing BPSK modulation with $m_k = 2$, $L = 2$, and $N_R = 8$, where it can be seen that G_{PA} increases with increasing K, which is expected, since increasing the number of layers will lead to an increase in the degrees of freedom. Therefore a better distribution for the power can be found, since the total number of layers over which the power can be distributed is increased.

We can summarize four possible Transmitter-Receiver configurations for LSSTC in Table 1.

Figure 10 plots the SER versus E_s/N_0 of the four possible configurations listed in Table 1. Looking at the SER around 10^{-5}, we can see that in the case of EPA using the post-ordered detector provides a gain of 1.2 dB compared to using the non-ordered detector, while in the case of OPA using the post-ordered detector provides a gain of 0.5 dB compared to using the non-ordered detector, because the OPA had already pre-ordered the detection of the sub-streams, therefore attempting to further order them by the post-ordered detector will not result in much gain. We can also note that the using the OPA is better than using the post-ordered detector, since it provides about 2.8 dB gain compared to 2.1 dB for the latter. In addition, the

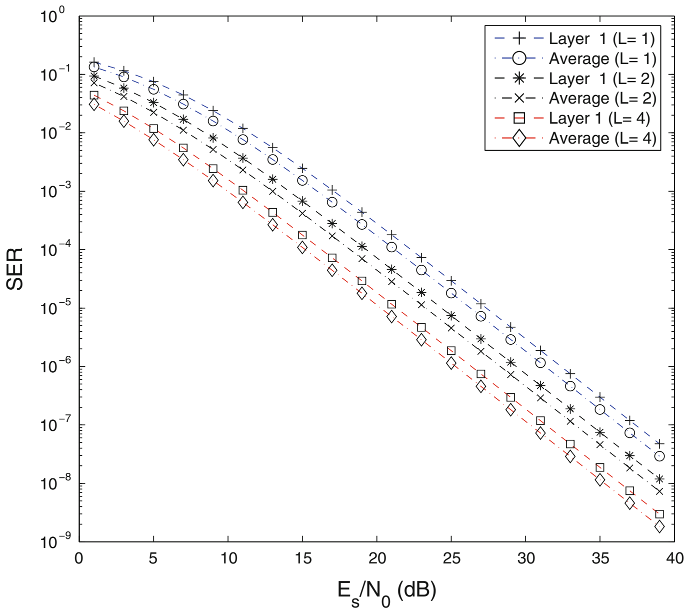

Fig. 11 First layer and average SER of EPA-LSSTC employing SGIC and BPSK modulation with $K = 2$, $N_R = 2$ and $m_k = 2$ (varying the number of beamforming elements (L))

post-ordered detector will require more processing at the user handset and that will consume the battery, on the other hand, the PA will be done at the base station where the processing and power is not an issue.

It has been mentioned earlier that the first layer dominates the probability of error, therefore we can approximate the average probability of error of the whole system by that of the first layer. In the following, we seek to further study the last statement, by comparing the SER of the first layer to the average SER, aiming to find which parameters or conditions will make this approximation much accurate.

Figure 11 shows the SER of LSSTC with varying the number of beamforming elements (L), where we can see that the gap between the first layer and the average doesn't change, and therefore it does not depend on L.

Figure 12 shows the SER of LSSTC with varying the number of layers (K), and we can see that the gap between the first layer and the average increases with increasing K.

In Fig. 13 the SER of LSSTC is plotted with varying the STBC size (m_k) which corresponds to the number of AAs per layer. We can see that the gap between the first layer and the average decreases with increasing m_k. We can also note that the gap becomes constant when m_k becomes high, this is because the diversity order becomes high and the improvement doesn't change much after further increase in m_k.

7 Complexity of OPA-LSSTC

It was observed that the OPA at high SNR provides a significant SNR gain with a little increase in the complexity of the signal. The main parameters that will be affected by the OPA processing are the feedback load and the number of operations per unit time.

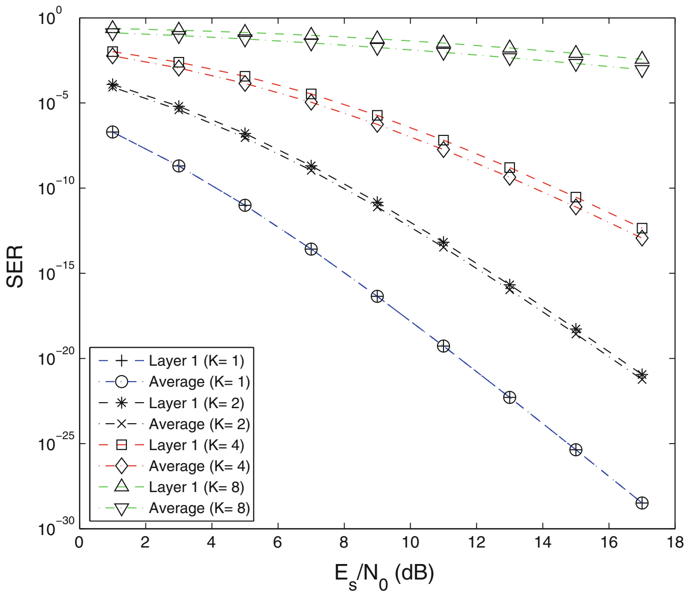

Fig. 12 First layer and average SER of EPA-LSSTC employing SGIC and BPSK modulation with $L = 2$, $m_k = 2$ and $N_R = 8$ (varying the number of layers (K))

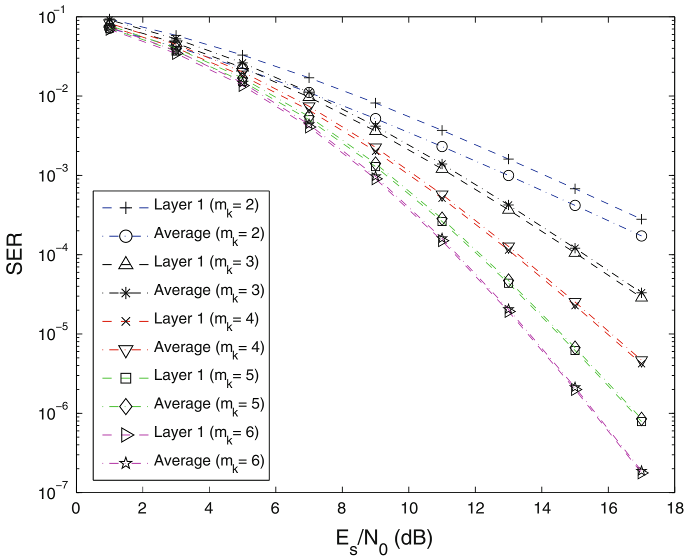

Fig. 13 First layer and average SER of EPA-LSSTC employing SGIC and BPSK modulation with $L = 2$, $K = 2$ and $N_R = 2$ (varying the STBC size (m_k))

The BS analyzes the channel state information (CSI) to optimize the performance by assigning the layer powers according to $\mathbf{K}_{opt}$. As a result, the number of operations will increase, and faster processors will be required. Observing the simulation results, the computational complexity was noted to be higher for small SNR values. The reason for that is hardware limitation, as the tiny difference between the optimal powers will require the step size δ to be very small. In such a case, finding the solution by numerical methods will require a huge number of operations. The step size used to solve the optimal PA equations ranged from 10^{-4} to 0.1, typically, values below 10^{-2} gave accepted results. Also it should be clear that finding the OPA will not improve the performance much in the low SNR range, and therefore no need to allocate powerful computational resources for it. For the high SNR range, few operations are enough to provide the optimal performance.

To speed up the convergence of finding $\mathbf{K}_{opt}$, the BS can have a database that contains the best initial guess of each SNR value. This way the number of operations required will be minimized and the system resources are used efficiently. The feedback load does not increase much when using OPA-LSSTC since we have assumed that the channel changes slowly, and the CSI need to be sent only if the channel state changes.

8 Conclusions

In this paper we investigated the performance of single-user PA-LSSTC. We have derived an expression for the probability of error for PA-LSSTC employing M-QAM modulation which includes the diversity gain of STBC and the SNR gain of beamforming. The analytical results have shown merely a perfect match with simulation results, which proves their validity. Also the benefits of PA-LSSTC in improving the performance were clearly demonstrated. Finally, the optimal PA performance for LSSTC was derived using Newton's method. It was shown that the OPA-LSSTC for some structure can provide about a 2.8 dB gain over the existing EPA-LSSTC of the same structure. This gain has been provided with merely no cost, the only cost is the OPA processing at the BS which is insignificant.

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