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A New Union Bound on the Error Probability of Binary Coded OFDM Systems in Wireless Environments

Salam A. Zummo · Matasm M. Hassan · Ping-Cheng Yeh

Abstract Orthogonal Frequency Division Multiplexing (OFDM) systems are commonly used to mitigate frequency-selective multipath fading and provide high-speed data transmission. In this paper, we derive new union bounds on the error probability of a coded OFDM system in wireless environments. In particular, we consider convolutionally coded OFDM systems employing single and multiple transmit antennas over correlated block fading (CBF) channels with perfect channel state information (CSI). Results show that the new union bound is tight to simulation results. In addition, the bound accurately captures the effect of the correlation between sub-carriers channels. It is shown that as the channel becomes more frequency-selective, the performance get better due to the increased frequency diversity. Moreover, the bound also captures the effect of multi-antenna as space diversity. The proposed bounds can be applied for coded OFDM systems employing different coding schemes over different channel models.

Keywords Diversity · Error probability · Rayleigh · Fading · Space-time block codes · MIMO · OFDM · Correlated block fading

1 Introduction

In wireless environments, traditional single-carrier mobile communication systems do not perform well because signals are usually impaired by multipath fading. In such channels,

S. A. Zummo · M. M. Hassan
Electrical Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

S. A. Zummo
e-mail: zummo@kfupm.edu.sa

P.-C. Yeh (✉)
Department of Electrical Engineering and Graduated Institute of Communication Engineering, National Taiwan University, Rm 248, EE Building, No. 1, Sec. 4, Roosevelt Rd., Taipei 106, Taiwan
e-mail: pcyeh@cc.ee.ntu.edu.tw

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inter-symbol interference (ISI) due to the frequency selectivity of the channel results in irreducible error floors. Multi-carrier modulation is a solution in such channels [1], of which orthogonal frequency division multiplexing (OFDM) is a special form [2,3]. OFDM transforms a wideband frequency-selective fading channel into multiple narrowband frequency-flat sub-carriers. Each sub-carrier is modulated at a much lower symbol rate. This makes the symbol duration much longer than the channel impulse response and thus reduces the ISI. As a result, OFDM has been applied in wireless local area networks (WLAN) standards [4]. It has also been adopted for wireless broadband access technologies such as WiMAX or LTE-Advanced and as the core technique for the fourth-generation (4G) wireless mobile communications.

By combining OFDM with different diversity techniques, the effect of multipath fading can be mitigated efficiently by providing the receiver with independently attenuated replicas of the signal. One common approach is to use multiple antennas at the transmitter or receiver [5,6]. Space-time block coding (STBC) was proposed by Alamouti [7] to provide diversity at the transmitter. This idea was soon generalized by Tarokh et al. [8] to a general number of transmit antennas. On the other hand, channel coding is also widely used and considered as another form of time diversity. In practice, OFDM is usually combined with coding to ensure that data in sub-carriers with low signal-to-noise ratio (SNR) are recovered correctly at the receiver through the decoding process. Hoshyar et al. [9] showed that the effective OFDM channel lies in the category of correlated block fading (CBF) channels, and space-time coded OFDM scheme with multiple transmitter and receiver antennas was proposed by Agrawal et al. [10]. The performance of coded OFDM with non-binary modulation can be found in [11].

In coded OFDM, the expression of the joint probability distribution of the effective CBF channel is very complex, making the performance analysis a difficult task. In the literature, there have been efforts in analyzing the error performance of coded OFDM systems, [9,12,13]. All these efforts were based on the impractical assumption that each sub-carrier carries only one symbol of a codeword to simplify the analysis, or the assumption that the pairwise error probability can only be computed by enumerating all non-zero codewords, which is very complex as in [9]. However, in practice, it is possible for the codeword to have more than one symbol carried over each sub-carrier, and enumerating all error codewords without any statistical assumptions is prohibitively complex. As a result, the performance analysis in those papers can only provide rough estimate to the actual coded OFDM performance.

In this paper, union bounds for single and multiple antennas coded OFDM systems using BPSK modulation are derived based on the uniform interleaving assumption [14]. The uniform interleaving assumption states that data are uniformly interleaved prior to the transmission over the OFDM system. This causes the errors to have a uniform distribution over the sub-carriers. The assumption is used to simplify the analysis and it gives the expected system performance averaged over all possible interleavers. The result is generally not too much different from the system performance when a specific interleaver is considered. In this work, we also assume perfect channel state information (CSI) is available at the receiver. Numerical results show that the new union bound is tight to simulation results. In addition, the bound accurately captures the effect of the correlation between sub-carriers channels.

This paper is organized as follows. In Sect. 2, the OFDM system models is described. New union bounds for coded OFDM systems employing single and multiple antennas are derived in Sect. 3. Results are presented in Sect. 4. Finally, conclusions are given in Sect. 5.

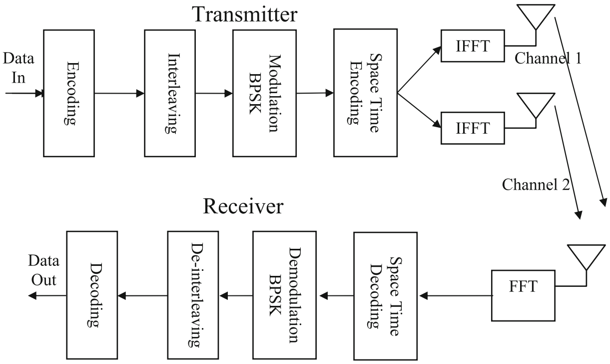

Fig. 1 Multi-antenna coded OFDM system with 2 transmit and 1 receive antennas

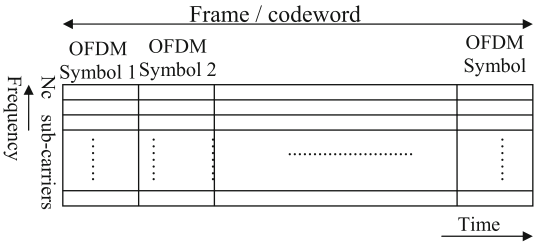

Fig. 2 A coded OFDM frame structure

2 OFDM System Model

2.1 Signal Model

A block diagram of a multiple-antenna coded OFDM system is shown in Fig. 1. The transmitter is equipped with n_t transmit antennas and there is a single receive antenna. At a transmitter, K data bits are encoded into L coded bits by a channel encoder, interleaved, and mapped to a BPSK signal which is converted to a STBC vector using the ST encoder. An N_{fft}-point inverse fast Fourier transform (IFFT) block converts every N_c coded bits to an OFDM symbol, which is transmitted using N_c orthogonal sub-carriers. Thus, each codeword of L bits is transmitted over N_c sub-carriers, resulting in every $m = \lceil L/N_c \rceil$ bits being transmitted over a sub-carrier as shown in Fig. 2.

In the case of single transmit antenna, the received signal corresponding to the l-th OFDM symbol can be written as [9]

$$r_{n,l} = \frac{1}{\sqrt{N_c}} \sum_{k=1}^{N_c} x_{k,l} h_k \exp\left(j2\pi \frac{n}{N_{fft}} p_k\right) + z_{n,l}, \qquad n = 0, \ldots, N_{fft} - 1, \quad l = 1, \ldots, m, \tag{1}$$

where $x_{k,l}$ is the BPSK signal corresponding to the kth data bit composing the l-th OFDM symbol, p_k determines the frequency index of the kth sub-carrier, $r_{n,l}$ is the nth sample, $z_{n,l}$ is a zero-mean additive white Gaussian noise (AWGN) noise sample affecting the nth sample, with a variance of $N_0/2$, and h_k is the frequency-domain fading coefficient affecting the kth sub-carrier throughout the whole codeword. The received samples $\{r_{n,l}\}_{n=0}^{N_{fft}-1}$ are converted using FFT [15] resulting in

$$y_{k,l} = \frac{\sqrt{N_c}}{N_{fft}} \sum_{n=0}^{N_{fft}-1} r_{n,l} \exp\left(-j2\pi \frac{n}{N_{fft}} p_k\right), \qquad k = 1, \ldots, N_c, \quad l = 1, \ldots, m. \tag{2}$$

By substituting (1) into (2) the received signals are simplified to

$$y_{k,l} = h_k x_{k,l} + n_{k,l}, \qquad k = 1, \ldots, N_c, \quad l = 1, \ldots, m, \tag{3}$$

where $n_{k,l}$ is the AWGN noise sample after the FFT process with zero-mean and a variance of $N_0/2$. These signals are deinterleaver and decoded using a Viterbi decoder [16].

For the multi-antenna case, we give the example with $n_t = 2$, in which the BPSK symbols are fed to the IFFT block using the orthogonal transmission matrix [7] given by

$$\mathbf{G} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}. \tag{4}$$

To be able to detect STBCs, the fading channel from each transmit antenna should remain constant for the duration of transmission of $\mathbf{G}$, i.e., for n_t symbol duration.

At the single-antenna receiver, FFT is applied and the received signals in the 1st and 2nd symbol periods can be compactly written as

$$\mathbf{y}_k = \sqrt{E_s}\mathbf{G}_k\mathbf{h}_k + \mathbf{z}_k, \tag{5}$$

where $\mathbf{G}_k$ is the $n_t \times n_t$ transmission matrix of the kth fading block, z_k is a length-n_t random vector with a distribution $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ and $\mathbf{I}$ is $n_t \times n_t$ identity matrix. In (5), $\mathbf{h}_k$ is the channel vector from the transmit antennas in kth fading block and it is modeled as a CBF channel

$$\begin{aligned} y_{k,l,1} &= h_{k,1}^* x_{k,l,1} + h_{k,2} x_{k,l,2}^* + n_{k,l,1} \\ y_{k,l,2} &= h_{k,2}^* x_{k,l,1} - h_{k,1} x_{k,l,2}^* + n_{k,l,2}, \qquad k = 1, \ldots, N_c, \end{aligned} \tag{6}$$

where $h_{k,1}$ and $h_{k,2}$ are the frequency-domain fading coefficients from transmit antennas 1 and 2 to the receive antenna affecting sub-carrier k, respectively. These signals are then de-interleaved and fed to the Viterbi decoder to find the most likely codeword.

2.2 Correlated Block Fading Channel

Because the sub-carriers in the OFDM system are closely located in the frequency domain, the fading channels affecting the sub-carriers are correlated, and the resulting effective channel is a CBF channel [9]. The frequency-domain fading coefficient affecting the kth sub-carrier is given by

$$h_k = \sum_{p=1}^{P} a_p \beta_p \exp\left(-j2\pi \frac{\tau_p}{N_{fft}} p_k\right), \qquad k = 1, \ldots, N_c, \tag{7}$$

where a_p is the gain of the pth path in the channel delay-spear profile with $\sum_{p=1}^{P} a_p^2 = 1$, τ_p the delay of the pth path and P is the total number of paths in the channel. In (7), the variables $\{\beta_p\}$ are independent complex Gaussian random variables with zero-mean and unit-variance.

Define the vectors $\mathbf{h} = [h_1, h_2, \ldots, h_{N_c}]^T$, $\mathbf{b} = [\beta_1, \beta_2, \ldots, \beta_P]^T$, then we can write $\mathbf{h} = \mathbf{A}\mathbf{b}$, where the coefficients of the transformation matrix $\mathbf{A}$ are given by $\mathbf{A}(k, p) = a_p \exp\left(-j2\pi \frac{\tau_p}{N_{fft}} p_k\right)$. The correlation matrix of the vector $\mathbf{h}$ is defined as $\mathbf{C_h} = \mathrm{E}[\mathbf{h}.\mathbf{h}^H]$, whose (i, k)-th coefficient represents the correlation between ith and jth sub-carriers and is given by

$$\mathbf{C_h}(i,k) = \sum_{p=1}^{P} \mathbf{A}(i,p)\mathbf{A}^*(p,k) = \sum_{p=1}^{P} a_p^2 e^{-j\tau_p \frac{2\pi}{N_{fft}}(p_i - p_k)}. \tag{8}$$

This correlation matrix will be used in the next section to derive the error performance of the coded OFDM system over CBF channels.

3 Union Bound

Throughout the paper, the subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. The bit error probability for a convolutional code is upper bounded [17] by

$$P_b \leq \sum_{d=d_{\min}}^{N} w_d \; P_u(d), \tag{9}$$

where $d_{\min}$ is the minimum Hamming distance of the convolutional code and w_d is the number of codewords with weight d. Here, $P_u(d)$ is the unconditional PEP defined as the probability of decoding a received sequence as a weight-d codeword given that the all-zero codeword is transmitted.

In CBF channels, $P_u(d)$ is a function of the distribution of the d nonzero error bits over the N_c sub-carriers. Denote the number of errors that occur in sub-carrier k by d_k. Due to the uniform interleaving of the coded bits prior to the transmission over the channel, it is possible for the d nonzero bits to distribute among the sub-carriers following any pattern $\mathbf{d} = \{d_k\}_{k=0}^{N_c}$ that satisfies $\sum_{k=1}^{N_c} d_k = d$. Conditioning on the distribution pattern $\mathbf{d}$, the unconditional PEP can be written as

$$P_u(d) = \sum_{d_1=0}^{d} \sum_{d_2=0}^{d-d_1} \sum_{d_3=0}^{d-d_1-d_2} \cdots \sum_{d_{N_c}=0}^{d-\sum_{i=1}^{N_c-1}} P_u(d|\mathbf{d}) p(\mathbf{d}), \tag{10}$$

where $P_u(d|\mathbf{d})$ is the unconditional PEP given a specific distribution pattern $\mathbf{d}$, which occurs with a probability given by

$$p(\mathbf{d}) = \frac{\prod_{i=1}^{N_c} \binom{m}{d_i}}{\binom{L}{d}}. \tag{11}$$

An upper bound on the bit error probability of convolutionally coded OFDM systems is obtained by substituting (10)–(11) in (9), and evaluating the term $P_u(d|\mathbf{d})$ in (10), which is derived below for systems employing single and multiple transmit antennas.

It should be noted that carefully designed interleavers may outperform the uniform interleaver. However, analyzing the performance of a specific interleaver is much more difficult and the uniform interleaver performance generally gives a good approximation to it. Also the number of summations involved in computing $P_u(d)$ in (10) increases as the number of carriers increase. Thus a good approximation to the union bound is obtained by truncating (9) to a distance $d_{\max} < L$. It should be noted that the low-weight terms in the union bound dominate the performance at high SNR values. Thus truncating the bound does not affect its accuracy especially at high SNR where simulation results are difficult to obtain.

3.1 Single-Antenna System

The decoder chooses the codeword $\mathbf{X}$ that maximizes the metric [14]

$$\mathbf{m}(\mathbf{Y},\mathbf{X}) = \sum_{k=1}^{N_c}\sum_{l=1}^{m} \mathrm{Re}\left\{y_{k,l}^{*} h_k x_{k,l}\right\}, \tag{12}$$

where $\mathbf{Y} = \left\{y_{k,l} | k = 1, \ldots, N_c, l = 1, \ldots, m\right\}$, $\mathbf{X} = \left\{x_{k,l} | k = 1, \ldots, N_c, l = 1, \ldots, m\right\}$, Re $\{.\}$ represents the real part of a complex number. The conditional PEP, denoted as $P_c(d|\mathbf{d})$ is defined as the probability of confusing the vector $\mathbf{X}$ with an error vector $\hat{\mathbf{X}}$ as

$$P_c(d|\mathbf{d}) = \Pr\left(\mathbf{m}(\mathbf{Y},\mathbf{X}) - \mathbf{m}(\mathbf{Y},\hat{\mathbf{X}}) < 0 | \mathbf{H}, \mathbf{X}, \mathbf{d}\right), \tag{13}$$

where $\mathbf{h} = \{h_k\}_{k=1}^{N_c}$. The unconditional PEP, $P_u(d|\mathbf{d})$ is found by substituting the metric (12) in (13), which can be written for BPSK signaling as follows

$$P_u(d|\mathbf{d}) = \mathrm{E}_{d_E^2}\left[\mathrm{Q}\left(\sqrt{2R_c\gamma_b d_E^2}\right)\right]. \tag{14}$$

where the averaging in (14) is over the distribution of the Euclidean distance d_E^2, defined as the distance between the correct and the estimated codewords and written as

$$d_E^2 = \sum_{k=1}^{N_c} d_k |h_k|^2 = \tilde{\mathbf{h}}^H \tilde{\mathbf{h}}, \tag{15}$$

where $\tilde{\mathbf{h}} = [\sqrt{d_1} h_1, \ldots, \sqrt{d_{N_c}} h_{N_c}]^H$ is the equivalent fading coefficient affecting the kth sub-carrier, and $(.)^H$ denotes as Hermition of the complex vector. Note that the dependency of (14) on d is implicitly induced from the relation of $\sum_{k=1}^{N_c} d_k = d$ of the $\{d_k\}$ in (15). The distance defined in (15) is a weighted sum of squared correlated complex Gaussian random variables, whose distribution is very complicated to perform the averaging in (14). However, the problem can be simplified by orthogonalizing the vector $\tilde{\mathbf{h}}$ as discussed below.

In (15), the Euclidean distance is expressed as the inner product of a correlated complex Gaussian random vector, i.e., $\tilde{\mathbf{h}}$. As a result, the averaging step in (14) can be obtained by converting the vector $\tilde{\mathbf{h}}$ into an uncorrelated random vector using the orthogonalization process [18]. To do so, define the correlation matrix of $\tilde{\mathbf{h}}$ as $\mathbf{C}_{\tilde{\mathrm{h}}} = \mathrm{E}\left[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H\right]$, and apply the eigenvalue decomposition to get

$$\mathbf{C}_{\tilde{\mathrm{h}}} = \mathbf{B}\boldsymbol{\Lambda}\mathbf{B}^H, \tag{16}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{N_c}\}$ is the diagonal matrix resulted from the eigenvalues of $\mathbf{C}_{\tilde{\mathrm{h}}}$, and $\mathbf{B}$ is a unitary matrix whose columns are the eigenvectors of $\mathbf{C}_{\tilde{\mathrm{h}}}$. Since $\mathbf{B}$ is unitary, we have $\mathbf{B}^H\mathbf{B}$ equal to the identity matrix $\mathbf{I}$. Now define a new random vector $\mathbf{g} = \mathbf{B}^H\tilde{\mathbf{h}}$, it can be shown [18] that the correlation matrix of $\mathbf{g}$ is diagonal and equal to $\boldsymbol{\Lambda}$. Hence, we can re-write d_E^2 in (15) as

$$d_E^2 = \mathbf{g}^T\mathbf{B}^T\mathbf{B}\mathbf{g} = \mathbf{g}^T\mathbf{g}. \tag{17}$$

Note that the coefficients of $C_{\mathbf{h}}$ and $C_{\tilde{\mathbf{h}}}$ are related as follows

$$C_{\tilde{\mathbf{h}}}(i,j) = C_{\mathbf{h}}(i,j)\sqrt{d_i d_j}. \tag{18}$$

Now, the averaging task in (14) becomes simple since the vector $\mathbf{g}$ is an uncorrelated complex Gaussian random vector. By using the integral expression of the Q-function $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \mathrm{e}^{-x^2/2sin^2\theta} d\theta$ [19], an exact expression of the PEP is derived to be

$$
\begin{aligned}
P_u(d|\mathbf{d}) &= \mathrm{E}_{\{g_k\}}\left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(\frac{-R_c\gamma_b}{\sin^2\theta}\sum_{k=1}^{L_\eta}\lambda_k|g_k|^2\right)d\theta\right] \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{k=1}^{L_\eta}\left(\frac{1}{1+R_c\gamma_b\lambda_k/\sin^2\theta}\right)d\theta,
\end{aligned} \tag{19}
$$

where L_η is the number of non-zero eigenvalues of the matrix $C_{\tilde{\mathrm{h}}}$.

3.2 Multi-Antenna System

In multiple transmit antenna systems, the ML decision rule employed in the Viterbi decoder chooses the vector $\mathbf{X}$ that maximizes the metric

$$
\mathbf{m}(\mathbf{Y},\mathbf{X}) = \sum_{k=1}^{N_c}\sum_{l=1}^{m}\mathrm{Re}\{\mathbf{y}_{k,l}^H\mathbf{G}_{k,l}\mathbf{h}_k\}. \tag{20}
$$

The conditional PEP for coded STBCs is found by substituting the metric (20) in (13) resulting in

$$
P_c(d|\mathbf{d}) = \Pr\left(\sum_{k=1}^{N_c}\sum_{l=1}^{m}\mathrm{Re}\{\mathbf{y}_{k,l}^H\mathbf{W}_{k,l}\mathbf{h}_k\} < 0|\mathbf{H},\mathbf{X},\mathbf{d}\right), \tag{21}
$$

here $\mathbf{W}_k = \mathbf{G}_k - \hat{\mathbf{G}}_k$ where $\mathbf{G}_k$ and $\hat{\mathbf{G}}_k$ are the transmission matrices of the kth fading block representing the all-zero codeword and a weight-d error codeword, respectively. Define $u_k = \mathrm{Re}\{\mathbf{y}_k^*\mathbf{W}_k\mathbf{h}_k\}$. The conditional mean and variance can be expressed as

$$
\mathrm{E}\left[u_k|\mathbf{G}_k,\mathbf{h}_k\right] = \sqrt{E_s}\mathrm{Re}\{\mathbf{h}_k^H\mathbf{W}_k^H\mathbf{W}_k\mathbf{h}_k\} = \sqrt{E_s}d_k\sum_{i=1}^{n_t}|h_k^i|^2, \tag{22}
$$

$$
\mathrm{Var}\left[u_k|\mathbf{G}_k,\mathbf{h}_k\right] = \mathrm{E}\left[\mathrm{Re}\{\mathbf{z}_k^H\mathbf{W}_k\mathbf{h}_k\mathbf{h}_k^H\mathbf{W}_k^H\mathbf{z}_k\}|\mathbf{G}_k,\mathbf{h}_k\right] = d_kN_o\sum_{i=1}^{n_t}|h_k^i|^2. \tag{23}
$$

Note that due the orthogonality of the rows in $\mathbf{W}_k$ the cross terms in (22) and (23) are zero. From (21)–(23), the conditional PEP can be expressed as

$$
P_c(d|\mathbf{d}) = \mathrm{Q}\left(\sqrt{\frac{R_c\gamma_b}{n_t}D_E^2}\right), \tag{24}
$$

where D_E^2 is the Euclidean distance defined as

$$
D_E^2 = \sum_{k=1}^{N_c}d_k\sum_{i=1}^{n_t}|h_k^i|^2 = \sum_{i=1}^{n_t}(\tilde{\mathbf{h}}^i)^H\tilde{\mathbf{h}}^i, \tag{25}
$$

with $\tilde{\mathbf{h}}^i = [\sqrt{d_1}h_1^i,\ldots,\sqrt{d_{N_c}}h_{N_c}^i]^H$. The vector $\tilde{\mathbf{h}}^i$ is a correlated Gaussian random vector. It can be diagonalized following the similar approach in the single-antenna case. The conditional PEP is then derived to be

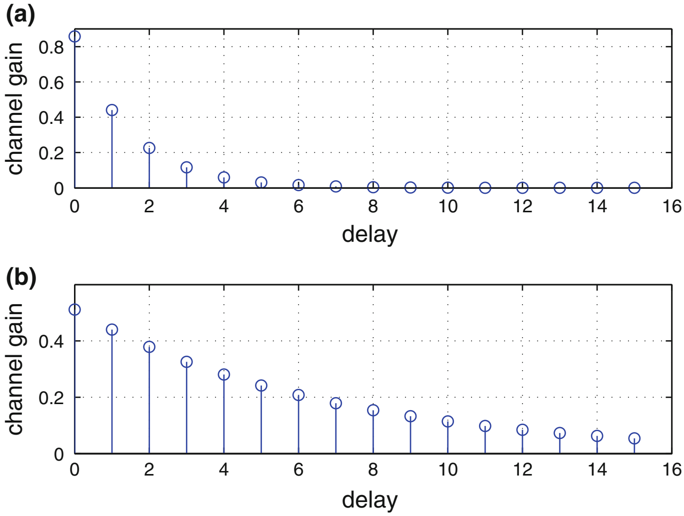

Fig. 3 Delay-spread profile of the exponential channels. **a** Channel 1, **b** Channel 2

$$P_c(d|\mathbf{d}) = \mathrm{Q}\left(\sqrt{\frac{2R_c\gamma_b}{n_t}\sum_{k=1}^{L_\eta}\lambda_k\sum_{i=1}^{n_t}|g_k^i|^2}\right). \tag{26}$$

The unconditional PEP, $P_u(d|\mathbf{d})$ can then be obtained by averaging the conditional PEP over the fading gains with the aid of the Q-function integral expression [19] again as

$$\begin{aligned} P_u(d|\mathbf{d}) &= \mathrm{E}_{\{g_k^i\}}\left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(\frac{-R_c\gamma_b}{n_t\sin^2\theta}\sum_{k=1}^{L_\eta}\lambda_k\sum_{i=1}^{n_t}|g_k^i|^2\right)d\theta\right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{k=1}^{L_\eta}\left(\frac{1}{1+R_c\gamma_b\lambda_k/(n_t\sin^2\theta)}\right)^{n_t}d\theta. \end{aligned} \tag{27}$$

4 Numerical Results

In this paper, a rate-1/2 (5,7) convolutional coded OFDM system over CBF channels is simulated and the union bound is evaluated for this system. Throughout the results, the frame size after encoding by the convolutional code is set to $N = 1024$ coded bits and the number of FFT point used by OFDM is $N_{fft} = 32$. Different numbers of sub-carriers $N_c = 4, 8, 16$ and 32 are considered in our simulation and bound evaluation. In the case of $N_c = 32$, the union bound is not computed because of the computational complexity involved. For the same reason, the union bound is truncated to a distance $d_{\max} \leq 12$. For illustration, we use two exponential channels whose delay-spread profiles are shown in Fig. 3. The channel interleaver is randomly replaced every 10 frames to simulate the effect of the random uniform interleaver.

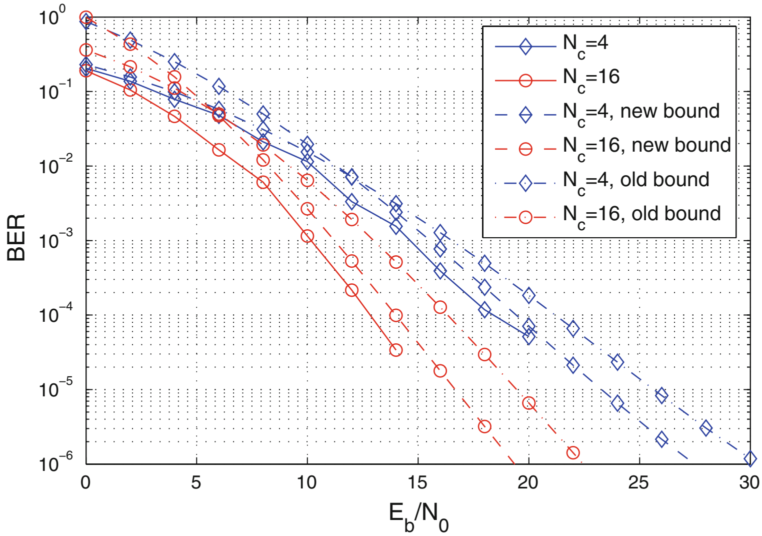

Fig. 4 Performance of a rate-$\frac{1}{2}$ convolutionally coded OFDM system over Channel 2, and number of sub-carriers $N_c = 4$, and 16

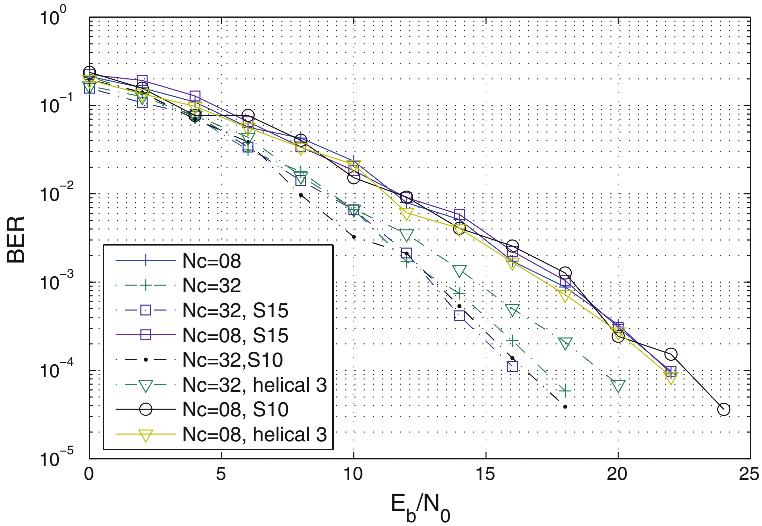

Fig. 5 Performance of a rate-$\frac{1}{2}$ convolutionally coded OFDM system over Channel 1 using different interleaver designs from literature and number of sub-carriers $N_c = 8$ and 32

Figure 4 shows a comparison of the proposed union bound with the existing bound from [9] evaluated for Channel 2. The delay-spread profile of the channel is shown in Fig. 3b. From the figure, we can observe that the new bound is much tighter to the simulation results. This tightness is due to to the fact that the proposed bound counts error codewords with their probabilities to occur, and the use of the exact formula of the Q-function. Figure 5 shows a comparison of the performance of coded OFDM system employing different interleaver designs, including the uniform interleaver used in deriving the proposed union bound. The

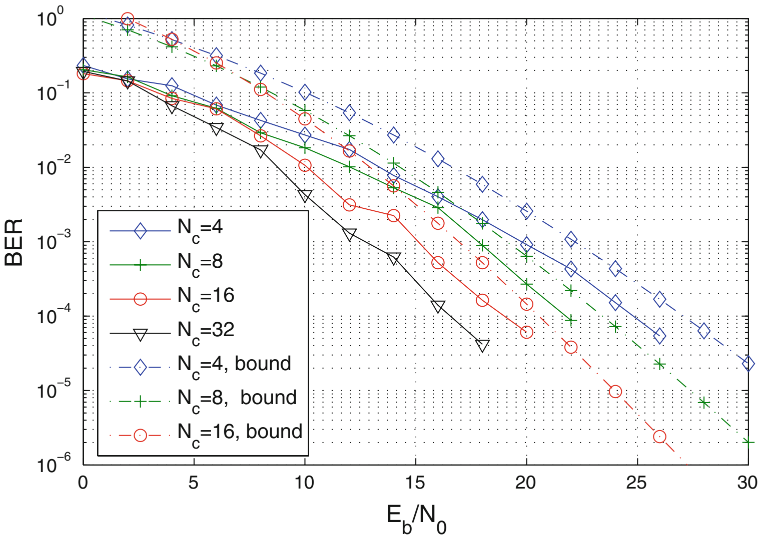

Fig. 6 Performance of a rate-$\frac{1}{2}$ convolutionally coded OFDM system over Channel 1, and number of sub-carriers $N_c = 4$, 8, 16 and 32

interleavers used are the S-Random interleaver with $S = 10, 15$ [20,21], and Helical interleaver [22]. The figure indicates clearly that the performance obtained from a uniform interleaver is not far away from the performance of other interleaver designs, and that the uniform interleaver performance falls in between the performances of other interleavers. Given the simplicity of the analysis under the uniform interleaver assumption, the value of our proposed bound becomes clear in prediction of the average performance of coded OFDM systems over CBF channels.

In Fig. 6, the performance of the coded OFDM system over Channel 1 is shown. The delay-spread profile of the channel is shown in Fig. 3a. Here, we observe that the system performance improves by increasing the number of sub-carriers because of the increased diversity order in the CBF. For instance, the 32 sub-carrier coded OFDM system needs only 16.4 dB to achieve BER $= 10^{-4}$ whereas the 8 sub-carrier coded OFDM system needs around 22.1 dB with a difference of 5.7 dB. Also, we observe that the union bound is very close to the simulation results for different CBF channels and sub-carrier number. In addition, comparing with Fig. 4, we observe that the system over Channel 2 performs better than over Channel 1. This is because the delay-spread profile of Channel 2 is longer, resulting in more frequency selectivity. For example, the 16 sub-carrier coded OFDM system over Channel 1 needs 19 dB to achieve BER $= 10^{-4}$ whereas over Channel 2 needs 12.8 dB.

The performance of the coded STBC OFDM system using $n_t = 2$ and 4 over Channel 1 are shown in Figs. 7 and 8, respectively. Numerical results are obtained using the analytical expressions derived in Sect. 3. In Figs. 6, 7 and 8, we observe that the multi-antenna systems perform better than single-antenna systems due to the space diversity of the multi-antenna systems. For example, the 32 sub-carrier multi-antenna system using $n_t = 2$ and 4 need 11.9 and 8.3 dB, respectively, to achieve BER $= 10^{-4}$ whereas single-antenna system needs 16.4 dB. Moreover, the performance of the coded system is improved by increasing the number of sub-carriers, and this improvement decreases as the space diversity order increases (more antennas). This is due to the fact that having more space diversity reduces the effect of

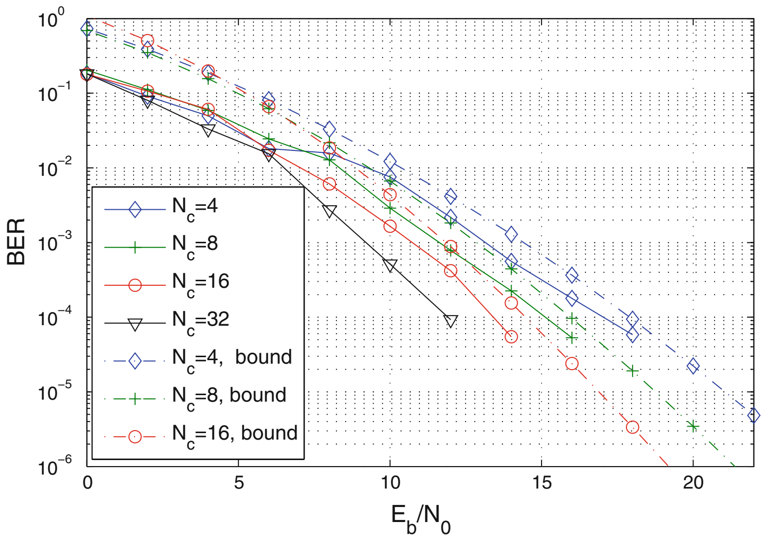

Fig. 7 Performance of a rate-$\frac{1}{2}$ convolutionally coded STBC OFDM system using $n_t = 2$ over Channel 1, and number of sub-carriers $N_c = 4$, 8, 16 and 32

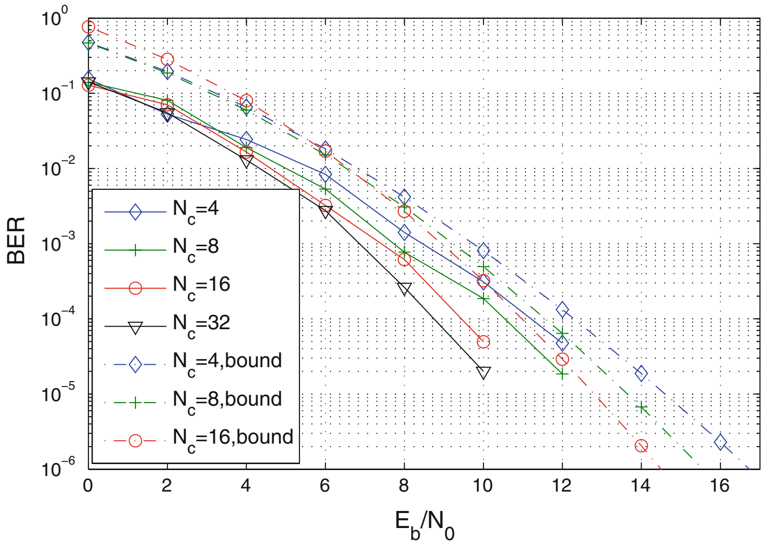

Fig. 8 Performance of a rate-$\frac{1}{2}$ convolutionally coded STBC OFDM system using $n_t = 4$ over Channel 1, and number of sub-carriers $N_c = 4$, 8, 16 and 32

frequency diversity provided by more sub-carriers. In general, the analytical results are very close to the simulation results.

The results of the coded STBC OFDM system using $n_t = 2$ over Channel 2 is shown in Fig. 9. We observe that the coded STBC OFDM system over Channel 1 performs worse than over Channel 2 since Channel 1 has less frequency selectivity due to its shorter delay-spread profile. For example, 16 sub-carriers multi-antenna system using $n_t = 2$ over Channel 1

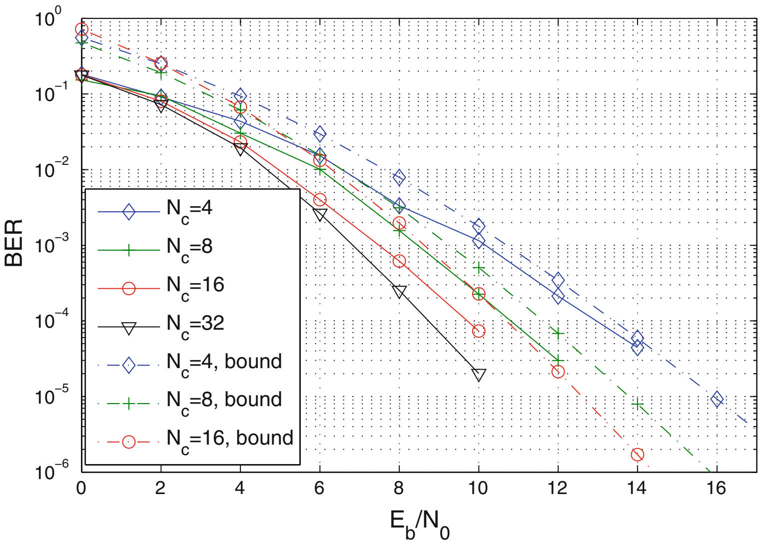

Fig. 9 Performance of a rate-$\frac{1}{2}$ convolutionally coded STBC OFDM system using $n_t = 2$ over Channel 2, and number of sub-carriers $N_c = 4$, 8, 16 and 32

needs 13.7 dB to achieve BER $= 10^{-4}$ whereas over Channel 2 needs 9.8 dB. Furthermore, the loss in performance due the less frequency selectivity in Channel 1 compared to Channel 2 is less noticed in the case of 2 antennas, compared to single-antenna systems. This is because more space diversity will reduce the effect of frequency diversity resulting from the CBF channel delay-spread profile.

5 Conclusion

In this paper, we have analyzed the error performance of binary coded OFDM system over wireless channels exhibiting CBF. Union bounds for convolutionally coded OFDM systems employing single and multiple antennas over CBF channel have been derived. The bound is a function of the eigenvalues for the correlation matrix between sub-carriers weighted with the errors in different sub-carriers. The bound has been verified using two CBF channels with different delay-spread profiles. Numerical results show that the proposed bound is tight to the simulation results. It is also observed that the coded OFDM systems perform better by increasing the numbers of sub-carriers due to the increased diversity order; and the delay-spread profile of the channel affects the performance of the system by providing different degrees of frequency selectivity.

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Author Biographies

Salam A. Zummo received his B.Sc. and M.Sc. degrees (with highest honors) in Electrical Engineering from King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, in 1998 and 1999, respectively. He received his Ph.D. degree from the University of Michigan at Ann Arbor, USA, in 2003. He is currently an Associate Professor of Electrical Engineering at KFUPM. His research interests include error control coding, iterative receivers, interference modeling and analysis, and cross-layer design of wireless communication networks.

Matasm M. Hassan received his B.Sc. degree in Electronics Engineering (with first class honours) from Sudan University of Science and Technology (SUST), Sudan, in 2002. He then received. M.Sc. in Telecommunication Engineering from King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, in 2008. Currently Matasm M. Hassan is an Instrumentation/Telecommunication Engineer in Zuhair Fayez Partnership (ZFP), Al-Khobar, Saudi Arabia.

Ping-Cheng Yeh received his B.Sc. degree in Mathematics and M.Sc. degree in Electrical Engineering from the National Taiwan University, Taiwan, in 1996 and 1998, respectively. He received his Ph.D. degree from the University of Michigan at Ann Arbor, USA, in 2005. He is currently an Assistant Professor in the Department of Electrical Engineering at the National Taiwan University. His research interests include channel coding, coded modulation, directional antennas, cooperative communications, and cross-layer design in wireless networks.