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Performance Evaluation of Generalized Selection Combiners Over Slow Fading with Estimation Errors

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Abstract In this paper we derive closed-form expressions for the single-user adaptive capacity of generalized selection combining (GSC) system, taking into account the effect of imperfect channel estimation at the receiver. The channel considered is a slowly varying spatially independent flat Rayleigh fading channel. The complex channel estimate and the actual channel are modelled as jointly Gaussian random variables with a correlation that depends on the estimation quality. Three adaptive transmission schemes are analyzed: (1) optimal power and rate adaptation; and (2) constant power with optimal rate adaptation, and (3) channel inversion with fixed rate. In addition to deriving an exact expression for the capacity of the aforementioned adaptive schemes, we analyze the impact of channel estimation error on the capacity statistics and the symbol error rate for GSC systems. The capacity statistics derived in this paper are the moment generating function, complementary cumulative distribution function and probability density function for arbitrary number of receive antennas. Moreover, exact closed-form expressions for M-PAM/PSK/QAM employing GSC are derived. As expected, the channel estimation error has a significant impact on the system performance.

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1 Introduction

An efficient way to combat multi-path fading and dramatically improve the system performance over fading channels is diversity. By applying diversity methods, multiple copies of the information signal can be sent over independent fading channels and combined by the receiver to maximize the power of received signal. The main principle of diversity is to ensure that the same information reaches the receiver on statistically independent channels. As the channels are statically independent, the probability that antennas are in a fading dip simultaneously is lower than the probability that one antenna is in a fading dip.

Diversity array combining techniques have attracted a lot of research attention in the last decade because they are capable of providing high speed wireless communications over a multipath environment. The receiver combines multiple copies of the desired signal, each of which being affected by an independent channel of the other signals. The most commonly used diversity combining techniques are equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), and a hybrid combination of SC and MRC, or called generalized selection combining (GSC). GSC is a technique to choose fixed subset of size M of a large number of available diversity channels of size L and then combine them using MRC which combines coherently the individually received branch signals so as to maximize the instantaneous output signal-to-noise ratio (SNR) [1–3]. It is well known that MRC is the optimal linear combining scheme. However, with receiver MRC, most of the system complexity concentrates at the receiver side. The main advantage of GSC over MRC is that GSC scheme reduces the number of radio frequency (RF) chains required at the receiver [2].

System performance analysis often considers perfect channel estimates under the assumption that the estimation errors are negligible. However, in practical systems, the branch SNR estimates are usually combined with noise which makes it difficult to estimate them perfectly. In practice, a diversity branch SNR estimate can be obtained either from a pilot signal or data signals (by applying a clairvoyant estimator) [4]. For example, if a pilot signal is inserted to estimate the channel, a Gaussian error may arise in due the large frequency separation or time dispersion. Thus, from a theoretical and practical viewpoints, it is important to quantify how GSC performance degrades due to the estimation error. Previous works on the analysis of imperfect channel estimation with no diversity can be found in [5–7]. In [4], Gans considered the channel estimation error of the MRC per branch SNR as being complex Gaussian and derives the probability density function (PDF) of the output combiner.

The pioneering work of Shannon [10] has established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a channel. Spectral efficiency of adaptive transmission techniques has received extensive interest in the last decade. In [12], the capacity of a single-user flat fading channel with perfect channel information at the transmitter and the receiver is derived for various adaptation policies, namely, (1) optimal rate and power adaptation (opra), (2)optimal rate adaptation (ora) and constant power, and (3) channel inversion with fixed rate (cifr). The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [12]. In [13], the general theory developed in [12] is applied to derive closed-form expressions for the capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques. Recently, there has been some work dealing with the channel capacity of different fading channels employing different adaptive schemes such as [14,15], and the references therein.

The objective of this paper is to investigate the impact of channel estimation error on the adaptive capacity, capacity statistics and the average symbol error rate (SER) for GSC over slow Rayleigh fading. The channel estimation error is quantified in terms of a correlation coefficient between the perfect channel and its estimate. This problem has been considered from SER point of view before in [8] and [9]. However, to the authors' best knowledge, this is the first attempt to derive the adaptive capacity, capacity statistics of GSC (L, M) with estimation errors for an arbitrary number of receive antennas. In this contribution, we investigate the adaptive capacity of GSC under different adaptive transmission in Rayleigh fading with estimation errors. Also, we derive the capacity statistics of GSC scheme which are valid for arbitrary number of receive antennas including moment generating function (MGF), cumulative distribution function (CDF) and, probability distribution function (PDF). The contributions of this paper are three-fold. First, we derive the capacity statistics of GSC subject to Rayleigh fading for an arbitrary number of receive antennas. Secondly, we derive closed-form expressions for the channel capacity of GSC in independent and identically distributed (i.i.d.) Rayleigh fading channels with the following adaptive transmission schemes: (1) opra; (2) ora with constant transmit power; and (3) cifr. Finally, we derive exact closed forms of SER for PAM/PSK/QAM with M-ary signaling.

The paper is organized as follows. In Sect. 2, the system model used in this paper is discussed. The capacity statistics are derived in Sect. 4. In Sect. 3, we derive closed-form expressions for the channel capacity under different adaptation schemes. Section 5 presents the closed-form expressions for average SER for M-PAM/PSK/QAM of GSC (L, M) in independent and identically distributed (i.i.d.) Rayleigh fading channels. Results are presented and discussed in Sect. 6. The main outcomes of the paper are summarized in Sect. 7.

2 System Model

In this section, we provide a brief description of GSC and the channel estimation model used throughout this paper. The channel estimation error used in this paper is characterized by the correlation between the true and estimated channel. Whereas the probability density function of effective SNR under channel estimation error can be expressed in terms of the correlation coefficient ρ. We consider an L-branch diversity receiver in slow fading channels. Assuming perfect timing and no inter-symbol interference (ISI), the received signal on the lth branch can be expressed as

$$r_l = g_l s + n_l, \quad l = 1, \ldots, L, \tag{1}$$

where g_l is a zero-mean complex Gaussian distributed channel gain, n_l is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$, and s is the data symbol taken from a normalized unit-energy signal set with an average power P_s. Under the assumption of channel estimation error at the receiver which employs GSC by combining the strongest M diversity branches out of a total of L diversity branches, the PDF of the received instantaneous SNR γ at the output of the combiner is given by [8]

$$p_\gamma(\gamma) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \times \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \exp\left(\frac{-\gamma}{\Lambda_l}\right), \tag{2}$$

where

$$D_l = \begin{cases} M, & \text{for } \ l=0 \\ 1, & \text{for } \ l \neq 0 \end{cases} \tag{3}$$

and the coefficients of partial fractions are given as follows: $\mu_0^0 = \binom{L}{M}$, $\mu_l^0 = (-1)^{M+l-1} \left(\frac{M}{M+l}\right)\left(\frac{M}{l}\right)^{M-1}\binom{L}{M}\binom{L-M}{l}$ for $l \neq 0$, $\mu_0^{M-k} = \sum_{i=l}^{L-M}(-1)^{i+M-k-1}\binom{L}{M}\binom{L-M}{i}\left(\frac{M}{i}\right)^{M-k}$ if $k < M < L$, and $(-1)^{M+l-1}\left(\frac{M}{M+l}\right)\left(\frac{M}{l}\right)^{M-1}\binom{L}{M}\binom{L-M}{l}$ for $l \neq 0$. The variable $\Lambda_l = \overline{\gamma}\left[(M + l(1-\rho^2))/(l+M)\right]$ and ρ denotes the correlation between the actual channel gains and their estimates and it can be expressed as

$$\rho = \frac{\text{cov}(g_l, \widehat{g_l})}{\sqrt{\text{var}(g_l)\text{var}(\widehat{g_l})}} = \sqrt{1-\epsilon^2}. \tag{4}$$

The actual channel gain g is related to the channel estimate $\widehat{g}$ [4] as follows

$$\widehat{g_l} = \sqrt{1-\epsilon^2} g_l + \epsilon z_l, \tag{5}$$

where z_l is a complex Gaussian random variable independent of $\widehat{g}$ with zero-mean and a unit variance and $\epsilon \in [0, 1]$ is a measure of the accuracy of the channel estimation. The actual channel gain is scaled to keep the covariance of the estimated channel and the true channel to be the same. For $\epsilon = 0$, the estimated channel is fully correlated with the true channel (perfect channel estimation $\rho = 1$). Note that for perfect channel estimation, the pdf of (2) reduces to the well-known PDF at the output of a GSC combiner which is given by

$$p_\gamma(\gamma) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k} \gamma^{k-1}(l+M)}{M(k-1)!\overline{\gamma}} \exp\left(\frac{-(l+M)\gamma}{M\overline{\gamma}}\right). \tag{6}$$

3 Adaptive Capacity Policies

In this section, we derive close-form expressions for the capacity of different adaptive schemes employing GSC over Rayleigh fading channels. In the derivation, we will rely on the main results from [13].

3.1 Power and Rate Adaptation

Given an average transmit power constraint, the channel capacity C_{opra} in (bits/s) of a fading channel [12,13] is given by

$$C_{\text{opra}} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma) \mathrm{d}\gamma, \tag{7}$$

where B (in hertz) is the channel bandwidth and γ_0 is the optimum cutoff SNR satisfying the following condition

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p_\gamma(\gamma)\mathrm{d}\gamma = 1. \tag{8}$$

To achieve the capacity in (7), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating high power levels and transmission rates for good channel conditions (large γ). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers from outage, whose probability P_{out} is defined as the probability of no transmission and is given by

$$P_{\text{out}} = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma)\mathrm{d}\gamma. \tag{9}$$

Substituting (2) into (8) yields the equality

$$\sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \times\left[\frac{\Lambda_l}{\gamma_0}\Gamma\left(n+1,\frac{\gamma_0}{\Lambda_l}\right)-\Gamma\left(n,\frac{\gamma_0}{\Lambda_l}\right)\right]=1. \tag{10}$$

In order to obtain the optimal cutoff SNR γ_0, in (10), we follow the following procedure. Let $x = \frac{\Lambda_l}{\gamma_0}$ and define $f_{\text{GSC}}(x)$ as

$$f_{\text{GSC}}(x) = \sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n} \times\sum_{n=0}^{k-1}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\left[\frac{\Gamma\left(n+1,x\right)}{x}-\Gamma\left(n,x\right)\right]. \tag{11}$$

Now, differentiating the function $f_{\text{GSC}}(x)$ with respect to x over the interval $(0, +\infty)$ results in

$$f_{\text{GSC}}^{'}(x) = \sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \times\left[\frac{\Gamma\left(n+1,x\right)}{x^2}\right] \tag{12}$$

Hence, $f_{\text{GSC}}^{'}(x) < 0, \forall x > 0$, meaning that $f_{\text{GSC}}^{'}$ is a strictly decreasing function of x. From (11) it can be observed that

(1) $\lim_{x\to 0} f_{\text{GSC}}(x) = +\infty$

(2) $\lim_{x\to +\infty} f_{\text{GSC}}(x) = \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n$

Note however that $f_{\text{GSC}}(x)$ is a continuous function of x, which leads to a unique positive γ_0 such that $f_{\text{GSC}}(x) = 0$. We thereby conclude that for each $\overline{\gamma} > 0$ there is a unique γ_0

satisfying (11). Numerical results using MATLAB shows that $\gamma_o \in [0, 1]$ as $\overline{\gamma}$ increases, and $\gamma_0 \to 1$ as $\overline{\gamma} \to \infty$.

Now, substituting (2) into (7) yields the channel capacity with opra scheme as follows

$$C_{\text{opra}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n \underbrace{\int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) \gamma^n \exp\left(\frac{-\gamma}{\Lambda_l}\right) \mathrm{d}\gamma}_{I_1}. \quad (13)$$

We evaluate I_1 by taking the help of the following identity [13] given by

$$J_s(\mu) = \int_1^{\infty} t^{s-1} \ln(t) e^{-\mu t} \mathrm{d}t = \frac{\Gamma(s)}{\mu^s} \left\{ E_1(\mu) + \sum_{k=1}^{s-1} \frac{1}{k} P_k(\mu) \right\}, \quad (14)$$

where E_1 denotes the Exponential integral of the first order [17] defined as

$$E_1(x) = \int_1^{\infty} \frac{e^{x\gamma}}{\gamma} \mathrm{d}\gamma, \quad x \geq 0, \quad (15)$$

and $P_k(\mu)$ denotes the Poisson distribution [17] given by

$$P_k(x) = \frac{\Gamma(k, x)}{\Gamma(k)} = e^{-x} \sum_{i=0}^{k-1} \frac{x^n}{n!}. \quad (16)$$

Upon substituting (14) into (13), the following closed-form expression for capacity C_{opra} per unit bandwidth (in bits/s/Hz) can be obtained as follows

$$C_{\text{opra}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \left\{ E_1\left(\frac{\gamma_0}{\overline{\gamma}}\right) + \sum_{k=1}^{n+1} \frac{1}{k} P_k\left(\frac{\gamma_0}{\overline{\gamma}}\right) \right\}. \quad (17)$$

The above capacity expression allows us to examine the limiting cases for ($M = L$, and $M = 1$) more conveniently for MRC and SC, respectively. For $M = L$, the $\mu_l^{(L-k)}$ coefficient becomes $\mu_0^0 = 1$ and $\mu_0^{(L-k)} = 0$ for all $k = 1, \ldots, L-1$. The opra capacity in (17) after some manipulation reduces to the closed-form expression for MRC capacity $C_{\text{opra}}^{\text{mrc}}$ per unit bandwidth (in bits/s/Hz) as follows

$$C_{\text{opra}}^{\text{mrc}} = \left[(1-\rho^2)\right]^{L-1} \sum_{n=0}^{L-1} \binom{L-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)}\right]^n \left\{ E_1\left(\frac{\gamma_0}{\overline{\gamma}}\right) + \sum_{k=1}^{n+1} \frac{1}{k} \mathcal{P}_k\left(\frac{\gamma_0}{\overline{\gamma}}\right) \right\}. \quad (18)$$

Letting $M = 1$ in (17) resulting in $\mu_0^0 = 1$, and $\mu_l^0 = (-1)^l \left(L/(l+1) \binom{L-1}{l} \right)$ for all $l = 1, \ldots, L-1$. Therefore, the capacity $C_{\text{opra}}^{\text{sc}}$ per unit bandwidth (in bits/s/Hz) after some manipulation can be obtained as follows

$$C_{\text{opra}}^{\text{sc}} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} E_1\left(\frac{(1+k)\gamma_0}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right) \exp\left(\frac{-\gamma_0(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right). \quad (19)$$

3.1.1 Asymptotic Approximation

We can obtain asymptotic approximation FOR C_{opra} using the series representation of Exponential integral of first order function [17] expressed as

$$E_1(x) = -E - \ln(x) - \sum_{i=1}^{+\infty} \frac{(-x)^i}{i.i!}, \quad (20)$$

where $E = 0.5772156659$ is the Euler–Mascheroni constant. Then, the asymptotic approximation C_{opra}^{∞} per unit bandwidth (in bits/s/hertz) can be shown to be

$$C_{\text{opra}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n$$
$$\times \left\{\left(-E - \ln\left(\frac{\gamma_0}{\overline{\gamma}}\right) + \left(\frac{\gamma_0}{\overline{\gamma}}\right)\right) + \sum_{k=1}^{n+1} \frac{1}{k} P_k\left(\frac{\gamma_0}{\overline{\gamma}}\right)\right\} \quad (21)$$

3.1.2 Upper Bound

The capacity expression of C_{opra} can be upper bounded by applying Jensen's inequality to (7) as follows

$$C_{\text{opra}}^{\text{UB}} = \ln\left(\mathbb{E}[\gamma]\right). \quad (22)$$

We evaluate the expression in (22) using the of pdf of γ given in (2) and the following identity [17]

$$\int_0^{\infty} x^n e^{-\mu x} \mathrm{d}x = n!\mu^{-n-1}, \quad \text{for } \operatorname{Re}[\mu] > 0, \quad (23)$$

and simplify the resulting expression to obtain the capacity (7) upper bound as follows

$$\frac{C_{\text{opra}}^{\text{UB}}}{B} = \left(\sum_{l=0}^{L-M} \sum_{k=1}^{D} \mu_l^{D_l-k} \Lambda_l^{2-k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\right.$$
$$\left. \times \sum_{n=0}^{k-1} \frac{(n+1)!}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\right). \quad (24)$$

3.2 Constant Transmit Power

By adapting the transmission rate to the channel fading condition with a constant power, the channel capacity C_{ora} [10,11] is given by

$$C_{\text{ora}} = \frac{B}{\ln 2} \int_0^{\infty} \ln(1+\gamma)\, p_{\gamma}(\gamma) \mathrm{d}\gamma. \quad (25)$$

Substituting (2) into (25) results in

$$\frac{C_{\text{ora}}}{B} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times \underbrace{\int_0^\infty \ln(1+\gamma)\,\gamma^n \exp\left(-\frac{\gamma}{\Lambda_l}\right) \mathrm{d}\gamma}_{I_3}. \tag{26}$$

The integral I_3 in (26) can be evaluated by taking use of equality [17]

$$\int_0^\infty \ln(1+y)y^{t-1}e^{xy}\mathrm{d}y = (t-1)!e^x \sum_{i=1}^{t} \frac{\Gamma(-t+i,x)}{x^i}, \tag{27}$$

where $\Gamma(.,.)$ is the complementary incomplete Gamma function which can be related to the exponential integral function $E_t(x)$ through [17]

$$E_t(x) = x^{t-1}\Gamma(1-l,x). \tag{28}$$

The expression in (27) can be further evaluated using (28) yielding

$$\int_0^\infty \ln(1+\gamma)\,\gamma^n \exp\left(\frac{-\gamma}{\Lambda_l}\right) \mathrm{d}\gamma = \exp\left(\frac{1}{\Lambda_l}\right) \sum_{j=1}^{n+1} \Lambda_l^{n+1} E_{n+2-j}\left(\frac{1}{\Lambda_l}\right). \tag{29}$$

Inserting (29) into (26) results in a closed-form expression for the capacity C_{ora} per unit bandwidth (in bits/s/hertz) given by

$$C_{\text{ora}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \frac{\Lambda_l}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \exp\left(\frac{1}{\Lambda_l}\right) \sum_{j=1}^{n+1} E_{n+2-j}\left(\frac{1}{\Lambda_l}\right). \tag{30}$$

The capacity C_{ora} can be expressed in another form as follows. It has been shown in [13] that the integral in (27) has the following form

$$\int_0^\infty \ln(1+y)y^{t-1}e^{xy}\mathrm{d}y = \frac{\Gamma(t)}{x^t} \left[P_t(-x)\,E_1(x) + \sum_{i=1}^{t-1} \frac{P_i(x)\,P_{t-i}(-x)}{i}\right]. \tag{31}$$

Substituting (31) into (26) yields another closed-form expression for the capacity C_{ora} per unit bandwidth (in bits/s/hertz)

$$C_{\text{ora}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \mu_l^{D_l-k} \Lambda_l^{1-k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \times \left[P_{n+1}\left(\frac{-1}{\Lambda_l}\right) E_1\left(\frac{1}{\Lambda_l}\right) + \sum_{i=1}^{n} \frac{P_i\left(\frac{1}{\Lambda_l}\right) P_{n+1-i}\left(\frac{-1}{\Lambda_l}\right)}{i}\right]. \tag{32}$$

For $M = L$, it yields the MRC capacity for ora policy per unit bandwidth (in bits/s/hertz)

$$C_{\text{ora}}^{\text{mrc}} = \left[(1-\rho^2)\right]^{L-1} \sum_{n=0}^{L-1} \binom{L-1}{n} \left[\frac{\rho^2}{(1-\rho^2)}\right]^n \times \left[P_{n+1}\left(\frac{-1}{\overline{\gamma}}\right) E_1\left(\frac{1}{\overline{\gamma}}\right) + \sum_{i=1}^{n} \frac{P_i\left(\frac{1}{\overline{\gamma}}\right) P_{n+1-i}\left(\frac{-1}{\overline{\gamma}}\right)}{i} \right]. \tag{33}$$

For $M = 1$, The ora capacity for SC is

$$\frac{C_{\text{ora}}^{\text{sc}}}{B} = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \exp\left(\frac{(1+k)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right) \times E_1\left(\frac{(1+k)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right). \tag{34}$$

3.2.1 Asymptotic Approximation

Following the same procedure in Sect. 3.1, the asymptotic approximation C_{ora}^{∞} per unit bandwidth (in bits/s/hertz) can be computed as

$$C_{\text{ora}}^{\infty} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \mu_l^{D_l-k} \Lambda_l^{1-k} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \times \left[P_{n+1}\left(\frac{-1}{\Lambda_l}\right)\left(-E - \ln\left(\frac{1}{\Lambda_l}\right) + \frac{1}{\Lambda_l}\right) + \sum_{i=1}^{n} \frac{P_i\left(\frac{1}{\Lambda_l}\right) P_{n+1-i}\left(\frac{-1}{\Lambda_l}\right)}{i} \right] \tag{35}$$

3.2.2 Upper Bound

The capacity C_{ora} can be upper bounded by applying Jensen's inequality to (7) as follows

$$\frac{C_{\text{ora}}^{\text{UB}}}{B} = \left(1 + \sum_{l=0}^{L-M} \sum_{k=1}^{D} \mu_l^{D_l-k} \Lambda_l^{2-k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \times \sum_{n=0}^{k-1} \frac{(n+1)!}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\right). \tag{36}$$

3.2.3 Higher SNR Region

The Shannon capacity can be approximated at high SNR region using the fact $\log_2(1+\gamma) = \log_2(\gamma)$ as $\gamma \to \infty$ for $x > 0$ yields an asymptotically tight bounds for (32) in high SNR region as follows

$$C^{\text{highSNR}} = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \mu_l^{D_l-k} \Lambda_l^{1-k} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \left[\psi(n+1) - \ln\left(\frac{1}{\Lambda_l}\right)\right], \tag{37}$$

where $\psi(x) = \frac{\mathrm{d}}{\mathrm{d}x}\ln(\Gamma(x))$. For integer values of x, the function is given by $\psi(x) = -E + \sum_{i}^{x-1}\frac{1}{i}$.

3.2.4 Lower SNR Region

We can approximate Shannon capacity in low SNR region by the square capacity of the argument (γ) in low SNR region as $\log_2(1+\gamma) \approx \sqrt{\gamma}$ [16]. Upon using this approximation along with definition of the incomplete Gamma function yields the approximated Shannon capacity at low SNR per unit bandwidth (in bits/s/hertz) as

$$C^{\mathrm{LowSNR}_1} \approx \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^{k-3/2}}\left[\overline{\gamma}(1-\rho^2)\right] \times \sum_{n=0}^{k-1}\frac{\Gamma(n+3/2)}{n!}\binom{k-1}{n}\left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n. \tag{38}$$

3.2.5 Lower SNR Region II

Another approximation for the Shannon capacity in low SNR region can be found by exploiting the fact $\log_2(1+\gamma) \approx \frac{1}{\ln(2)}(\gamma - \frac{1}{2}\gamma^2)$, which results the approximated Shannon capacity in low SNR region per unit bandwidth (in bits/s/hertz) as

$$C^{\mathrm{LowSNR}_2} \approx \frac{1}{\ln(2)}\sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}\Lambda_l^{2-k}}{n!}\left[\overline{\gamma}(1-\rho^2)\right]\sum_{n=0}^{k-1}\binom{k-1}{n} \times \left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n\left[\Gamma(n+2)-\frac{1}{2}\Gamma(n+3)\Lambda_l\Big)\right]. \tag{39}$$

3.3 Channel Inversion with Fixed Rate

In this subsection, we consider two schemes; namely, truncated channel inversion with fixed rate referred to as tifr, and channel inversion with fixed rate with no truncation, which we shall refer to as cifr. Channel inversion is an adaptive transmission technique whereby the transmitter uses the channel information feedback by the receiver in order to invert the fading channel. Accordingly, the channel appears to the encoder/decoder as a time-invariant AWGN channel. As a result, channel inversion suffers a large capacity penalty compared to the previous adaptation techniques (opra and ora), although it is much less complex to implement. The channel inversion technique requires a fixed code design and fixed rate modulation. In this case, the channel capacity C_{cifr} can be derived from the capacity of an AWGN channel with a received SNR and is given by [12,13]

$$C_{\mathrm{cifr}} = B\ln\left(1 + \frac{1}{\int_0^\infty \frac{1}{\gamma}p_\gamma(\gamma)\mathrm{d}\gamma}\right). \tag{40}$$

The channel capacity for the truncation scheme [13] C_{tifr} is given by

$$C_{\text{tifr}} = B \ln \left(1 + \frac{1}{\int_{\gamma_0}^{\infty} \frac{1}{\gamma} p_\gamma(\gamma) \mathrm{d}\gamma} \right) (1 - P_{\text{out}}), \tag{41}$$

where P_{out} is the outage probability. Note that, the cutoff level γ_0 can be chosen to either maximize C_{tifr} or achieve a specific P_{out}.

3.3.1 tifr

The integral in (41) can be evaluated using the PDF of the combiner output SNR given in (2) as

$$\int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma) \mathrm{d}\gamma = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \frac{1}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n \Gamma\left(n, \frac{\gamma_0}{\Lambda_l}\right). \tag{42}$$

Furthermore, the outage probability, P_{out} is derived from the CDF of the combiner output SNR, and is given by

$$\begin{aligned} P_{\text{out}} = \text{Prob}\,(\gamma \leq \gamma_0) &= \int_0^{\gamma_0} p_\gamma(\gamma) \mathrm{d}\gamma = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma) \mathrm{d}\gamma \\ &= 1 - \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^{k-1}} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \frac{1}{n!} \\ &\quad \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n \Gamma\left(n+1, \frac{\gamma_0}{\Lambda_l}\right). \end{aligned} \tag{43}$$

Substituting (42) and (43) in (41) yields a closed-form expression for the tifr capacity per unit bandwidth (in bits/s/Hz) that is given by

$$C_{\text{tifr}} = B \ln \left(1 + \frac{1}{\sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \Gamma\left(n, \frac{\gamma_0}{\Lambda_l}\right)} \right) \times \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^{k-1}} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \Gamma\left(n+1, \frac{\gamma_0}{\Lambda_l}\right). \tag{44}$$

The tifr capacity per unit bandwidth (in bits/s/Hz) for the case $M = L$ is

$$\frac{C_{\text{tifr}}^{\text{mrc}}}{B} = \left(1 + \frac{1}{\frac{1}{\overline{\gamma}} \left[(1-\rho^2)\right]^{L-1} \sum_{n=0}^{L-1} \binom{L-1}{n} \left[\frac{\rho^2}{(1-\rho^2)}\right]^n \Gamma\left(n, \frac{\gamma_0}{\overline{\gamma}}\right)} \right) \times \left[(1-\rho^2)\right]^{L-1} \sum_{n=0}^{L-1} \binom{L-1}{n} \left[\frac{\rho^2}{(1-\rho^2)}\right]^n \Gamma\left(n+1, \frac{\gamma_0}{\overline{\gamma}}\right). \tag{45}$$

The capacity C_{cifr} per unit bandwidth (in bits/s/hertz) for SC is expressed as:

$$\frac{C_{\text{tifr}}^{\text{sc}}}{B} = \ln\left(1 + \frac{\overline{\gamma}\left[k+1-k\rho^2\right]}{\sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1}\left[k+1-k\rho^2\right]E_1\left(\frac{\gamma_0(k+1)}{[k+1]}\right)}\right.$$
$$\left.\times\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\overline{\gamma}\left[k+1-k\rho^2\right]}\exp\left(\frac{\gamma_0(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right)\right). \quad (46)$$

Moreover, the expression in (46) can be used to derive an asymptotic approximation of C_{tifr}^{∞} as follows

$$\frac{C_{\text{tifr}}^{\text{sc}\infty}}{B} = \ln\left(1 + \frac{\overline{\gamma}\left[k+1-k\rho^2\right]}{\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\left[k+1-k\rho^2\right]\left(-E-\log\left(\frac{\gamma_0(k+1)}{[k+1]}\right)+\left(\frac{\gamma_0(k+1)}{[k+1]}\right)\right)}\right.$$
$$\left.\times\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\overline{\gamma}\left[k+1-k\rho^2\right]}\exp\left(\frac{\gamma_0(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right)\right). \quad (47)$$

3.3.2 cifr

If we set $\overline{\gamma} = 0$, we get the cifr capacity, where in this case the P_{out} is equivalent to zero. From (40) and (2), we get

$$\int_0^\infty \frac{1}{\gamma}p(\gamma)\mathrm{d}\gamma = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n. \quad (48)$$

By inserting (48) and (2) in (40) yields the cifr capacity per unit bandwidth (in bits/s/Hz) as follows

$$\frac{C_{\text{cifr}}}{B} = \left(1 + \frac{1}{\lim_{\gamma\to 0}\sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n}\right). \quad (49)$$

For MRC ($M = L$), the cifr capacity per unit bandwidth (in bits/s/Hz) can be obtained as

$$\frac{C_{\text{cifr}}}{B} = \left(1 + \frac{\overline{\gamma}}{\lim_{\gamma\to 0}\frac{1}{\gamma}\left[(1-\rho^2)\right]^{M-1}\sum_{n=0}^{L-1}\binom{L-1}{n}\left[\frac{\rho^2}{(1-\rho^2)}\right]^n\Gamma(n)}\right). \quad (50)$$

The cifr capacity for SC is

$$\frac{C_{\text{cifr}}}{B} = \ln\left(1 + \frac{\overline{\gamma}\left[k+1-k\rho^2\right]}{\lim_{\gamma\to 0}\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\left[k+1-k\rho^2\right]E_1\left(\frac{(k+1)}{[k+1]}\right)}\right). \quad (51)$$

4 Capacity Statistics

In this section, we focus on deriving the exact analytical expressions for capacity statistics of GSC over Rayleigh fading channels, assuming perfect channel knowledge at the receiver and no channel knowledge at the transmitter with average input-power constraint. The non-ergodic capacity of GSC system is given in [bit/s/Hz] by

$$C = \log_2(1+\gamma). \tag{52}$$

4.1 Moment Generating Function

The MGF of GSC capacity system in the presence of Gaussian channel estimation errors is given by

$$\Phi_C(\tau) = E\left[e^{\tau C}\right] = E\left[(1+\gamma)^{\frac{\tau}{\ln(2)}}\right]. \tag{53}$$

Expressing the expectation in an integral form over the PDF $p_\gamma(\gamma)$ and inserting (2), we obtain

$$\Phi_C(\tau) = \int_0^\infty (1+\gamma)^{\frac{\tau}{\ln(2)}} p_\gamma(\gamma) \mathrm{d}\gamma = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \underbrace{\int_0^\infty (1+\gamma)^{\frac{\tau}{\ln(2)}} \gamma^n \exp\left(-\frac{\gamma}{\Lambda_l}\right) \mathrm{d}\gamma}_{I_4}. \tag{54}$$

In order to evaluate the integral I_4, we make the change-of-variables $x = 1+\gamma$ to yield

$$\begin{aligned}
\int_0^\infty (1+\gamma)^{\frac{\tau}{\ln(2)}} \gamma^n \exp\left(\frac{-\gamma}{\Lambda_l}\right) \mathrm{d}\gamma &= \exp\left(\frac{1}{\Lambda_l}\right) \int_0^\infty x^{\frac{\tau}{\ln(2)}} (x-1)^{k-1} \exp\left(\frac{-x}{\Lambda_l}\right) \mathrm{d}x \\
&= \exp\left(\frac{1}{\Lambda_l}\right) \sum_{i=0}^{n} \binom{k-1}{i} (-1)^{k-i-1} \int_1^\infty x^{\left(\frac{\tau}{\ln(2)}+i\right)} \exp\left(\frac{-x}{\Lambda_l}\right) \mathrm{d}x. \\
&= \exp\left(\frac{1}{\Lambda_l}\right) \sum_{i=0}^{n} \binom{k-1}{i} (-1)^{k-i-1} \Gamma\left(n+1, \frac{\tau}{\ln(2)}+i\right) \Lambda_l^{\left(\frac{\tau}{\ln(2)}+i+1\right)}.
\end{aligned} \tag{55}$$

The result in (55) is obtained by the taking the use of binomial expression and the help of (23). Substituting (55) into (54) results in a closed-form expression for the MGF which is expressed as

$$\Phi_C(\tau) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times \exp\left(\frac{1}{\Lambda_l}\right) \sum_{i=0}^{n} \binom{k-1}{i} (-1)^{k-i-1} \Gamma\left(n+1, \frac{\tau}{\ln(2)}+i\right) \Lambda_l^{\left(\frac{\tau}{\ln(2)}+i\right)}. \tag{56}$$

For the case $M = L$, the MGF can be finally expressed as follows

$$\Phi_C^{\text{mrc}}(\tau) = \frac{1}{\overline{\gamma}}\left[(1-\rho^2)\right]^{L-1}\sum_{n=0}^{L-1}\frac{1}{n!}\binom{L-1}{n}\left[\frac{\rho^2}{\overline{\gamma}(1-\rho^2)}\right]^n \times \exp\left(\frac{1}{\overline{\gamma}}\right)\sum_{i=0}^{n}\binom{n-1}{i}(-1)^{n-i-1}\Gamma\left(n+1,\frac{\tau}{\ln(2)}+i\right)\overline{\gamma}^{\left(\frac{\tau}{\ln(2)}+i+1\right)}. \tag{57}$$

For $M = 1$, MGF in (56) reduces to

$$\Phi_C^{\text{sc}}(\tau) = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\left(\frac{(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right)^{\frac{\tau}{\ln(2)}}\exp\left(\frac{(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right) \times\Gamma\left(\frac{\tau}{\ln(2)}+1,\left(\frac{(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right)\right). \tag{58}$$

Furthermore, the integral in (54) can be expressed in another form with help of the integral representation of the confluent hypergeometric function $\Psi(a, b; z)$ [17]

$$\Psi(a,b;z) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}\mathrm{d}t, \tag{59}$$

which results in the new closed-form expression for the MGF given by

$$\Phi_C(\tau) = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times\Psi\left(n+1,\frac{\tau}{\ln(2)}+2+n;\frac{1}{\Lambda_l}\right). \tag{60}$$

Note that using an alternative notation for $\Psi(a,b;z) = z^{-a}{}_2F_0(a, 1+a-b; .; -1/z)$ where ${}_2F_0(., ; . : .)$ is a generalized hypergeometric serious [17], the MGF of the capacity can be simply written as

$$\Phi_C(\tau) = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^{k-1}}\left[\overline{\gamma}(1-\rho^2)\right]\sum_{n=0}^{k-1}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \times{}_2F_0\left(n+1,-\frac{\tau}{\ln(2)};-\Lambda_l\right). \tag{61}$$

4.2 Complementary Cumulative Distribution Function (CCDF)

We evaluate the CDF of (52) in the presence of Gaussian channel estimation errors as follows

$$F_{\overline{C}} = \text{Prob}\left(\overline{C} \leq C\right) = \int_0^{2^C-1} p_\gamma(\gamma)\mathrm{d}\gamma = 1 - \int_{2^C-1}^{\infty} p_\gamma(\gamma)\mathrm{d}\gamma$$
$$= 1 - \sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^{k-1}} \left[\overline{\gamma}(1-\rho^2)\right] \sum_{n=0}^{k-1} \frac{1}{n!}$$
$$\times \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \Gamma\left(n+1, \frac{2^C-1}{\Lambda_l}\right). \quad (62)$$

Thus, the complementary CDF can be obtained from (62) as follows

$$\overline{F}_{\overline{C}} = \sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^{k-1}} \left[\overline{\gamma}(1-\rho^2)\right]^{L-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n$$
$$\times\Gamma\left(n+1, \frac{2^C-1}{\Lambda_l}\right). \quad (63)$$

For $M = L$, the CCDF of the capacity simplifies to

$$\overline{F}_C^{\text{MRC}}(C) = \left[(1-\rho^2)\right]^{L-1} \sum_{n=0}^{L-1} \binom{L-1}{n} \left[\frac{\rho^2}{(1-\rho^2)}\right]^n \Gamma\left(n+1, \frac{2^C-1}{\overline{\gamma}}\right). \quad (64)$$

For $M = 1$, the CCDF in (63) reduces to

$$\overline{F}_C^{\text{SC}}(C) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \exp\left(\frac{2^C-1(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]}\right). \quad (65)$$

4.3 Probability Density Function

The PDF of the capacity is defined as the derivative of the CCDF with respect to C, which can be written as

$$p_C(C) = 2^C \ln(2) \sum_{l=0}^{L-M}\sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^{k}} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n}$$
$$\times \left[\frac{M\rho^2(2^C-1)}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \exp\left(-\frac{(2^C-1)}{\Lambda_l}\right). \quad (66)$$

Note that (66) can also be obtained from (2) by performing a transformation of random variables $\gamma \to C$.

1) $M = L$:

$$p_C^{\text{mrc}}(C) = 2^C \ln(2) \frac{1}{\overline{\gamma}} [(1-\rho^2)]^{M-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n}$$
$$\times \left[\frac{M\rho^2(2^C-1)}{\overline{\gamma}(1-\rho^2)}\right]^n \exp\left(-\frac{(2^C-1)}{\overline{\gamma}}\right). \quad (67)$$

2) $M = 1$:

$$p_C^{\mathrm{sc}}(C) = 2^C \ln(2) \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left(\frac{k+1}{\overline{\gamma}\left[k+1-k\rho^2\right]} \right) \exp\left(\frac{(2^C-1)(k+1)}{\overline{\gamma}\left[k+1-k\rho^2\right]} \right) \tag{68}$$

5 Symbol Error Rate

In wireless communications, the symbol error rate (SER) is one of the prime factors in determining the performance of the system over fading channels. In this section, we derive the SER for different modulation schemes employing GSC with channel estimation errors. For a given instantaneous SNR γ over an AWGN channel, the SER for different modulation schemes with M-ary signaling is given by $P_{e,\mathrm{PAM}}(\gamma) = a_{\mathrm{PAM}} Q(\sqrt{g_{\mathrm{PAM}}\gamma})$, $P_{e,\mathrm{PSK}}(\gamma) = a_{\mathrm{PSK}} Q(\sqrt{g_{\mathrm{PAM}}\gamma})$, and $P_{e,\mathrm{QAM}}(\gamma) = 1 - [1 - a_{\mathrm{QAM}} Q(\sqrt{g_{\mathrm{QAM}}\gamma})]^2$, where $a_{\mathrm{PAM}} = 2\left(1 - \frac{1}{M_d}\right)$, $g_{\mathrm{PAM}} = \frac{6}{M_d^2-1}$, $a_{\mathrm{PSK}} = 2$, $g_{\mathrm{PSK}} = 2\sin^2\left(\frac{\pi}{M_d}\right)$, $a_{\mathrm{QAM}} = 2\left(1 - \frac{1}{\sqrt{M_d}}\right)$, and $g_{\mathrm{QAM}} = \frac{3}{M_d-1}$. In order to obtain the SER in fading channels, the conditional SER is averaged over the PDF of γ. Thus, we obtain the following expression for SER of GSC using MPAM modulation

$$\begin{aligned} \mathrm{SER} = & \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \\ & \times \sum_{n=0}^{k-1} \frac{1}{n!} \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)} \right]^n \underbrace{\int_0^\infty Q(\sqrt{g_{\mathrm{PAM}}\gamma})\gamma^n \exp\left(\frac{\gamma}{\Lambda_l}\right) \mathrm{d}\gamma}_{I_5}. \end{aligned} \tag{69}$$

In order to compute I_5 in (69) we use the following formula

$$I(p,q,m) = \frac{\Gamma(m)}{q^m} \int_0^\infty Q\left(\sqrt{px}\right) e^{-qx} x^{m-1} \mathrm{d}x. \tag{70}$$

By applying Craig's formula given by $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp(-\frac{x^2}{2\sin^2\theta})\mathrm{d}\theta$, and the equality $\int_0^\infty x^{v-1} e^{-\mu x}\mathrm{d}x$, the integral in (70) can be evaluated. Upon using the Gaussian hypergeometric function ${}_2F_1(a,b;c;z) = \sum_{k=0}^\infty \frac{(a)_k(b)_k}{(c)_k}\frac{z^k}{k!}$, the integral in (70) can be simplified to

$$I(p,q,m) = \frac{1}{2\sqrt{\pi}(1+s)^m} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)} \times {}_2F_1\left(m, \frac{1}{2}; m+1; \frac{1}{s+1}\right), \tag{71}$$

where $s = \frac{p}{2q}$. Using the fact that ${}_2F_1(a,b;c;z) = (1-z)^{c-a-b}{}_2F_1(c-a,c-b;c;z)$, (71) can be expressed as

$$I(p,q,m) = \frac{1}{2\sqrt{\pi}(1+s)^m} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)} \left(\frac{s}{s+1}\right)^{\frac{1}{2}} \times {}_2F_1\left(1, m, +\frac{1}{2}; m+1; \frac{1}{s+1}\right). \tag{72}$$

Substituting $p = g_{\text{PAM}}$, $q = \frac{1}{\Lambda_l}$, and $m = n+1$ in (72), the following expression is obtained

$$I(g_{\text{PAM}}, b, k) = \frac{1}{2\sqrt{\pi}\,(2+\Lambda_l g_{\text{PAM}})^{n+1}} \frac{\Gamma(n+\frac{3}{2})}{\Gamma(n+2)} \left(\frac{\Lambda_l g_{\text{PAM}}}{\Lambda_l g_{\text{PAM}}+2}\right)^{\frac{1}{2}} \times {}_2F_1\left(1, n+1, +\frac{1}{2}; n+2; \frac{1}{\frac{g_{\text{PAM}}}{2b}+1}\right). \tag{73}$$

In order to get the SER for MPAM modulation, (73) is substituted in (69), and the final expression for the SER of MPAM using GSC is given by

$$\text{SER}_{\text{PAM}} = a_{\text{PAM}} \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times \frac{\left(\frac{\Lambda_l g_{\text{PAM}}}{\Lambda_l g_{\text{PAM}}+2}\right)^{\frac{1}{2}}}{2\sqrt{\pi}\,(2+\Lambda_l g_{\text{PAM}})^{n+1}} \frac{\Gamma(n+\frac{3}{2})}{\Gamma(n+2)} {}_2F_1\left(1, n+1, +\frac{1}{2}; n+2; \frac{1}{\Lambda_l g_{\text{PAM}}+2}\right). \tag{74}$$

Similarly, the SER for MPSK modulation can be expressed as

$$\text{SER}_{\text{PSK}} = a_{\text{PSK}} \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times \frac{\left(\frac{\Lambda_l g_{\text{PSK}}}{\Lambda_l g_{\text{PSK}}+2}\right)^{\frac{1}{2}}}{2\sqrt{\pi}\,(2+\Lambda_l g_{\text{PSK}})^{n+1}} \frac{\Gamma(n+\frac{3}{2})}{\Gamma(n+2)} {}_2F_1\left(1, n+1, +\frac{1}{2}; n+2; \frac{1}{\Lambda_l g_{\text{PSK}}+2}\right). \tag{75}$$

Assuming matched filter detection and perfect channel estimation for M-QAM system, the average SER for M-QAM with GSC diversity reception in Rayleigh fading environment can be obtained by averaging its conditional SER over the PDF of γ at the output of GSC diversity receiver resulting in

$$\text{SER}_{\text{QAM}} = 1 - \left\{1 - a_{\text{QAM}} \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \left[\overline{\gamma}(1-\rho^2)\right] \times \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times \frac{\left(\frac{\Lambda_l g_{\text{QAM}}}{\Lambda_l g_{\text{QAM}}+2}\right)^{\frac{1}{2}}}{2\sqrt{\pi}\,\left(2+\Lambda_l g_{\text{QAM}}\right)^{n+1}} \frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma(n+2)} {}_2F_1\left(1, n+1, +\frac{1}{2}; n+2; \frac{1}{\Lambda_l g_{\text{QAM}}+2}\right)\right\}^2. \tag{76}$$

Equation (72) can be further simplified when m is restricted to a positive integer value in (71) to give

$$I(p,q,m)=\frac{1}{2}\left[1-\mu\sum_{j=0}^{m-1}\binom{2j}{j}\left(\frac{1-\mu^2}{4}\right)^j\right], \tag{77}$$

where $\mu=\sqrt{\frac{p}{p+2q}}$. Therefore, the integral I_5 in (69) can be substituted by (77) to yield

$$\begin{aligned}\mathrm{SER_{PSK}}&=\frac{a_{\mathrm{PSK}}}{2}\sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n\\&\quad\times\left[1-\sqrt{\frac{\Lambda_l g_{\mathrm{PSK}}}{\Lambda_l g_{\mathrm{PSK}}+1}}\sum_{j=0}^{n}\binom{2j}{j}\left(\frac{1}{4(\Lambda_l g_{\mathrm{PSK}}+1)}\right)^j\right].\end{aligned} \tag{78}$$

The SER for MQAM is given by

$$\begin{aligned}\mathrm{SER_{QAM}}=1-\Bigg\{1-\frac{a_{\mathrm{QAM}}}{2}\sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\\\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n\\\left[1-\sqrt{\frac{\Lambda_l g_{\mathrm{QAM}}}{\Lambda_l g_{\mathrm{QAM}}+1}}\sum_{j=0}^{n}\binom{2j}{j}\left(\frac{1}{4(\Lambda_l g_{\mathrm{QAM}}+1)}\right)^j\right]\Bigg\}^2.\end{aligned} \tag{79}$$

We remark that as $\overline{\gamma}\to\infty$, ${}_2F_1(.)\to 1$. For example, the $\mathrm{SER_{PSK}}$ asymptotically approaches the following expression

$$\begin{aligned}\mathrm{SER_{PSK}}&=a_{\mathrm{PSK}}\sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}\left[\overline{\gamma}(1-\rho^2)\right]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n\\&\quad\times\frac{\left(\frac{\Lambda_l g_{\mathrm{PSK}}}{\Lambda_l g_{\mathrm{PSK}}+2}\right)^{\frac{1}{2}}}{2\sqrt{\pi}\,(2+\Lambda_l g_{\mathrm{PSK}})^{n+1}}\frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma(n+2)}.\end{aligned} \tag{80}$$

For $M=L$, the SER for MSPK in (75) reduces to

$$\begin{aligned}\mathrm{SER_{PSK}}&=a_{\mathrm{PSK}}\left[\frac{1}{\overline{\gamma}}(1-\rho^2)\right]\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{\rho^2}{\overline{\gamma}(1-\rho^2)}\right]^n\\&\quad\times\frac{\left(\frac{\overline{\gamma}g_{\mathrm{PSK}}}{\overline{\gamma}g_{\mathrm{PSK}}+2}\right)^{\frac{1}{2}}}{2\sqrt{\pi}\,(2+\overline{\gamma}g_{\mathrm{PSK}})^{n+1}}\frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma(n+2)}{}_2F_1\left(1,n+1,+\frac{1}{2};n+2;\frac{1}{\overline{\gamma}g_{\mathrm{PSK}}+2}\right)\\&=\frac{a_{\mathrm{PSK}}}{2}\left[\frac{1}{\overline{\gamma}}(1-\rho^2)\right]\sum_{n=0}^{k-1}\frac{1}{n!}\binom{k-1}{n}\left[\frac{\rho^2}{\overline{\gamma}(1-\rho^2)}\right]^n\\&\quad\times\left[1-\sqrt{\frac{\overline{\gamma}g_{\mathrm{PSK}}}{\overline{\gamma}g_{\mathrm{PSK}}+1}}\sum_{j=0}^{n}\binom{2j}{j}\left(\frac{1}{4(\overline{\gamma}g_{\mathrm{PSK}}+1)}\right)^j\right].\end{aligned} \tag{81}$$

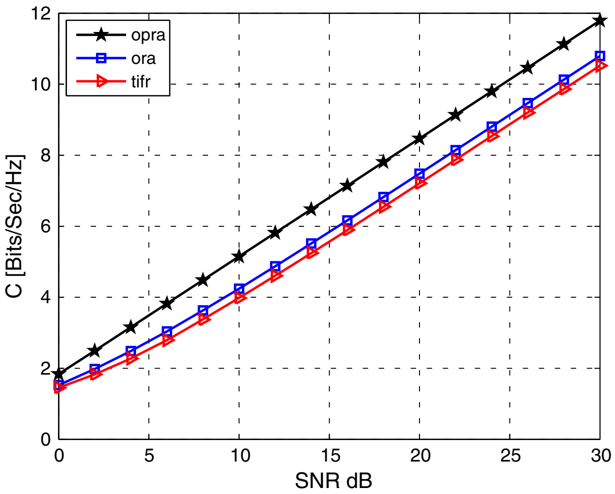

Fig. 1 Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$, $M = 3$) for different adaptation. Schemes with perfect estimation $\rho^2 = 1$

For MQAM, it can be represented as

$$\mathrm{SER_{QAM}} = 1 - \left\{ 1 - a_{\mathrm{QAM}} \left[\frac{1}{\overline{\gamma}}(1-\rho^2) \right] \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{\rho^2}{\overline{\gamma}(1-\rho^2)} \right]^n \right.$$
$$\left. \times \frac{\left(\frac{\overline{\gamma} g_{\mathrm{QAM}}}{\overline{\gamma} g_{\mathrm{QAM}}+2} \right)^{\frac{1}{2}}}{2\sqrt{\pi} \left(2 + \Lambda_l g_{\mathrm{QAM}} \right)^{n+1}} \frac{\Gamma\left(n+\frac{3}{2}\right)}{\Gamma(n+2)} {}_2F_1\left(1, n+1, +\frac{1}{2}; n+2; \frac{1}{\overline{\gamma} g_{\mathrm{QAM}}+2}\right) \right\}^2$$
$$= 1 - \left\{ 1 - \frac{a_{\mathrm{QAM}}}{2} \left[\frac{1}{\overline{\gamma}}(1-\rho^2) \right] \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \left[\frac{\rho^2}{\overline{\gamma}(1-\rho^2)} \right]^n \right.$$
$$\left. \times \left[1 - \sqrt{\frac{\overline{\gamma} g_{\mathrm{QAM}}}{\overline{\gamma} g_{\mathrm{QAM}}+1}} \sum_{j=0}^{n} \binom{2j}{j} \left(\frac{1}{4(\overline{\gamma} g_{\mathrm{QAM}}+1)} \right)^j \right] \right\}^2. \quad (82)$$

For SC, i.e., when $M = 1$, the SER for MPSK

$$\mathrm{SER_{PSK}} = a_{\mathrm{PSK}} \sum_{k=0}^{M-1} (-1)^k \binom{M}{k+1} \left[1 - \sqrt{\frac{1}{1 + \frac{(k+1)}{g_{\mathrm{PSK}}\overline{\gamma}(k+1-k\rho^2)}}} \right], \quad (83)$$

and the SER for MQAM,

$$\mathrm{SER_{QAM}} = 1 - \left\{ 1 - a_{\mathrm{QAM}} \sum_{k=0}^{M-1} (-1)^k \binom{M}{k+1} \left[1 - \sqrt{\frac{1}{1 + \frac{(k+1)}{g_{\mathrm{QAM}}\overline{\gamma}(k+1-k\rho^2)}}} \right] \right\}^2. \quad (84)$$

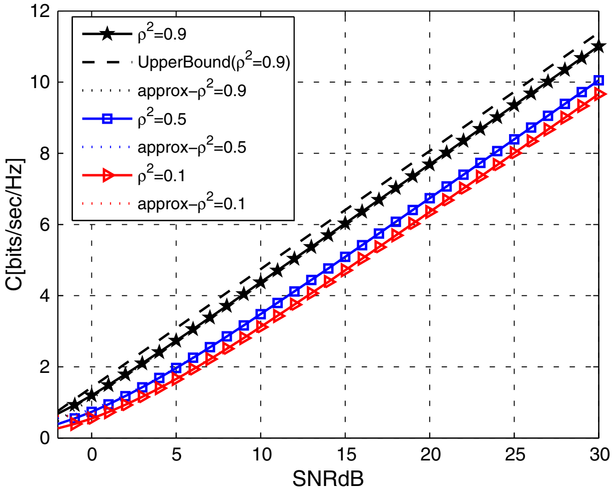

Fig. 2 Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$, $M = 3$) and various values of different ρ^2 under power and rate adaptation

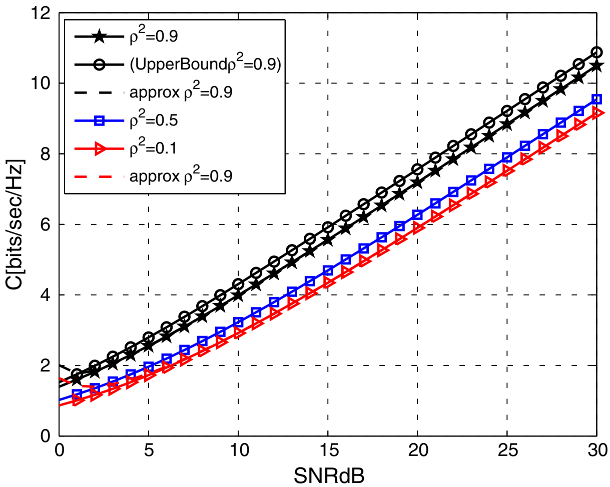

Fig. 3 Capacity per unit bandwidth for a Rayleigh fading channel with SCD ($L = 4$, $M = 3$) and various values of different ρ^2 under rate adaptation and constant power

6 Numerical Results

In this section, we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR $\overline{\gamma}$ in dB for different adaptation policies with GSC over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions.

Figure 1 shows the comparison of the capacity per unit bandwidth for opra, ora and tifr policies for GSC ($L = 4$, $M = 3$). The result indicates how the opra policy achieves the highest capacity for any average receive SNR, $\overline{\gamma}$. From the same figure, it can be noticed

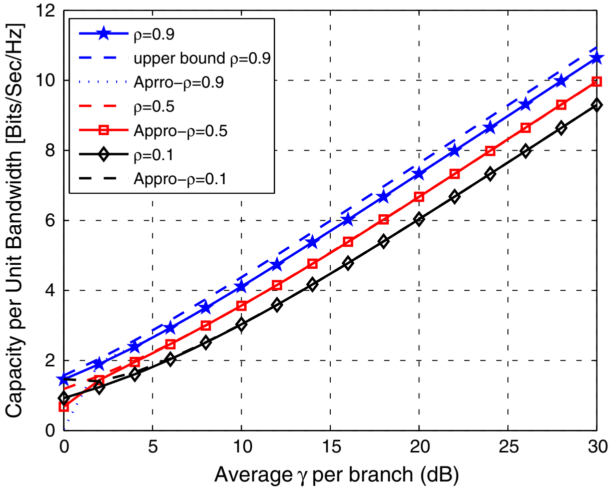

Fig. 4 Capacity per unit bandwidth for a Rayleigh fading channel with SCD ($L = 4$, $M = 3$) and various values of different ρ^2 under rate adaptation and constant power

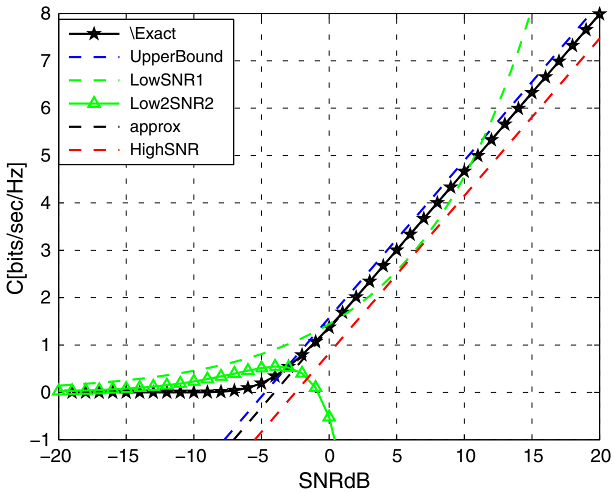

Fig. 5 Exact and approximated capacity per unit bandwidth for a Rayleigh fading with GSC diversity for ($L = 4$, $M = 3$) under imperfect channel estimation $\rho^2 = 0.9$

that ora achieves less capacity than opra. However, both opra and ora achieve the same result when there is no power adaptation implemented at the transmitter as in opra. As expected, Fig. 1 shows that the tifr scheme achieves less capacity compared to the other adaptation policies. The results in Fig. 1 is plotted for the case of fully estimated channel ($\rho^2 = 1$). Figure 2 compares C_{opra} for different values of correlation between the channel and its estimate; namely, $\rho^2 = 0.1$, $\rho^2 = 0.5$, and $\rho^2 = 0.9$. It can be noticed that the highest C_{opra} that can be achieved is when $\rho^2 = 1$ as shown in Fig. 1 with perfect estimation. Furthermore, C_{opra} decreases when the value of ρ^2 decreases where in this case the weight error increases. It can be observed from Fig. 2 that there is almost a 3 dB difference in C_{opra} between $\rho^2 = 0.9$

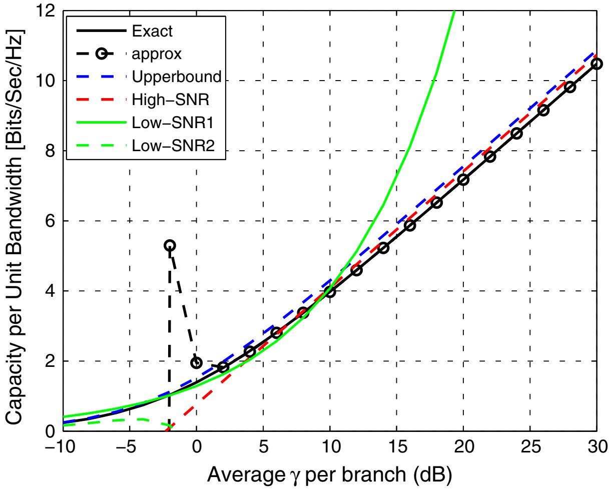

Fig. 6 Exact and approximated capacity per unit bandwidth for a Rayleigh fading with diversity for ($L = 4$, $M = 1$) under imperfect channel estimation $\rho^2 = 0.9$

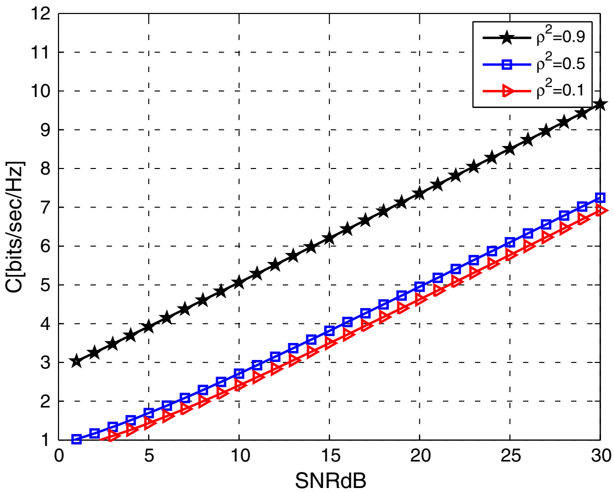

Fig. 7 Capacity per unit bandwidth for a Rayleigh fading with GCD diversity ($L = 4$, $M = 3$) and various values of ρ^2 under truncated channel inversion

and $\rho^2 = 0.1$. In Fig. 2, we showed the asymptotic capacity approximation as expressed in (21) and the upper bound in (24). We observe that the asymptotic capacity provide a good match to the exact capacity values which can be taken as a tight measure in case that the exact capacity can not be obtained analytically. Figure 3 shows the plot of C_{ora} as well as its asymptotic approximation and upper bound as a function of the average received SNR $\overline{\gamma}$ for ($L = 4$, $M = 3$). As can be seen from the same figure that the ora policy is more sensitive to the estimation error than the opra policy by 1 dB difference between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure 4 considers a special case of GSC with ($L = 4$, $M = 1$) in which the capacity shown in Fig. 3 is collapsed into the well known SC scheme. Figure 5 depicts

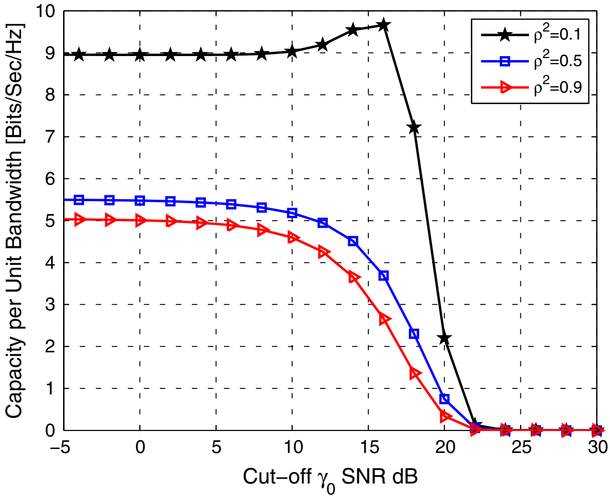

Fig. 8 Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$, $M = 3$) and various values of ρ^2 versus optimal cutoff SNR γ_0 with truncated channel inversion with $\overline{\gamma} = 15dB$

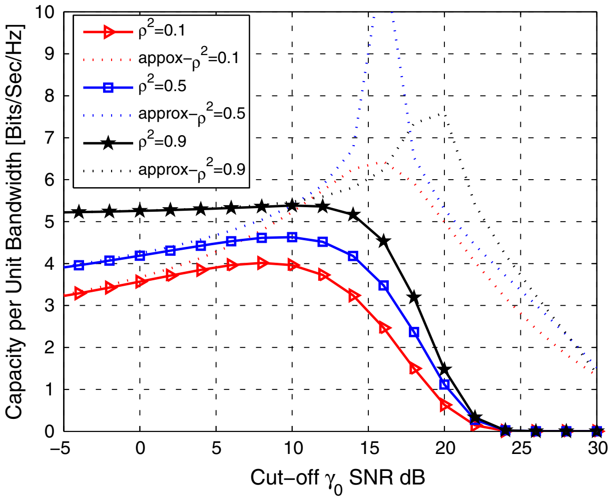

Fig. 9 Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$, $M = 1$) and various values of ρ^2 versus optimal cutoff SNR γ_0 with truncated channel inversion with $\overline{\gamma} = 15$ dB

the exact closed-form expression capacity of (32) for ($L = 4$, $M = 3$) and $\rho = 0.9$ as well as the corresponding approximations given in (35–39). It can be observed that the upper bound for SC and the high-SNR approximation for the SC scheme match on each other for SNR $\geq$ 5 dB which show a tight approximation of the exact average capacity. Furthermore, the approximations for low SNR region for the SC scheme are twofold: (1) the low-SNR approximation becomes tight for SNR values less than −5 dB, whereas; (2) the expression the low-SNR approximation II becomes tight to exact capacity between 0 and 10 dB as shown in Fig. 6. Figure 7 illustrates the capacity of the tifr policy for different values of ρ^2. The tifr policy performs the worst compared to the other adaptation policies. However,

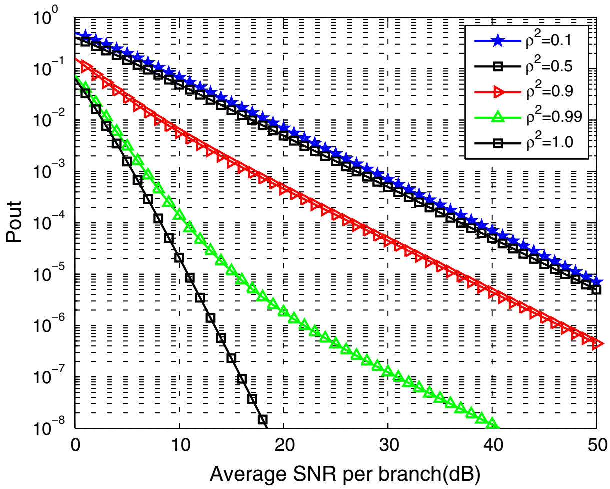

Fig. 10 Probability of outage for a Rayleigh fading with with GCD diversity ($L = 4$, $M = 3$) and various values of ρ^2 and $\gamma_0 = 0.7$

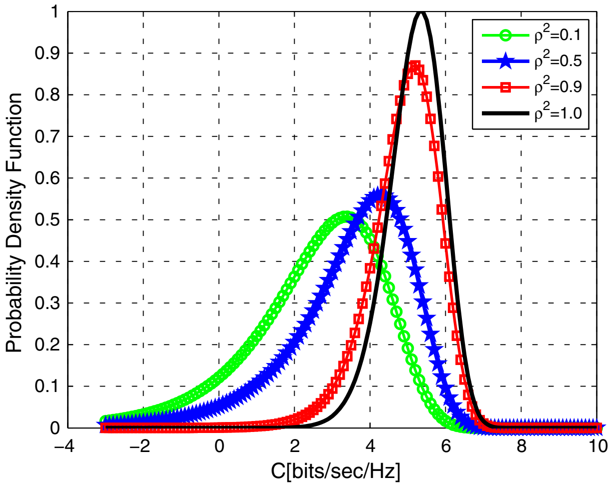

Fig. 11 Probability density function $P_C(C)$ for a GSC with ($L = 4$, $M = 3$) at $\overline{\gamma} = 15$ dB for different values of ρ^2

the loss due to the estimation errors is very high, which is almost 15 dB difference between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure 8 depicts the behavior of C_{tifr} against the optimal cutoff SNR, γ_0 for different values of ρ^2 and GSC ($L = 4$, $M = 3$), and Fig. 9 shows the same behavior for the case GSC ($L = 4$, $M = 1$), which illustrates the C_{tifr} performance for SC scheme as expressed in (46). As can be seen that C_{tifr} becomes almost flat when the value of ρ^2 decreases which experiences a large capacity loss. Figure 10 shows the behavior of the outage probability for different values of ρ^2. It can be observed that when there is no data to be transmitted because of the outage event, the tifr policy suffers an outage probability which is larger than the outage probability suffered by the opra policy. In addition, we note

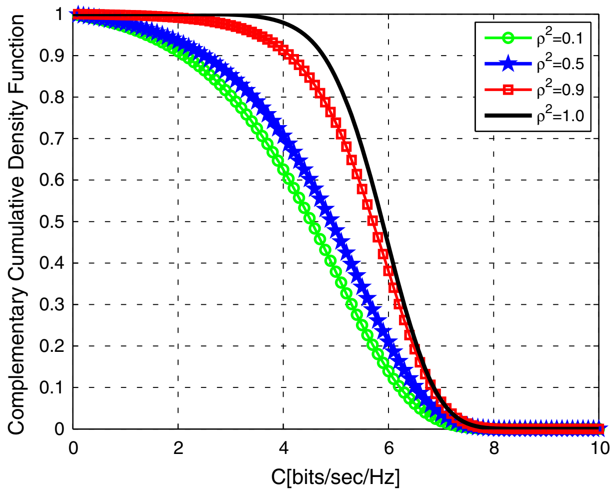

Fig. 12 Cumulative distribution function $F_C(C)$ for a GSC with $(L = 4, M = 3)$ at $\overline{\gamma} = 15$ dB for different values of ρ^2

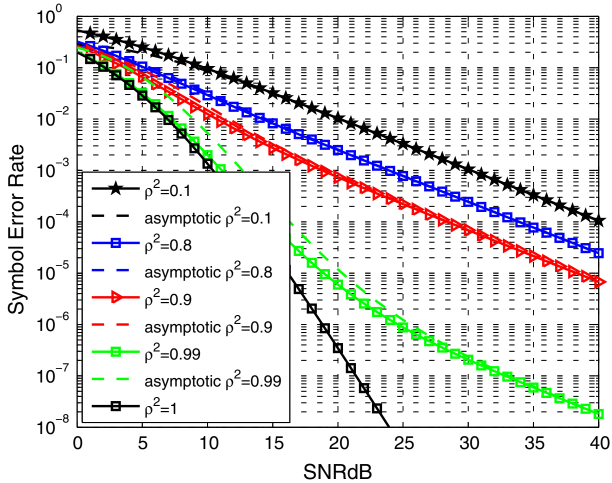

Fig. 13 Symbol error probability performance for QPSK modulation with GSC with $(L = 4, M = 3)$ and various values of ρ^2

that the correlation ρ^2 has a greater influence on tifr than it has on opra and the decrease in outage probability with increase in $\overline{\gamma}$ is faster for opra than tifr.

Figure 11 depicts the PDF curves for different values of the correlation coefficient squared ρ^2, considering an average SNR per branch of $\overline{\gamma} = 15$ dB and $(L = 4, M = 1)$. This figure shows that the capacity distribution has a Gaussian-like shape even in the presence of channel estimation errors. As expected, the distribution of C shift towards the left indicating a decreasing value of its mean as the value of ρ^2 decreases. Figure 12 considers the same setting in Fig. 11 and depicts the CCDF curves for different values of ρ^2, with very similar observations. The expressions in (74–76 , 78), and (79) generalize the M-PSK and the square M-QAM average symbol error rates results with GSC (L, M) over slow Rayleigh fading

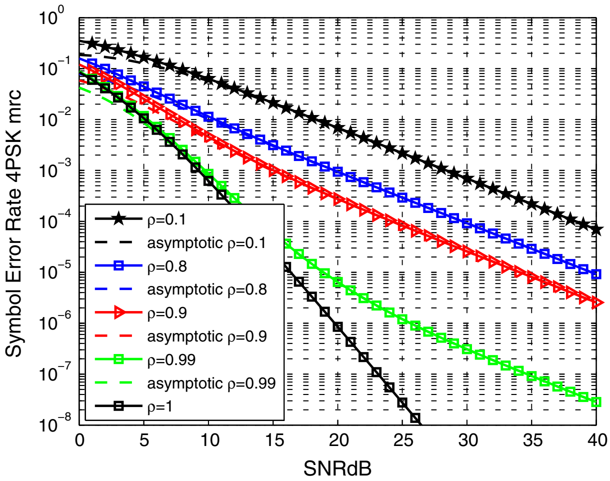

Fig. 14 Symbol error probability performance for QPSK modulation under various values of ρ^2 for a GSC with ($L = 4$, $M = 1$) given by (75)

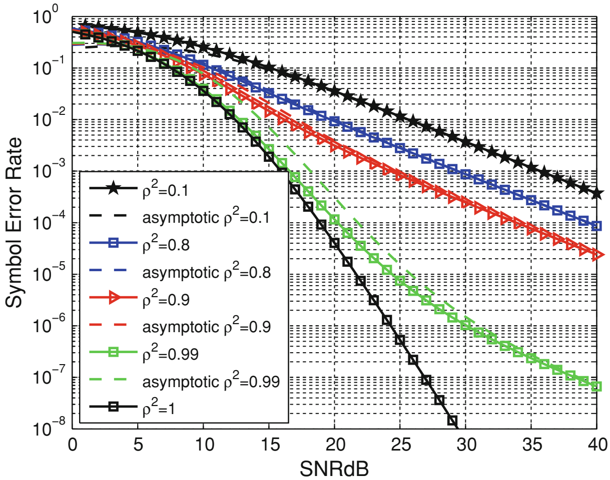

Fig. 15 Symbol error probability performance for 16-QAM modulation under various values of ρ^2 for GSC with ($L = 4$, $M = 3$)

with weight estimation errors. These expressions are collapsed into M-PSK and the square M-QAM average SER results for MRC and conventional SC. For instance, for particular case of $M = L$ (i.e. MRC), we showed the average SER for M-PSK and the square M-QAM SER over Rayleigh fading with estimation error in (81) and (82), respectively. Similarly, for ($M = 1$) (i.e. SC) the average SER for M-PSK and the square M-QAM are expressed in (81) and (82), respectively.

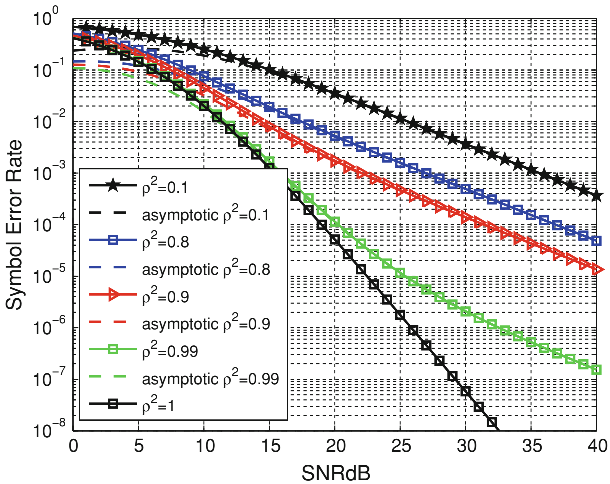

Fig. 16 Symbol error probability performance for 16-QAM modulation under various values of ρ^2 for a special case of GSC with ($L = 4$, $M = 4$) given by (82)

7 Conclusions

The closed-form expressions for the channel capacity per unit bandwidth for three different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for GSC (L, M) with estimation error is derived. Furthermore, we presented an upper bound as well as asymptotically tight approximations for ora policy for the high and low SNR regions. The result showed that C_{opra} outperforms C_{ora} and C_{tifr}, and is less sensitive to the estimation error compared to other policies. However, C_{tifr} performs the worst among the other policies because it suffers a large capacity penalty due to the estimation error whereas it is less complex to implement. It is worth to mention that the derived capacity expressions represent general formulas for GSC (L, M) with estimation error over slow Rayleigh fading channel from which those spacial limited cases of GSC (i.e, $M = L$, MRC; $M = 1$, SC; $\rho^2 = 1$, perfect estimation; $\rho^2 = 0$ no diversity) can be derived. Also, The second part of this paper analyzed the impact of channel estimation error on the capacity statistics and SER performance of GSC over slow Rayleigh fading channels. Starting with PDF of instantaneous branch SNR, we derived exact closed-forms for capacity statistics: moment generating function, probability density function and cumulative distribution function. In addition, the exact analytical expressions of SER for M-PAM/PSK/QAM modulation were obtained. The derived expressions are valid for arbitrary number of receivers.

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References

1. Proakis, J. G. (1995). *Digital communications* (3rd ed.). New York: McGraw.
2. Stüber, G. L. (1996). *Principles of mobile communications*. Norwell, MA: Kluwar.
3. Jakes, W. C. (1994). *Micro wave mobile communication, 2ed Piscataway*. New York: IEEE Press.

4. Gans, M. J. (1971). The effect of Gaussian error in maximal ratio cominaters. *IEEE Transactions on Communication Technology, COM-19*(4), 492–500.
5. Aghamohammadi, A., & Meyr, H. (1990). On the error probablity of linearly modulated singals on Rayleigh frequancy-flat fading channels. *IEEE Transactions on Communications, 38*(11), 1966–1970.
6. Dong, X., & Beaulieu, N. C. (2003). SER of two-dimensional signaling in Rayleigh fading with channel estimation error. In *Proceedings of international conference on communication* (ICC), Anchorage, AK, pp. 2763–2767.
7. Annamalali, A. Jr., (2001). The effect of Gaussian error in selection diveristy combiners. *Wireless Communications and Mobile Computing, 1*, 419–439.
8. Annamalali, A., & Tellambura, C. (2002). Analysis of hybrid selection/maximal-ratio diversity with Gaussian errors. *EEE Transactions on Communications, 1*(3), 498–511.
9. Ma, Y., Zhang, D., & Schober, R. (2005). Effect of channel estimation errors on M-QAM with GSC diversity in fading channels. In *Proceddings of IEEE International Conference on Communications*. Seoul, Korea, pp. 2190–2194.
10. Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal, 27*, 397–423.
11. Fochini, G., & Gans, M. (1998). On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Communications, 6–3*, 311–335.
12. Goldsmith, A. J., & Varaiya, P. (1997). Capacity of Rayleigh fading Channel with channel side information. *IEEE Transactions on Information Theory, 43*(6), 1986–1992.
13. Alouini, M.-S., & Goldsmith, A. J. (1999). Capacity of Rayleigh fading channel under different adaptive transmission and diversity combining techniques. *IEEE Transactions Vehicular Technology, 47*(4), 488–492.
14. Bhaskar, V. (2007). Spectral efficiency evaluation for MRC diversity schemes over generalized Rician fading channels. *International Journal of Wireless Information Networks, 14*(3), 209–223.
15. Mallik, R. K., Win, M. Z., Shao, J. W., Alouini, M. S., & Goldmsith, A. J. (2004). Channel capacity of adaptive transmission with maximal ratio combining in correlated Rayeigh fading. *IEEE Transactions on Wireless Communications, 13*(4), 1124–1133.
16. Dohelr, M., & Aghvami, H. (2005). On the approximation of MIMO capacity. *EEE Transactions on Wireless Communications, 4*(1), 30–34.
17. Gradshteyn, I. S., & Ryzhik, I. M. (2000). *Tables of Intergrals, serious and Products* (6th ed.). San Diago: Acadamic Press.

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