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Spectral efficiency of maximum ratio combining (MRC) over slow fading with estimation errors

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ABSTRACT

In this paper, we derive closed-form expressions for capacity statistics of the maximal ratio combining (MRC) system, taking into account the effect of imperfect channel estimation at the receiver. The channel considered is a slowly varying flat Rayleigh fading that is also spatially independent. The combiner weights are assumed to be affected by Gaussian errors at the receiver. In particular, we derive the moment generating function (MGF), complementary cumulative distribution function (CDF) and the probability density function (PDF) of the capacity. Furthermore, we derive closed-form expressions for the system capacity when employing different adaptive transmission schemes such as (1) optimal power and rate adaptation (*opra*); (2) constant power with optimal rate adaptation (*ora*); and (3) channel inversion with fixed rate (*cifr*). Analytical results show accurately the impact of the channel estimation error on the achievable spectral efficiency.

1. Introduction

It is widely accepted that using diversity at the transmitter or at the receiver of a wireless communication system can improve significantly the performance of wireless links. Diversity combining, which skillfully combines multiple replicas of received signals has long been as one of the most efficient techniques to overcome the destructive effects of multipath fading in wireless communication systems. There are several diversity combining methods employed in communication receivers including maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), and a combination of MRC and SC, called generalized selection combining (GSC). By definition, MRC combiner linearly combines the individually received branch signals so as to maximize the instantaneous output signal-to-noise ratio (SNR) [1–3].

Most system designs assume that perfect channel estimation is available at the receiver. In practice, however, the channel gains have to be estimated at the receiver for diversity combining which can be obtained either from a pilot signal or data signals (by applying a clairvoyant estimator). The work in [4] analyzed the performance of MRC with pilot tone-based weighting on frequency-selective Rayleigh fading channels. The pilot tone was assumed to be separated from the data signal and the resulting channel-estimation error was shown be to be Gaussian. Previous work on the analysis of imperfect channel estimation with no diversity can be found in [5] and [6]. In [7], Gans modeled the channel estimation errors as complex Gaussian and derived the distribution of the SNR statistics which has been used by Tomiuk in [8] to obtain the average probability of error for the MRC diversity schemes.

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The pioneering work of Shannon [9] has established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a channel. In [11], the capacity of a single user flat fading channel with perfect channel information at the transmitter and the receiver is derived for various adaptation policies, namely, (1) optimal rate and power adaptation (*opra*), (2) optimal rate adaptation and constant power (*ora*), and (3) channel inversion with fixed rate (*cifr*). The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [11]. The general theory developed in [11] was applied to derive closed-form expressions for the capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques [12]. Recently, there has been some work dealing with the channel capacity of different fading channels employing different adaptive schemes such as [13,14], and the references therein.

In this paper, we extend the results in [12] to obtain closed-form expressions for the single-user capacity of MRC system, in the presence of Gaussian channel estimation errors. In addition, we investigate the capacity statistics of MRC scheme which are valid for arbitrary number of receive antennas including moment generating function (MGF), cumulative distribution function (CDF) and probability density function (PDF). The contributions of this paper are two-fold. Firstly, we derive the capacity statistics of MRC receiver subject to Rayleigh fading for arbitrary number of diversity branches, in the presence of Gaussian estimation errors. Secondly, we derive closed-form expressions for the channel capacity of MRC in independent and identically distributed (i.i.d.) Rayleigh fading channels with the following adaptive transmission schemes (1) *opra*; (2) *ora* with constant transmit power; and (3) *cifr*. The paper is organized as follows. In Section 2, the system model is discussed. The capacity statistics are derived in Section 3. In Section 4, we derive closed-form expressions for the channel capacity under different adaptation schemes. Results are presented and discussed in Section 5. The main outcomes of the paper are summarized in Section 6.

2. System model

Consider an L-branch diversity receiver in slow fading channels. Assuming perfect timing and inter-symbol interference (ISI) free transmission, the received signal on the lth branch due to the transmission of a symbol s can be expressed as

$$r_l = g_l s + n_l, \quad l = 1, \dots, L, \tag{1}$$

where g_l is a zero-mean complex Gaussian distributed channel gain, n_l is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$, and s is the data symbol taken from a normalized unit-energy signal set with an average power P_s. The actual channel gains of L diversity branches are i.i.d. random variables. The PDF of the received instantaneous SNR was firstly derived by Gans [7] and revisited by Tomiuk et al. [8]. The pdf of the received instantaneous SNR is expressed as

$$p_\gamma(\gamma) = \sum_{k=1}^{L} \binom{L-1}{k-1} \frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left(\frac{1}{\gamma_t}\right)^k \gamma^{k-1} e^{\frac{-\gamma}{\gamma_t}}, \tag{2}$$

where $\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}$ is representing the weighting coefficients in the sum of L branches and it is known as Bernstein polynomials, and γ_t is the average SNR per bit per branch ($\gamma_t = \frac{E_b}{N_0}$). The factor ρ denotes the correlation between the actual channel coefficients g_l and their estimates $\widehat{g}_l$. The actual channel gain can be related to the channel estimate by

$$g_l = \rho \widehat{g}_l + z_l, \tag{3}$$

where z_l is a complex Gaussian random variable independent of $\widehat{g}_l$ with zero mean and variance σ_z^2 defined as $\rho = \frac{\mathrm{cov}(g_l, \widehat{g}_l)}{\sqrt{\mathrm{var}(g_l)\mathrm{var}(\widehat{g}_l)}}$. In a system with no estimation errors, $\rho^2 = 1$, and hence the distribution of γ in (2) reduces to

$$p_\gamma(\gamma) = \left(\frac{1}{\gamma_t}\right)^L \frac{\gamma^{L-1}}{\Gamma(L)} e^{\frac{-\gamma}{\gamma_t}}. \tag{4}$$

3. Capacity statistics

In this section, we focus on deriving the exact analytical expressions for capacity statistics of MRC over Rayleigh fading channels, assuming perfect channel knowledge at the receiver and no channel knowledge at the transmitter with average input-power constraint. The non-ergodic capacity of MRC system is given in [bit/s/Hz] by [9] as

$$C = \log_2(1+\gamma). \tag{5}$$

3.1. Moment generating function (MGF)

The MGF of the capacity of MRC system in the presence of Gaussian channel estimation errors in the branch weights is given by

$$\Phi_C(\tau)=E\left[e^{\tau C}\right]=E\left[(1+\gamma)^{\frac{\tau}{\ln(2)}}\right]. \tag{6}$$

Expressing the expectation in an integral form over the PDF $p_\gamma(\gamma)$ and inserting (2), we obtain

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}\left(\frac{1}{\gamma_t}\right)^k\times\underbrace{\int_0^\infty(1+\gamma)^{\frac{\tau}{\ln(2)}}\gamma^{k-1}\exp\left(\frac{-\gamma}{\gamma_t}\right)d\gamma}_{I_1}. \tag{7}$$

The integral I_1 can be obtained by making the change of variables $x=1+\gamma$ and the integral region to yield

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}\left(\frac{1}{\gamma_t}\right)^k\exp\left(\frac{1}{\gamma_t}\right)\times\underbrace{\int_1^\infty x^{\frac{\tau}{\ln(2)}}(x-1)^{k-1}\exp\left(\frac{-x}{\gamma_t}\right)dx}_{I_2}. \tag{8}$$

In order to obtain a closed-form MGF expression, we need to evaluate the above integral I_2. We first expand the term $(x-1)^{k-1}$ as a finite sum, resulting

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}\left(\frac{1}{\gamma_t}\right)^k\exp\left(\frac{1}{\gamma_t}\right)\sum_{i=0}^{n}\binom{k-1}{i}(-1)^{k-i-1}\int_1^\infty x^{(\frac{\tau}{\ln(2)}+i)}\exp\left(\frac{-x}{\gamma_t}\right)dx. \tag{9}$$

With the help of the equality [16]

$$\int_u^\infty x^{n-1}e^{-\mu x}\,dx=\mu^{-n}\Gamma(n,u\mu), \tag{10}$$

we obtain MGF expression in closed-form as

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}\left(\frac{1}{\gamma_t}\right)^k e^{(\frac{1}{\gamma_t})}\sum_{i=0}^{k-1}\binom{k-1}{i}(-1)^{k-i-1}\gamma_t^{(\frac{\tau}{\ln(2)}+i+1)}\Gamma\left(\left(\frac{\tau}{\ln(2)}+i+1\right),\frac{1}{\gamma_t}\right), \tag{11}$$

where $\Gamma(a,x)=\int_x^\infty t^{a-1}e^{-t}\,dt\ \forall\{a,x\}\geqslant 0$ denotes the upper incomplete Gamma function.

Furthermore, the integral in (7) can be evaluated in another form with help of the integral representation of the confluent hypergeometric function $\Psi(a,b;z)$ [16]

$$\Psi(a,b;z)=\frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}\,dt. \tag{12}$$

The MGF can be expressed in closed form as

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\left(\frac{1}{\gamma_t}\right)^k}{\Gamma(k)}\Psi\left(k,\frac{\tau}{\ln(2)}+1+k;\frac{1}{\gamma_t}\right). \tag{13}$$

Note that using alternative notation for $\Psi(a,b;z)=z_2^{-a}F_0(a,1+a-b;.;-1/z)$ where ${}_2F_0(.,;.:.)$ is a generalized hypergeometric series [16], the MGF of C can simply be written as

$$\Phi_C(\tau)=\sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}{}_2F_0\left(k,-\frac{\tau}{\ln(2)}-1;-\gamma_t\right). \tag{14}$$

3.2. Complementary cumulative distribution function (CCDF)

The CDF of C is defined as follows

$$F_C(C)=\mathrm{Prob}(\bar{C}\leqslant C)=\int_0^{2^C-1}p_\gamma(\gamma)\,d\gamma. \tag{15}$$

Averaging over the distribution of received instantaneous SNR *gamma* in (2) results in

$$F_C(C) = \sum_{k=1}^{L} \binom{L-1}{k-1} \frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left(\frac{1}{\gamma_t}\right)^k \int_0^{2^C-1} \gamma^{k-1} e^{\frac{-\gamma}{\gamma_t}}\, d\gamma. \tag{16}$$

In order to evaluate the integral with respect to γ, we make use of the following equality [16]

$$\int_0^x s^{\nu-1} e^{-\mu s}\, ds = \mu^{-\nu}\gamma(\nu, \mu x), \tag{17}$$

where $\gamma(.,.)$ is the incomplete Gamma function. Since the factor ν in (17) is integer, we can replace the incomplete Gamma function by its finite sum representation [16] given by

$$\gamma(\nu, \mu x) = (\nu-1)!\left[1 - e^{-x\mu}\left(\sum_{m=0}^{\nu-1} \frac{x^m}{m!\mu^m}\right)\right]. \tag{18}$$

After replacing the integral in (16) with its closed-form and simplifying, we obtain the expression for the CDF as

$$F_C(C) = \sum_{k=1}^{L} \binom{L-1}{k-1} \frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left[\frac{(k-1)!}{\gamma_t^k} - e^{-\frac{2^C-1}{\gamma_t}} \sum_{n=0}^{k-1} \frac{(k-1)!}{n!} \frac{(2^C-1)^n}{\gamma_t^{k-n}}\right]. \tag{19}$$

Thus, the complementary CDF can be obtained from (16) as follows

$$\overline{F}_C(C) = 1 - \sum_{k=1}^{L} \binom{L-1}{k-1} \frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left[\frac{(k-1)!}{\gamma_t^k} - e^{-\frac{2^C-1}{\gamma_t}} \sum_{n=0}^{k-1} \frac{(k-1)!}{n!} \frac{(2^C-1)^n}{\gamma_t^{k-n}}\right]. \tag{20}$$

3.3. Probability density function (PDF)

The PDF of C is defined as the derivative of $F_C(C)$ with respect to C. Taking the derivative of $F_C(C)$ in (19) results in

$$P_C(C) = \frac{d}{dC} F_C(C) = 2^C \ln(2) \sum_{k=1}^{L} \binom{L-1}{k-1} \frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left(\frac{1}{\gamma_t}\right)^k \left(2^C-1\right)^{k-1} e^{\frac{-(2^C-1)}{\gamma_t}}. \tag{21}$$

Note that (21) can also be obtained from (2) by performing the transformation of random variable $\gamma \to C$. The Jacobian of such transformation is $J(\gamma) = \frac{d}{d\gamma} = \frac{1}{\ln(2)(1+\gamma)}$, $P_C(C)$ can be easily obtained as $P_C(C) = 2^C \ln(2) p_\gamma(2^C-1)$ which leads to the same result in (21).

4. Adaptive capacity policies

In this section, we derive closed-form expressions for different adaptive schemes employing MRC with imperfect channel estimation over Rayleigh fading channels. In the derivation, we will rely on the main results from [12].

4.1. Power and rate adaptation (opra)

Given an average transmit power constraint, the channel capacity C_{opra} in (bits/s) of a fading channel [11,12] is given by

$$C_{opra} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma)\, d\gamma, \tag{22}$$

where B (in hertz) is the channel bandwidth and γ_0 is the optimum cutoff SNR satisfying [11]

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p_\gamma(\gamma)\, d\gamma = 1. \tag{23}$$

To achieve the capacity in (22), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating power levels and transmission rate for good channel condition (large γ). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers outage whose probability P_{out} is defined the probability of no transmission and is given by

$$P_{\text{out}} = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma)\, d\gamma. \tag{24}$$

However, C_{opra} in (22) can be expressed in terms of the CDF of γ by applying integration by-parts resulting in

$$\frac{C_{opra}}{\ln(2)}B = -\int_{\gamma_0}^{\infty} \frac{1}{\gamma} F_\gamma(\gamma)\, d\gamma. \tag{25}$$

Substituting (2) into (23) yields the equality

$$\sum_{k=1}^{L} \frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)} \left(\frac{1}{\gamma_t}\right)^k \left[\int_{\gamma_0}^{\infty} \frac{\gamma^{k-1}}{\gamma_0} e^{\frac{-\gamma}{\gamma_t}} - \int_{\gamma_0}^{\infty} \gamma^{k-2} e^{\frac{-\gamma}{\gamma_t}}\right] d\gamma = 1. \tag{26}$$

Using (26) along with the fact in (10), it is found that optimal cutoff SNR, γ_0 has to satisfy the following equality

$$\sum_{k=1}^{L} \binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \left[\frac{\gamma_t}{\gamma_0} \Gamma\left(k, \frac{\gamma_0}{\gamma_t}\right) - \Gamma\left(k-1, \frac{\gamma_0}{\gamma_t}\right)\right] = \Gamma(k)\gamma_t^k. \tag{27}$$

To obtain the optimal cutoff SNR γ_0, in (26), we follow the following procedure. Let $x = \frac{\gamma_t}{\gamma_0}$ and define $f_{\mathrm{MRC}}(x)$ as

$$f_{\mathrm{MRC}}(x) = \sum_{k=1}^{L} \binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \left[\frac{\Gamma(k,x)}{x} - \Gamma(k-1,x)\right] - \Gamma(k)\gamma_t^k. \tag{28}$$

Now, differentiating the function $f_{\mathrm{MRC}}(x)$ with respect to x over the interval $]0, +\infty[$ results in

$$f'_{\mathrm{MRC}}(x) = -\frac{\sum_{k=1}^{L} \binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}(\rho^2)\Gamma(k,x)}{x^2}. \tag{29}$$

Hence, $f'_{\mathrm{MRC}}(x) < 0, \forall x > 0$, meaning that f'_{MRC} is a strictly decreasing function of x. From (28) it can be observed that $\lim_{x\to 0} f_{\mathrm{MRC}}(x) = +\infty$ and $\lim_{x\to +\infty} f_{\mathrm{MRC}}(x) = -\Gamma(k)\gamma_t^k$. Note however that $f_{\mathrm{MRC}}(x)$ is a continuous function of x, which leads to a unique positive γ_0 such that $f_{\mathrm{MRC}}(x) = 0$. We thereby conclude that for each $\gamma_t > 0$ there is a unique γ_0 satisfying (28). Numerical results using MATLAB shows that $\gamma_o \in [0,1]$ as γ_t increases, and $\gamma_o \to 1$ as $\gamma_t \to \infty$.

Now, substituting (2) into (22) yields the channel capacity with *opra* scheme as follows

$$\frac{C_{opra}}{B\ln(2)} = \int_{\gamma_0}^{\infty} \sum_{k=1}^{L} \binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \ln\left(\frac{\gamma}{\gamma_0}\right) \frac{\gamma^{k-1}}{\Gamma(k)\gamma_t^k} e^{-\frac{\gamma}{\gamma_t}}\, d\gamma. \tag{30}$$

The summation in (30) is of finite order, and hence, the order of summation and integral can be inverted to yield

$$\frac{C_{opera}}{B\ln(2)} = \sum_{k=1}^{L} \binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \underbrace{\int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) \gamma_t \frac{(\gamma_t\gamma)^{k-1}}{\Gamma(k)} e^{-\frac{\gamma}{\gamma_t}}\, d\gamma}_{I_3}. \tag{31}$$

The integral I_3 can be evaluated using the identity [12]

$$J_s(\mu) = \int_{1}^{\infty} t^{s-1} \ln(t) e^{-\mu t}\, dt = \frac{\Gamma(s)}{\mu^s} \left\{E_1(\mu) + \sum_{k=1}^{s-1} \frac{1}{k} P_k(\mu)\right\}, \tag{32}$$

where E_1 denotes the exponential integral of the first order [16] defined as

$$E_1(x) = \int_{1}^{\infty} \frac{e^{x\gamma}}{\gamma}\, d\gamma, \quad x \geqslant 0, \tag{33}$$

and $P_k(\mu)$ denotes the Poisson distribution [16] given by

$$P_k(x) = \frac{\Gamma(k,x)}{\Gamma(k)} = e^{-x} \sum_{i=0}^{k-1} \frac{x^n}{n!}. \tag{34}$$

Substituting (32) into (31) implies that closed-form expression for capacity C_{opra} per unit bandwidth (in bits/s/Hz) can be expressed

$$\frac{C_{opra}}{B\ln(2)} = \left[E_1\left(\frac{\gamma_0}{\gamma_t}\right) + \sum_{k=1}^{L} \left\{\binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \times \sum_{i=1}^{k-1} \frac{P_i\left(\frac{\gamma_0}{\gamma_t}\right)}{i}\right\}\right]. \tag{35}$$

4.1.1. Asymptotic approximation

We can obtain asymptotic approximation of C_{opra} as $\gamma_t \to \infty$ using the series representation of exponential integral of the first order [16] given by

$$E_1(x) = -E - \ln(x) - \sum_{i=1}^{+\infty} \frac{(-x)^i}{i.i!}, \tag{36}$$

where $E = 0.5772156659$ is the Euler–Mascheroni constant. Then, the *opra* capacity per unit bandwidth (in bits/s/Hz) is approximated asymptotically as

$$\frac{C_{opra}^{\infty}}{B\ln(2)} \simeq \left[\left(-E - \ln\left(\frac{\gamma_0}{\gamma_t}\right) + \left(\frac{\gamma_0}{\gamma_t}\right)\right) + \sum_{k=1}^{L}\left\{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k} \times \sum_{i=1}^{k-1}\frac{P_i(\frac{\gamma_0}{\gamma_t})}{i}\right\}\right]. \tag{37}$$

4.1.2. Upper bound

The capacity expression of C_{opra} can be upper bounded by applying Jensen's inequality to (22) as $C_{opra}^{\mathrm{UP}} = \ln(\mathrm{E}[\gamma])$. Then, we evaluate C_{opra}^{UP} using the pdf of γ given in (2) and the identity defined in (10) for $\mathrm{Re}[\mu] > 0$. Simplifying the resulting expression, we obtain an upper bound on the *opra* capacity which is given by

$$\frac{C_{opra}^{\mathrm{UB}}}{B} = \ln\left(\sum_{k=0}^{L-1}\frac{1}{\gamma_t}\binom{L-1}{k}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\right). \tag{38}$$

4.2. Constant transmit power

By adapting the code rate to channel fading state with the transmission power being constant, the channel capacity of this scheme, referred to as optimal rate adaptation *ora* is given by [9,10]

$$C_{ora} = \frac{B}{\ln 2} = \int_0^{\infty} \ln(1+\gamma)p_{\gamma}(\gamma)\,d\gamma. \tag{39}$$

Inserting (2) into (39) yields

$$C_{ora} = \sum_{k=1}^{L-1}\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\underbrace{\int_0^{\infty}\ln(1+\gamma)\frac{\gamma^{k-1}}{\Gamma(k)\gamma_t^k}e^{\frac{-\gamma}{\gamma_t}}\,d\gamma}_{I_4}. \tag{40}$$

An integral similar to the integral I_4 was evaluated in [12] using Poisson distribution, and the result is given by

$$I_4 = P_k\left(\frac{-1}{\gamma_t}\right)E_1\left(\frac{1}{\gamma_t}\right) + \sum_{i=1}^{k-1}\frac{P_i(\frac{-1}{\gamma_t})P_{k-i}(\frac{-1}{\gamma_t})}{i}. \tag{41}$$

Substituting (41) in (40), results in a closed-form expression for the *ora* capacity with MRC and channel estimation errors per unit bandwidth (in bits/s/Hz), which can be expressed as

$$C_{ora} = \sum_{k=1}^{L}\left\{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k} \times \left[P_k\left(\frac{-1}{\gamma_t}\right)E_1\left(\frac{1}{\gamma_t}\right) + \sum_{i=1}^{k-1}\frac{P_i(\frac{-1}{\gamma_t})P_{k-i}(\frac{-1}{\gamma_t})}{i}\right]\right\}. \tag{42}$$

4.2.1. Asymptotic approximation

Following a similar argument to one used to asymptotically approximate the *opra* capacity, the approximated *ora* capacity C_{ora}^{∞} per unit bandwidth (in bits/s/Hz) is obtained as

$$\frac{C_{ora}^{\infty}}{B} = \sum_{k=1}^{L}\left\{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\left[P_k\left(\frac{-1}{\gamma_t}\right)\left[-E - \ln\left(\frac{1}{\gamma_t}\right) + \frac{1}{\gamma_t}\right] + \sum_{i=1}^{k-1}\frac{P_i(\frac{-1}{\gamma_t})P_{k-i}(\frac{-1}{\gamma_t})}{i}\right]\right\}. \tag{43}$$

4.2.2. Upper bound

The capacity C_{ora} can be upper bounded by applying Jensen's inequality to (22) as follows

$$C_{ora}^{\mathrm{UB}} = \ln\bigl(1 + \mathbb{E}[\gamma]\bigr), \tag{44}$$

and the upper bound can be written as

$$\frac{C_{opra}^{\mathrm{UB}}}{B} = \ln\left(1 + \sum_{k=0}^{L-1}\gamma_t\binom{L-1}{k}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\right). \tag{45}$$

4.2.3. Higher SNR region

The Shannon capacity can be approximated at high SNR region using the fact $\log_2(1+\gamma) = \log_2(\gamma)$ as $\gamma \to \infty$ for x > 0 yields an asymptotically tight bounds for (42) in high SNR per unit bandwidth (in bits/s/hertz) as

$$\frac{C_{\text{est-error}}^{\text{high}}}{B} = \sum_{k=0}^{L-1} \binom{L-1}{k} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \left[\psi(k) - \ln\left(\frac{1}{\gamma_t}\right)\right] \tag{46}$$

where $\psi(x)$ denotes Psi function defined as $\psi(x) = \frac{d}{dx}\ln(\Gamma(x))$. For integer values of x, Psi can be represented as $\psi(x) = -E + \sum_i^{x-1} \frac{1}{i}$.

4.2.4. Lower SNR region

We can approximate Shannon capacity in low SNR region by the square capacity of the argument (*gamma*) in low SNR region as $\log_2(1+\gamma) \approx \sqrt{\gamma}$ [15]. Upon using this approximation along with definition of incomplete gamma function yields the approximated Shannon at low SNR per unit bandwidth (in bits/s/hertz) as

$$\frac{C_{\text{est-error}}^{\text{low1}}}{B} \approx \sum_{k=0}^{L-1} \frac{1}{\gamma_t} \binom{L-1}{k} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \frac{\Gamma(k+\frac{1}{1})}{\gamma(k)} \sqrt{\gamma_t}. \tag{47}$$

4.2.5. Lower SNR region II

The Shannon capacity can be approximated as well in low SNR region by exploiting the fact $\log_2(1+\gamma) \approx \frac{1}{\ln(2)}(\gamma - \frac{1}{2}\gamma^2)$ gives the approximated Shannon capacity in low SNR region per unit bandwidth (in bits/s/hertz)

$$\frac{C_{\text{est-error}}^{\text{low1}}}{B} \approx \sum_{k=0}^{L-1} \frac{1}{\gamma_t} \binom{L-1}{k} (\rho^2)^{k-1} (1-\rho^2)^{L-k} \gamma_t \left[k - \left(\frac{(k+1)\gamma_t}{t}\right)\right]. \tag{48}$$

4.3. Channel inversion with fixed rate

We consider two schemes: truncated channel inversion with fixed rate, referred to as *tifr*, and channel inversion with fixed rate with no truncation, which we shall refer to as *cifr*. Channel inversion is an adaptive transmission technique where the transmitter uses the channel information feedback by the receiver in order to invert the channel fading. Accordingly, the channel appears to the encoder/decoder as a time invariant AWGN channel. As a result, channel inversion suffers a large capacity penalty compared to the previous adaptation techniques (*opra* and *ora*), although it is much less complex to implement. The channel inversion technique requires a fixed code design and fixed rate modulation. In this case, the channel capacity C_{cifr} can be derived from the capacity of an AWGN channel with a received SNR and is given by [11,12]

$$C_{cifr} = B \ln\left(1 + \frac{1}{\int_0^\infty \frac{1}{\gamma} p_\gamma(\gamma)\, d\gamma}\right). \tag{49}$$

The channel capacity with the truncation scheme C_{tifr} is given by [12]

$$C_{tifr} = B \ln\left(1 + \frac{1}{\int_{\gamma_0}^\infty \frac{1}{\gamma} p_\gamma(\gamma) d\gamma}\right)(1 - P_{\text{out}}), \tag{50}$$

where P_{out} is the outage probability. Note that, the cutoff level γ_0 can be chosen to either maximize C_{tifr} or achieve a specific P_{out}.

4.3.1. tifr

The integral in (50) can be evaluated using the PDF of the combiner output SNR given in (2) as

$$\int_{\gamma_0}^\infty \frac{1}{\gamma} p(\gamma)\, d\gamma = \sum_{k=1}^{L} \frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)\gamma_t} \int_{\gamma_0}^\infty \gamma^{k-2} e^{\frac{-\gamma}{\gamma_t}}\, d\gamma = \frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)\gamma_t} \Gamma\left(k-1, \frac{\gamma_0}{\gamma_t}\right). \tag{51}$$

Furthermore, the outage probability, P_{out} is derived from the CDF of the combiner output SNR, and is given by

$$P_{\text{out}} = 1 - \sum_{k=1}^{L} \binom{L-1}{k-1} (\rho^2)^{k-1} (1-\rho^2)^{L-k} e^{-(\frac{\gamma_0}{\gamma_t})} \sum_{i=0}^{k-1} \frac{(\frac{\gamma_0}{\gamma_t})}{i!}. \tag{52}$$

Combining (51) and (52), the following closed-form expression for the *tifr* capacity per unit bandwidth (in bits/s/Hz) is obtained

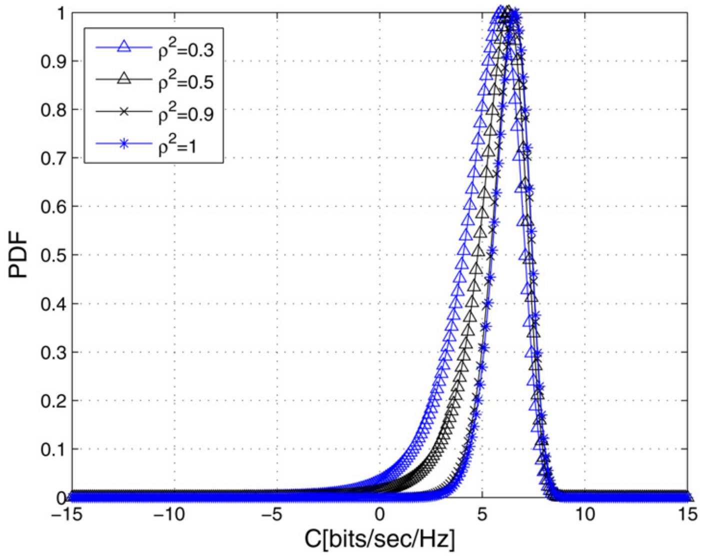

Fig. 1. The PDF of the channel capacity of MRC systems with $L = 3$ at $\gamma_t = 15$ dB and different values of ρ^2.

$$\frac{C_{tifr}}{B} = \left(1 + \frac{\gamma_t}{\sum_{k=1}^{L}\{\binom{L-1}{k-1}\frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}P_{k-1}(\frac{\gamma_0}{\gamma_t})\}}\right)\sum_{k=1}^{L}\left\{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}P_k\left(\frac{\gamma_0}{\gamma_t}\right)\right\}. \tag{53}$$

On the other hand, when $\rho^2 = 1$, the capacity C_{tifr} is given by

$$\frac{C_{tifr}}{B} = \left(1 + \frac{\gamma_t(M-1)}{P_{k-1}(\frac{\gamma_0}{\gamma_t})}\right)P_k\left(\frac{\gamma_0}{\gamma_t}\right) \tag{54}$$

which leads to the same result obtained in [12].

4.3.2. cifr

If we set $\gamma_t = 0$, we get the *cifr* capacity, where in this case the P_{out} is equivalent to zero. From (49) and (2)

$$\int_0^\infty \frac{1}{\gamma}p(\gamma)\,d\gamma = \sum_{k=1}^{L}\frac{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)\gamma_t}P_{k-1}\left(\frac{\gamma}{\gamma_t}\right). \tag{55}$$

By inserting (55) and (2) in (49) yields the *cifr* capacity per unit bandwidth (in bits/s/Hz) as follows

$$\frac{C_{cifr}}{B} = \left(1 + \frac{\gamma_t}{\lim_{\gamma\to 0}\sum_{k=1}^{L}\{\binom{L-1}{k-1}\frac{(\rho^2)^{k-1}(1-\rho^2)^{L-k}}{\Gamma(k)}P_{k-1}(\frac{\gamma}{\gamma_t})\}}\right). \tag{56}$$

Note that $\lim_{\gamma\to 0} P_{k-1}(x) = 1$, and hence, the *cifr* capacity reduces to

$$\frac{C_{cifr}}{B} = \left(1 + \frac{(M-1)\gamma_t}{\sum_{k=1}^{L}\{\binom{L-1}{k-1}(\rho^2)^{k-1}(1-\rho^2)^{L-k}\}}\right). \tag{57}$$

The result in (57) can be expressed for perfect channel estimation $\rho^2 = 1$ as follows

$$\frac{C_{cifr}}{B} = \left(1 + (M-1)\gamma_t\right), \tag{58}$$

which is consistent with the result obtained in [12].

5. Numerical results

In this section we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR (γ_t) in dB for different adaptation policies with MRC over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions, (14), (20), (21), (35), (37), (38), (46), (47), (48), (42), (43), (45), and (53).

Fig. 1 depicts the PDF curves for different values of the correlation coefficient squared ρ^2, considering an average SNR per branch of $\gamma_t = 15$ dB and $L = 3$. This figure shows that the capacity distribution has a Gaussian-like shape even in the

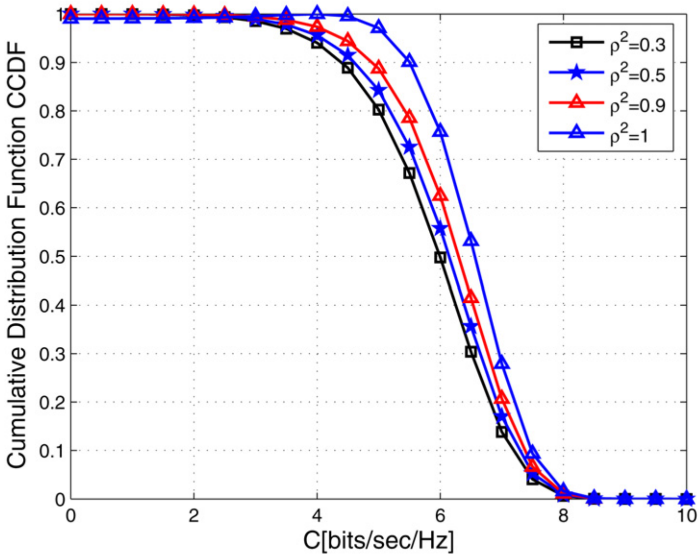

Fig. 2. The CCDF of the channel capacity of MRC systems with $L = 3$ at $\gamma_t = 15$ dB and different values of ρ^2.

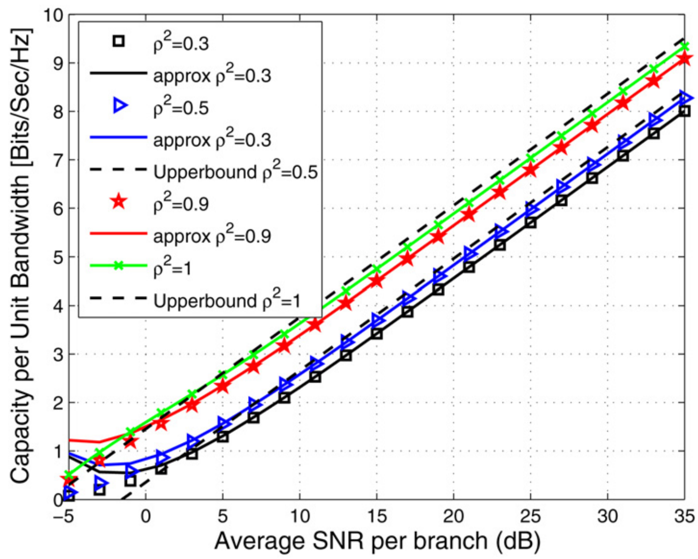

Fig. 3. Capacity per unit bandwidth for MRC systems with $L = 3$ and different values of ρ^2 under the *opra* adaptation policy.

presence of channel estimation errors. As expected, the distribution of C shift towards the left indicating a decreasing value of its mean as the value of ρ^2 decreases. Fig. 2 considers the same setting in Fig. 1 and depicts the CCDF curves for different values of ρ^2, with very similar observations.

Fig. 3 compares C_{opra} for different values of correlation between the channel and its estimate; namely, $\rho^2 = 0.3$, $\rho^2 = 0.5$, $\rho^2 = 0.7$, $\rho^2 = 0.9$ and $\rho^2 = 1$. It can be noticed the highest C_{opra} that can be achieved is when $\rho^2 = 1$. Furthermore, C_{opra} decreases when the value of ρ^2 decreases where in this case the weight error increases. It can be observed from Fig. 3 that there is almost a 7 dB difference in C_{opra} between $\rho^2 = 1$ and $\rho^2 = 0.3$.

In addition, Fig. 3 depicts the asymptotic capacity approximation expressed in (37) and the upper bound expressed in (38). As expected, both approximation and upper bound show a good match to the exact capacity values. Figs. 4 and 5 show the capacity of *opra* and *tifr* schemes for different values of ρ^2, respectively.

In Fig. 4, the exact, asymptotic, and upper bound of the average capacity C_{ora} are plotted against γ_t for different values of ρ^2 {0.3, 0.5, 0.9 and 1} when $L = 3$. As it can be observed from Fig. 4 that the difference in the capacity of *ora* between $\rho^2 = 1$ and $\rho^2 = 0.3$ is increasing along with the increase of the average of received SNR per branch γ_t which makes it more sensitive to the estimation errors than *opra* policy. However, both *opra* and *ora* achieve the same result when there is no power adaptation implemented at the transmitter as in *opra*.

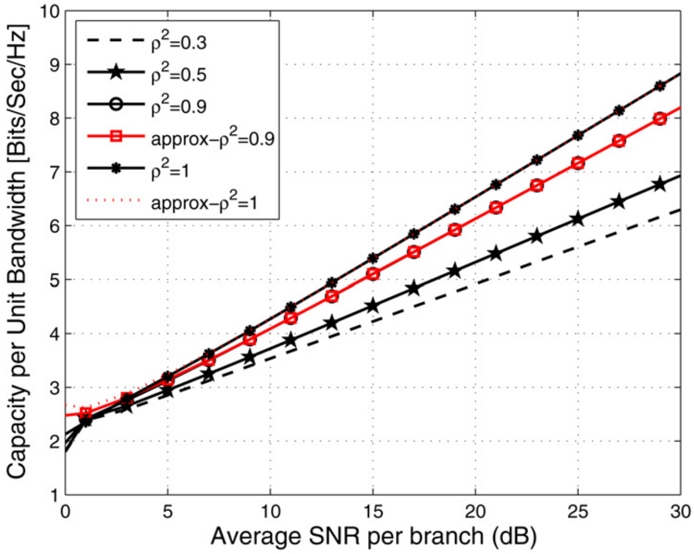

Fig. 4. Capacity per unit bandwidth for MRC systems with $L = 3$ and different values of ρ^2 under the *ora* adaptation policy.

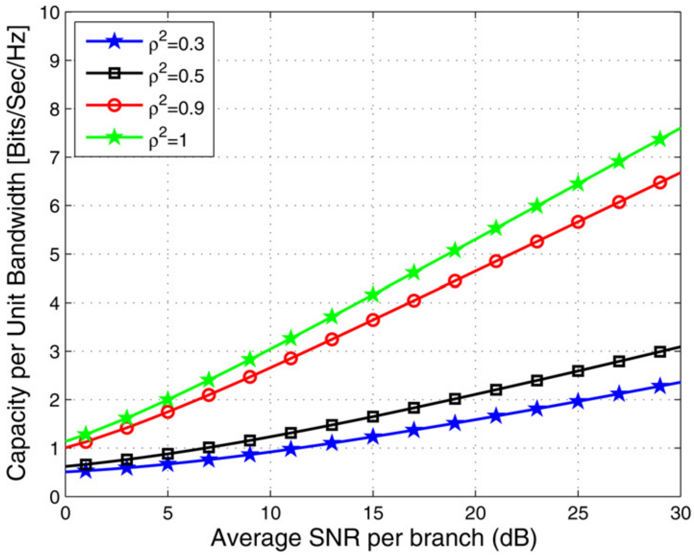

Fig. 5. Capacity per unit bandwidth for MRC systems with $L = 3$ and different values of ρ^2 under the *tifr* adaptation policy.

Fig. 5 depicts the capacity of the *tifr* policy for different values of ρ^2. The *tifr* policy performs the worst compared to the other adaptation policies. However, the loss due to the estimation errors is very high, which is almost 20 dB difference between $\rho^2 = 1$ and $\rho^2 = 0.3$. It can be observed that when there is no data to be transmitted because of the outage event, the *tifr* policy suffers an outage probability which is larger than the outage probability suffered by the opra policy. In addition, we note that the correlation ρ^2 has a greater influence on *tifr* than it has on *opra* and the decrease in outage probability with increase in γ_t is faster for *opra* than *tifr*. Fig. 6 compares the capacity C_{tifr} for different number of diversity branches; namely, $L = 2, 3, 4, 5$ for $\gamma_t = 15$ dB and different values of γ_0. Fig. 7 depicts the behavior of C_{tifr} against the optimal cutoff SNR, γ_0 for different values of ρ^2 and fixed $L = 3$. As can be seen that C_{tifr} becomes almost flat when the value of ρ^2 decreases which experiences a large capacity loss. As L increases, C_{tifr} achieves a small increase and this small increase diminishes as ρ^2 decreases and the average SNR γ_t increases.

6. Conclusion

The channel capacity statistics of MRC including the probability density function (PDF) and cumulative distribution function (CDF) as well as the moment generating function (MGF) is studied. Furthermore, the channel capacity of unit bandwidth

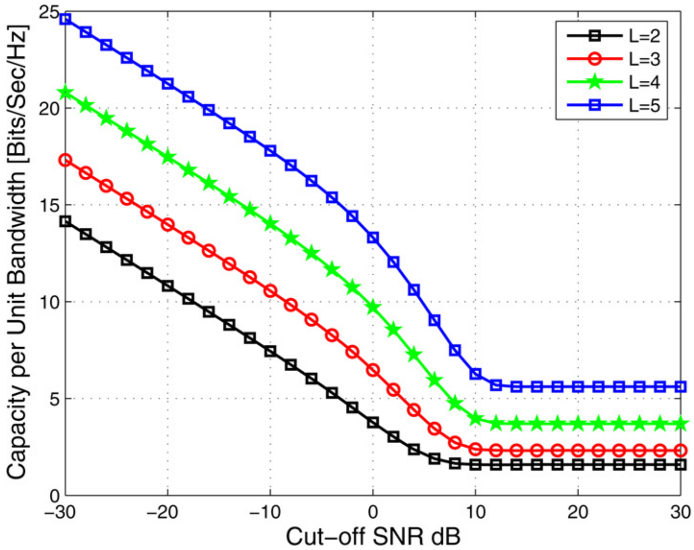

Fig. 6. Capacity per unit bandwidth for MRC systems with different diversity orders and $\rho^2 = 1$ under the *tifr* adaptation policy at $\gamma_t = 15$ dB.

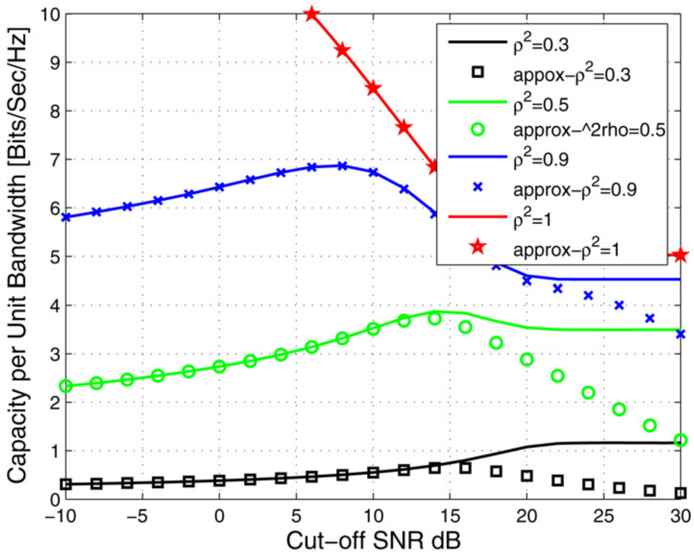

Fig. 7. Capacity per unit bandwidth for MRC systems with $L = 3$ and different values of ρ^2 versus optimal cutoff SNR, γ_0 under the *tifr* adaptation policy at $\gamma_t = 15$ dB.

for different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for MRC with estimation error was discussed. Our numerical results showed that for the same bandwidth, the capacity increases with an increase of the diversity order L and an increase of the average γ_t per branch. Also, the result showed that C_{opra} outperforms C_{ora} and C_{tifr}, and is less sensitive to the estimation error when compared to other policies. However, C_{tifr} performs the worst among the other policies because it suffers a large capacity penalty due to the estimation error whereas it is less complex to implement.

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