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Spectral efficiency evaluation for selection combining diversity (SCD) scheme over slow fading

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Abstract: In this paper we derive closed-form expressions for the single-user capacity of selection combining diversity (SCD) system, taking into account the effect of imperfect channel estimation at the receiver. The channel considered is a slowly varying spatially independent flat Rayleigh fading channel. The complex channel estimate and the actual channel are modelled as jointly Gaussian random variables with a correlation that depends on the estimation quality. Three adaptive transmission schemes are analysed: 1) optimal power and rate adaptation *opra*; 2) constant power with optimal rate adaptation *ora*; and 3) channel inversion with fixed rate *cifr*. Furthermore, we derive in this paper analytical results for capacity statistics including moment generating function (MGF), complementary cumulative distribution function (CDF) and probability density function (PDF). These statistics are valid for arbitrary number of receive antennas. Our numerical results show the effect of Gaussian channel estimation error on the achievable spectral efficiency.

1 Introduction

It is well known that information bearing signals transmitted over wireless channels experience multipath fading that introduces both random phase shift and amplitude variation [1], resulting in a serious degradation in communication and increased bit error rate (BER). Diversity can help effectively in recovering the signal by providing the receiver with a multiple faded replica of information bearing signal [1–3]. In particular, selection combining diversity (SCD) has been the most commonly implemented scheme in wireless communication systems owing to its simplicity.

Most system designs assume that perfect channel estimation is available at the receiver. However, in practical systems, the branch signal-to-noise ratio (SNR) estimates are usually combined with noise which makes it difficult to estimate them perfectly. In practice, a diversity branch SNR estimate can be obtained from either a pilot signal or data signals (by applying a clairvoyant estimator) [4]. For example, if a pilot signal is inserted to estimate the channel, a Gaussian error may arise because of the large frequency separation or time dispersion. Previous work on the analysis of imperfect channel estimation with no diversity can be found in [5, 6]. In [7], a new closed-form expression for the probability density function (PDF) of the SCD combiner output with imperfect channel estimation was derived, based on the derivation of [4]. The author focused on deriving the average error probability, where it was shown that the degradation due to imperfect channel estimation induces error floors at relatively high SNR values.

Shannon's benchmark paper [8] established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a communication channel. The Shannon capacity of fading channels under different assumptions about the knowledge of the channel information at the transmitter and the receiver was presented in [9, 10], respectively. In [11], the capacity of a single-user flat fading channels with perfect channel information at the transmitter and the receiver was derived for various adaptation policies, namely, (i) optimal rate and power adaptation (opra), (ii) optimal rate adaptation and constant power (ora) and (iii) channel inversion with fixed rate (cifr). The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [11]. In [12], the general theory developed in [11] was applied to derive closed-form expressions for the capacity of Rayleigh fading channels under different adaptive transmission and diversity combining techniques. Recently, there has been some work dealing with the channel capacity of different fading channels employing different adaptive schemes such as [13, 14], and the references therein. To the knowledge of the authors, the capacity of SCD receivers with estimation errors has not been derived.

In this paper, we extend the results in [12] to obtain closed-form expressions for the single-user capacity of an SCD system, in the presence of Gaussian channel estimation errors. In addition, we investigate the capacity statistics of the SCD scheme, which are valid for an arbitrary number of receiver antennas including moment generating function (MGF), cumulative distribution function (CDF) and PDF. The contributions of this paper are two-fold. Firstly, we derive closed-form expressions for the channel capacity of SCD in independent and identically distributed (i.i.d.) Rayleigh fading channels with the following adaptive transmission schemes: (i) opra; (ii) ora with constant transmit power and (iii) cifr. Secondly, we derive the capacity statistics of an SCD receiver subject to Rayleigh fading for an arbitrary number of diversity branches, in the presence of Gaussian estimation errors.

The paper is organised as follows. In Section 2, the system model used in this paper is discussed. In Section 3, we derive closed-form expressions for the channel capacity under different adaptation schemes. The capacity statistics are derived in Section 4. Results are presented and discussed in Section 5. The main outcomes of the paper are summarised in Section 6.

2 System model

Consider an L-branch diversity receiver in slow fading channels. Assuming perfect timing and no intersymbol interference (ISI), the received signal on the lth branch owing to the transmission of a symbol s can be expressed as

$$r_l = g_l s + n_l, \quad l = 1, \ldots, L \tag{1}$$

Where g_l is a zero-mean complex Gaussian distributed channel gain, n_l is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$ and s is the data symbol taken from a normalised unit-energy signal set with an average power P_s. An SCD receiver tracks the amplitude of the channel estimate $\widehat{g}_l$ from the L diversity branches, and selects the branch yielding the largest fading amplitude. Thus, if the SCD is employed with equal noise mean power at all branches, the decision criteria reduce to

$$m = \arg \max_{l=1,\ldots,L} \{|\widehat{g}_l|\} \tag{2}$$

where $\widehat{g}_m$ is the magnitude of the selected diversity branch gain at the output of the combiner. The channel estimate $\widehat{g}$ and the channel gain g can be accurately approximated as jointly complex Gaussian [4]. We further assume the actual channel gains of the L diversity branches are i.i.d. as well as the channel estimates. The actual channel gain g is related to the channel estimate $\widehat{g}$ [4] as follows:

$$g_l = \rho \widehat{g}_l + z_l \tag{3}$$

where ρ is a complex number representing the normalised correlation between g and $\widehat{g}$, and z_l is a complex Gaussian random variable independent of $\widehat{g}$ with zero mean and a variance of σ_z^2. The PDF of the SCD receiver with imperfect channel estimation is given by [7]

$$p_\gamma(\gamma) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \frac{k+1}{\gamma_t(k+1-k\rho^2)} \times \exp\left(\frac{-\gamma(k+1)}{\gamma_t(k+1-k\rho^2)}\right) \tag{4}$$

where $\gamma_t = P_s/N_0$ is the average SNR per receive branch. In the following, the PDF in (4) is used to derive the channel capacity with SCD and channel estimation errors.

3 Adaptive capacity policies

In this section, we derive closed-form expressions for different adaptive schemes with SCD over Rayleigh fading channels. In the derivation, we will rely on the main results from [12].

3.1 Power and rate adaptation

Given an average transmit power constraint, the channel capacity C_{opra} in (b/s) of a fading channel [11, 12] is given by

$$C_{\text{opra}} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma)\, d\gamma \tag{5}$$

where B (in Hz) is the channel bandwidth and γ_0 is the optimum cutoff SNR satisfying the following condition:

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p_\gamma(\gamma)\, d\gamma = 1 \tag{6}$$

To achieve the capacity in (5), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating high power levels and transmission rates for good channel conditions (large γ). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers from outage, whose probability P_{out} is defined as the probability of no transmission and is given by

$$P_{\text{out}} = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma)\, d\gamma \tag{7}$$

However, C_{opra} in (5) can be expressed in terms of the CDF

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of γ by applying integration by-parts resulting in

$$\frac{C_{\text{opra}}\ln(2)}{B} = -\int_{\gamma_0}^{\infty}\frac{1}{\gamma}F(\gamma)\,\mathrm{d}\gamma \tag{8}$$

Substituting (4) into (6) yields the equality

$$\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\gamma_t[k+1-k\rho^2]} \times\left\{\int_{\gamma_0}^{\infty}\frac{1}{\gamma_0}\exp\left(-\frac{\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\mathrm{d}\gamma - \int_{\gamma_0}^{\infty}\frac{1}{\gamma}\exp\left(\frac{-\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\right\}\mathrm{d}\gamma = 1 \tag{9}$$

The second term of (9) can be evaluated by making use of exponential integral function of first order [16] defined as

$$E_1(x) = \int_1^{\infty}\frac{\mathrm{e}^{-xt}}{t}\,\mathrm{d}t \tag{10}$$

Upon substitution of (10) into (9), it is found that the optimal cutoff SNR, γ_0, has to satisfy the following equality:

$$\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}(k+1) \times\left[\exp\left(\frac{\gamma_0(1+k)}{\gamma_t[k+1-k\rho^2]}\right)\left(\frac{[k+1-k\rho^2]\gamma_t}{(1+k)\gamma_0}\right) - E_1\left(\frac{(k+1)\gamma_0}{\gamma_t[k+1-k\rho^2]}\right)\right] = \gamma_t[k+1-k\rho^2] \tag{11}$$

To obtain the optimal cutoff SNR, γ_0 in (11), we follow the following procedure. Let $x = (\gamma_0/\gamma_t)$ and define the function $f_{sc}(x)$ as

$$f_{sc}(x) = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\times\left[\exp\left(\frac{-x(1+k)}{\gamma_t[k+1-k\rho^2]}\right) \times\left(\frac{[k+1-k\rho^2]}{(1+k)x}\right) - E_1\left(\frac{(k+1)x}{\gamma_t[k+1-k\rho^2]}\right)\right] - \frac{\gamma_t[k+1-k\rho^2]}{k+1} \tag{12}$$

Making change of variable where $\mu = (k+1)/(\gamma_t[k+1-k\rho^2])$ and applying the first-order derivative to (12) with respect to x, it yields

$$f'_{sc}(x) = -\sum_{k=1}^{L-1}\binom{L}{k+1}\exp\left(\frac{-\mu x}{\mu^2x^2}\right) \tag{13}$$

Hence, $f'_{sc}(x) < 0$, $\forall x > 0$, meaning that $f'_{sc}(x)$ is a strictly decreasing function of x. Also, observing that

$$\lim_{x\to 0^+} f_{sc}(x) = \infty \quad \text{and} \quad \lim_{x\to\infty^+} f_{sc}(x) = -\frac{\gamma_t[k+1-k\rho^2]}{k+1} \tag{14}$$

and noting that, $f_{sc}(x)$ is a continuous function of x, which leads to a unique positive γ_0 such that $f_{sc}(x) = 0$. Therefore it is concluded that for each $\gamma_t > 0$ there is a unique γ_0 satisfying (12). Numerical results using MATLAB show that $\gamma_0 \in [0, 1]$ as γ_t increases, and $\gamma_0 \to 1$ as $\gamma_t \to \infty$.

Now, substituting (4) into (5) yields the channel capacity with the opra scheme as follows

$$\frac{C_{\text{opra}}}{B} = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\gamma_t[k+1-k\rho^2]} \times\underbrace{\int_{\gamma_0}^{\infty}\ln\left(\frac{\gamma}{\gamma_0}\right)\exp\left(\frac{-\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\mathrm{d}\gamma}_{I_1} \tag{15}$$

where the integral I_1 in the above expression can be computed using the fact from [12], which states the following:

$$\int_0^1 \ln x\,\exp(-\mu x) = E_1(\mu)/\mu \tag{16}$$

Inserting (16) into (15) implies that the capacity C_{opra} per unit bandwidth (in b/s/Hz) can be expressed as

$$\frac{C_{\text{opra}}}{B} = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}E_1\left(\frac{(1+k)\gamma_0}{\gamma_t[k+1-k\rho^2]}\right) \times\exp\left(\frac{-\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \tag{17}$$

(1) *Asymptotic approximation:* We can obtain asymptotic approximation C_{opra} using the series representation of exponential integral of first-order function [15] expressed as

$$E_1(x) = -E - \ln(x) - \sum_{i=1}^{+\infty}\frac{(-x)^i}{i.i!} \tag{18}$$

where $E = 0.5772156659$ is the Euler–Mascheroni constant. Then, the asymptotic approximation C^{∞}_{opra} per unit bandwidth (in b/s/Hz) can be shown to be

$$\frac{C_{\text{opra}}}{B} = \sum_{k=0}^{M-1}(-1)^k\binom{M}{k+1}\left(-E - \ln\left(\frac{(1+k)}{\gamma_t[k+1-k\rho^2]}\right) + \left(\frac{\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\right)\exp\left(\frac{-\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \tag{19}$$

(2) *Upper bound:* The capacity expression of C_{opra} can be upper bounded by applying Jensen's inequality to (5) as follows:

$$\frac{C^{\text{UB}}_{\text{opra}}}{B} = \ln(\mathbb{E}[\gamma]) \tag{20}$$

where $\mathbb{E}[.]$ is the expectation operator. The expression in (20) can be evaluated by averaging it over the PDF in (4) and using [15], resulting in

$$\int_0^\infty x^n \mathrm{e}^{-\mu x}\,\mathrm{d}x = n!\mu^{-n-1} \tag{21}$$

for $\mathrm{Re}[\mu] > 0$. The resulting expression can be further simplified to obtain the upper bound for C_{opra} as follows:

$$\frac{C_{\mathrm{opra}}^{\mathrm{UB}}}{B} = \ln\left(\sum_{k=0}^{L-1}\frac{(-1)^k}{\gamma_0}\binom{L}{k+1}\frac{\gamma_t[k+1-k\rho^2]}{(k+1)}\right) \tag{22}$$

3.2 Constant transmit power

By adapting the transmission rate to the channel fading condition with a constant power, the channel capacity C_{ora} [8, 9] is given by

$$C_{\mathrm{ora}} = \frac{B}{\ln 2} = \int_0^\infty \ln(1+\gamma)p_\gamma(\gamma)\,\mathrm{d}\gamma \tag{23}$$

Substituting (4) into (23) results in

$$\frac{C_{\mathrm{ora}}}{B} = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}[1-\frac{k\rho^2}{k+1}]^{-1} \times \underbrace{\int_0^\infty \ln(1+\gamma)\exp(\frac{-\gamma(k+1)}{\gamma_t[k+1-k\rho^2]})\,\mathrm{d}\gamma}_{I_2} \tag{24}$$

The integral I_2 can be computed conveniently by using the change of variable $x = 1+\gamma$ and applying (16), resulting in a closed-form expression for the capacity C_{ora} per unit bandwidth (in b/s/Hz) given by

$$\frac{C_{\mathrm{ora}}}{B} = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\exp\left(\frac{(1+k)}{\gamma_t[k+1-k\rho^2]}\right) \times E_1\left(\frac{(1+k)}{\gamma_t[k+1-k\rho^2]}\right) \tag{25}$$

(1) *Asymptotic approximation:* Following the same procedure in Section 3.1, the asymptotic approximation C_{ora}^∞ per unit bandwidth (in b/s/Hz) can be computed as

$$\frac{C_{\mathrm{ora}}^\infty}{B} = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\exp\left(\frac{(1+k)}{\gamma_t[k+1-k\rho^2]}\right) \times\left(-E-\ln\left(\frac{(1+k)\gamma_0}{\gamma_t[k+1-k\rho^2]}\right)+\left(\frac{\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\right) \tag{26}$$

(2) *Upper bound:* The capacity C_{ora} can be upper bounded by applying Jensen's inequality to (5) as follows:

$$C_{\mathrm{ora}}^{\mathrm{UB}} = \ln(1+\mathbb{E}[\gamma]) \tag{27}$$

and the upper bound can be written as

$$\frac{C_{\mathrm{ora}}^{\mathrm{UB}}}{B} = \ln\left(1+\sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{\gamma_t[k+1-k\rho^2]}{(k+1)}\right) \tag{28}$$

3.3 Channel inversion with fixed rate

We consider two schemes: truncated channel inversion with fixed rate referred to tifr , and channel inversion with fixed rate with no truncation, referred to as cifr. Channel inversion is an adaptive transmission technique whereby the transmitter uses the channel information feedback by the receiver in order to invert the channel fading. Accordingly, the channel appears to the encoder/decoder as a time-invariant AWGN channel. As a result, channel inversion suffers a large capacity penalty compared to the previous adaptation techniques (opra and ora), although it is much less complex to implement. The channel inversion technique requires a fixed code design and fixed rate modulation. In this case, the channel capacity C_{cifr} can be derived from the capacity of an AWGN channel with a received SNR and is given by [11, 12]

$$C_{\mathrm{cifr}} = B\ln\left(1+\frac{1}{\int_0^\infty (1/\gamma)p_\gamma(\gamma)\,\mathrm{d}\gamma}\right) \tag{29}$$

The channel capacity with the truncation scheme [12] C_{tifr} is given by

$$C_{\mathrm{tifr}} = B\ln\left(1+\frac{1}{\int_{\gamma_0}^\infty (1/\gamma)p_\gamma(\gamma)\,\mathrm{d}\gamma}\right)(1-P_{\mathrm{out}}) \tag{30}$$

Note that the cutoff SNR γ_0 can be selected to achieve a certain value of P_{out} or to increase C_{tifr}. From (30) and (4), we can show that

$$\int_{\gamma_0}^\infty \frac{1}{\gamma}p(\gamma)\,\mathrm{d}\gamma = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\gamma_t[k+1-k\rho^2]} \times\int_{\gamma_0}^\infty\frac{1}{\gamma}\exp\left(\frac{\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\mathrm{d}\gamma \tag{31}$$

Inserting (16) into (31) yields

$$\int_{\gamma_0}^\infty \frac{1}{\gamma}p(\gamma)\,\mathrm{d}\gamma = \sum_{k=0}^{L-1}(-1)^k\binom{L}{k+1}\frac{k+1}{\gamma_t[k+1-k\rho^2]} \times E_1\left(\frac{\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \tag{32}$$

Based on the cutoff SNR γ_0, we can obtain P_{out}, which is

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given by [7]

$$P_{\mathrm{out}} = \int_0^{\gamma_0} P_\gamma(\gamma) = 1 - \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \times \frac{k+1}{\gamma_t[k+1-k\rho^2]} \exp\left(\frac{\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (33)$$

The capacity C_{cifr} per unit bandwidth (in b/s/Hz) can be obtained by inserting (31) and (33) into (30) resulting in the tifr capacity per unit bandwidth (in b/s/Hz) as follows (as shown at the bottom of the page)

Using the first series expansion in (18) and substituting it into (34) yields the asymptotic approximation of $C_{\mathrm{tifr}}^{\infty}$ expressed as (as shown at the bottom of the page)

If we set $\gamma_t = 0$, we get the capacity for channel inversion with fixed rate and without truncation, where in this case the P_{out} is equivalent to zero. Inserting (4) into (29), thus, capacity C_{cifr} per unit bandwidth (in b/s/Hz) becomes (as shown at the bottom of the page)

Asymptotic approximation of $C_{\mathrm{cifr}}^{\infty}$ can be expressed as (as shown at the bottom of the page)

4 Capacity statistics

In this section, we focus on deriving the exact analytical expressions for capacity statistics of SCD over Rayleigh fading channels, assuming perfect channel knowledge at the receiver and no channel knowledge at the transmitter with average input-power constraint. The non-ergodic capacity of the SCD system is given in (b/s/Hz) by

$$C = \log_2(1+\gamma) \quad (38)$$

4.1 Moment generating function

The MGF of the SCD capacity system in the presence of Gaussian channel estimation errors is given by [16]

$$\Phi_C(\tau) = E[\mathrm{e}^{\tau C}] = E[(1+\gamma)^{\tau/\ln(2)}] \quad (39)$$

Expressing the expectation in an integral form over the distribution of $p_\gamma(\gamma)$ we obtain

$$\Phi_C(\tau) = \int_0^{\infty} (1+\gamma)^{\tau/\ln(2)} p_\gamma(\gamma)\,\mathrm{d}\gamma \quad (40)$$

$$\frac{C_{\mathrm{tifr}}}{B} = \ln\left(1 + \frac{\gamma_t[k+1-k\rho^2]}{\sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1}[k+1-k\rho^2]E_1\left(\frac{\gamma_0(k+1)}{[k+1]}\right)}\right) \sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1} \frac{k+1}{\gamma_t[k+1-k\rho^2]} \exp\left(\frac{\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (34)$$

$$\frac{C_{\mathrm{tifr}}^{\infty}}{B} = \ln\left(1 + \frac{\gamma_t[k+1-k\rho^2]}{\sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1}[k+1-k\rho^2]\left(-E - \log\left(\frac{\gamma_0(k+1)}{[k+1]}\right) + \left(\frac{\gamma_0(k+1)}{[k+1]}\right)\right)}\right) \times \sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1} \frac{k+1}{\gamma_t[k+1-k\rho^2]} \exp\left(\frac{\gamma_0(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (35)$$

$$\frac{C_{\mathrm{cifr}}}{B} = \ln\left(1 + \frac{\gamma_t[k+1-k\rho^2]}{\lim_{\gamma_0\to 0}\sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1}[k+1-k\rho^2]E_1\left(\frac{\gamma_0(k+1)}{[k+1]}\right)}\right) \quad (36)$$

$$\frac{C_{\mathrm{cifr}}^{\infty}}{B} \simeq \ln\left(1 + \frac{\gamma_t[k+1-k\rho^2]}{\lim_{\gamma_0\to 0}\sum_{k=0}^{L-1}(-1)^k \binom{L}{k+1}[k+1-k\rho^2]\left(-E - \log\left(\frac{\gamma_0(k+1)}{[k+1]}\right) + \left(\frac{\gamma_0(k+1)}{[k+1]}\right)\right)}\right) \quad (37)$$

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Inserting (4) into (40) yields

$$\Phi_C(\tau) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} [1 - \frac{k\rho^2}{k+1}]^{-1} \times \underbrace{\int_0^\infty (1+\gamma)^{\tau/\ln(2)} \exp\left(\frac{-\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \mathrm{d}\gamma}_{I_3} \quad (41)$$

The integral I_3 can be obtained by using the change of variable $x = 1 + \gamma$ resulting in

$$I_3 = \exp\left(\frac{(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \int_0^\infty x^{\tau/\ln(2)} \times \exp\left(\frac{-x(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \mathrm{d}x \quad (42)$$

With the help of identity [15]

$$\int_u^\infty x^{n-1} \mathrm{e}^{-\mu x} \,\mathrm{d}x = \mu^{-n}\Gamma(n, u\mu) \quad (43)$$

Hence, the MGF of the capacity can be computed as

$$\Phi_C(\tau) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left(\frac{(k+1)}{\gamma_t[k+1-k\rho^2]}\right)^{-\tau/\ln(2)} \times \exp\left(\frac{(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \times \Gamma\left(\frac{\tau}{\ln(2)} + 1, \left(\frac{(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\right) \quad (44)$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} \mathrm{e}^{-t} \,\mathrm{d}t \,\forall\{a, x\} \geq 0$ denotes the upper incomplete Gamma function.

Moreover, I_3 in (41) can be shown in another form using the integral representation of the confluent hypergeometric function $\psi(a; b; z)$ [15]

$$\Psi(a, b; z) = \frac{1}{\Gamma(a)} \int_0^\infty \mathrm{e}^{-zt} t^{a-1} (1+t)^{b-a-1} \,\mathrm{d}t \quad (45)$$

The MGF can be expressed as

$$\Phi_C(\tau) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left(\frac{(k+1)}{\gamma_t[k+1-k\rho^2]}\right)^{-\tau/\ln(2)} \times \Psi\left(1, \frac{\tau}{\ln(2)} + 2; -\left(\frac{-\gamma(k+1)}{\gamma_t[k+1-k\rho^2]}\right)\right) \quad (46)$$

Note that using alternative notation for $\Psi(a, b; z) = z^{-a} {}_2F_0(a, 1+a-b; .; -1/z)$, where ${}_2F_0(.,;.:.)$ is a generalised hypergeometric series [15], the MGF of C can simply be written as

$$\Phi_C(\tau) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \left(\frac{(k+1)}{\gamma t[k+1-k\rho^2]}\right)^{[\tau/\ln(2)]^{-1}} \times {}_2F_0\left(1, \frac{\tau}{-\ln(2)}; -\left(\frac{-\gamma_t[k+1-k\rho^2]}{\gamma(k+1)}\right)\right) \quad (47)$$

4.2 Complementary cumulative distribution function (CCDF)

The CDF of C is defined as follows:

$$F_C(C) = \mathrm{Prob}(C \leq C) = \int_0^{2^C - 1} p_\gamma(\gamma) \,\mathrm{d}\gamma \quad (48)$$

Averaging over the distribution of gamma in the presence of Gaussian channel estimation errors results in

$$F_C(C) = 1 - \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \exp\left(\frac{2^C - 1(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (49)$$

Thus, the complementary CDF can be obtained from (48) as follows:

$$\overline{F}_C(C) = 1 - F_C(C) = \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \times \exp\left(\frac{2^C - 1(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (50)$$

4.3 Probability density function

The PDF of C is defined as the derivative of $F_C(C)$ with respect to C. Taking the derivative of $F_C(C)$ in (45) results in

$$p_C(C) = \frac{\mathrm{d}}{\mathrm{d}C} F_{C_{\mathrm{ora}}}(C) = 2^C \ln(2) \sum_{k=0}^{L-1} (-1)^k \binom{L}{k+1} \times \frac{k+1}{\gamma_t[k+1-k\rho^2]} \exp\left(\frac{(2^C-1)(k+1)}{\gamma_t[k+1-k\rho^2]}\right) \quad (51)$$

Note that (51) can also be obtained from (4) by performing a transformation of random variables $\gamma \to C$. The Jacobian of such transformation being $J(\gamma) = \mathrm{d}/\mathrm{d}\gamma = [1/\ln(2)(1+\gamma)]$ $p_C(C)$ can be easily obtained as

$$p_C(C) = 2^C \ln(2) p_\gamma(2^C - 1) \quad (52)$$

which is consistent with the result obtained in (51).

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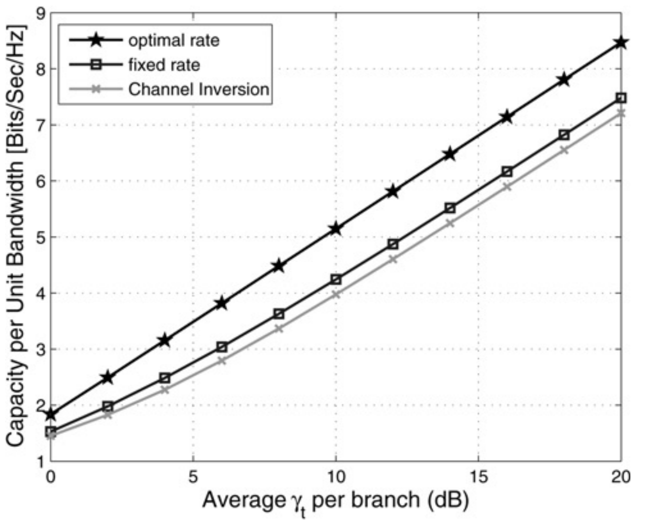

Figure 1 *Capacity per unit bandwidth for a Rayleigh fading with SCD diversity (L = 3) for different adaptation schemes*

5 Numerical results

In this section, we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR (γ_t) in dB for different adaptation policies with SCD over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions (17), (19), (22), (25), (26), (28), (34)–(37), (44), (50) and (51).

Fig. 1 shows the comparison of the capacity per unit bandwidth for opra, ora and tifr policies. The results indicate how the opra policy achieves the highest capacity for any average receiver SNR, γ_t. From the same figure, it can be noticed that ora achieves less capacity than opra.

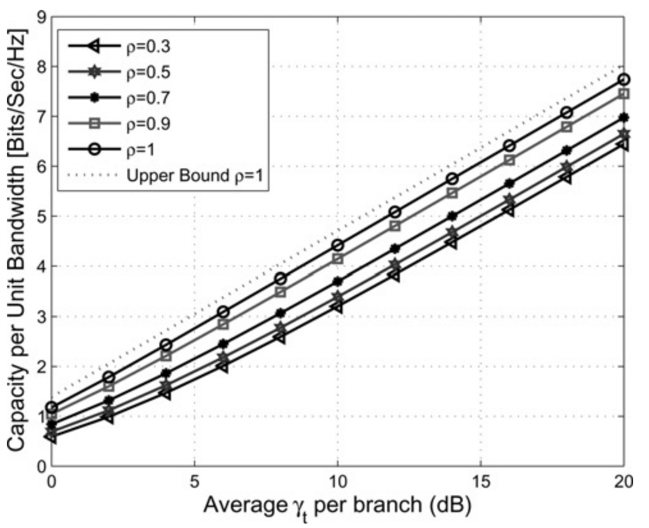

Figure 2 *Capacity per unit bandwidth for a Rayleigh fading with SCD diversity (L = 3) and various values of different ρ under power and rate adaptation*

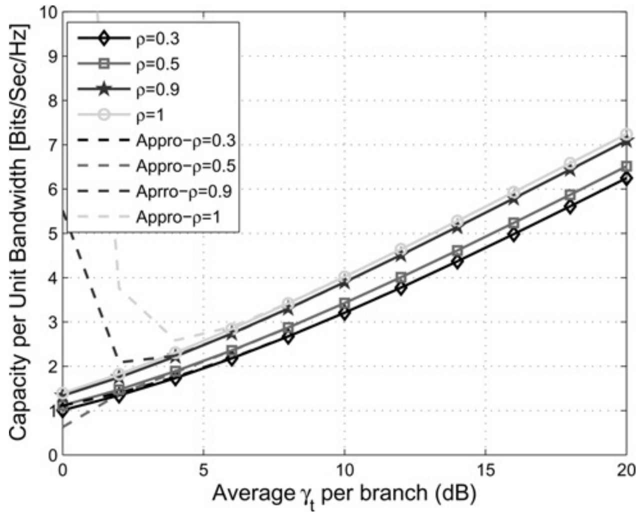

Figure 3 *Capacity per unit bandwidth for a Rayleigh fading channel with SCD (L = 3) and various values of different ρ under rate adaptation and constant power*

However, both opra and ora achieve the same result when there is no power adaptation implemented at the transmitter as in opra. As expected, Fig. 1 shows that the tifr scheme achieves less capacity compared to the other adaptation policies. The results in Fig. 1 are plotted for the case of a fully estimated channel ($\rho = 1$).

Fig. 2 compares C_{opra} for different values of correlation between the channel and its estimate; namely, $\rho = 0.3, 0.5, 0.7, 0.9$ and 1. It can be noticed that the highest C_{opra} that can be achieved when $\rho = 1$. Furthermore, C_{opra} decreases when the value of ρ decreases where in this case the weight error is increases. It can be observed from Fig. 2 that there is almost a 5 dB difference in C_{opra} between $\rho = 1$ and 0.3. Figs. 3 and 4 show the capacity of opra and tifr schemes for different values of ρ, respectively. It can be

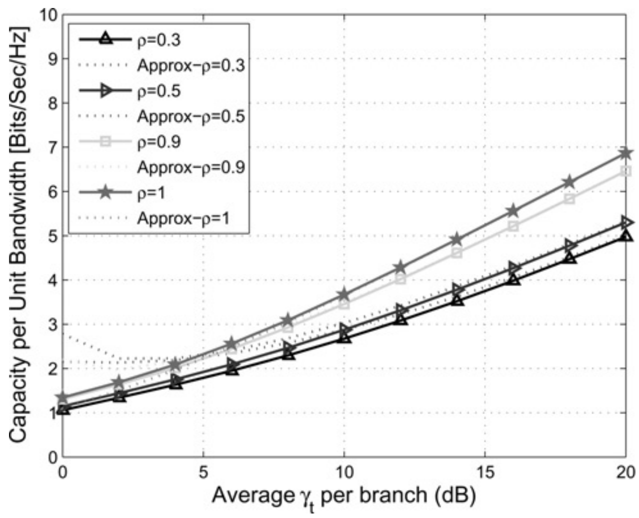

Figure 4 *Capacity per unit bandwidth for a Rayleigh fading with SCD diversity (L = 3) and various values of different ρ for truncated inverse channel and fixed rate*

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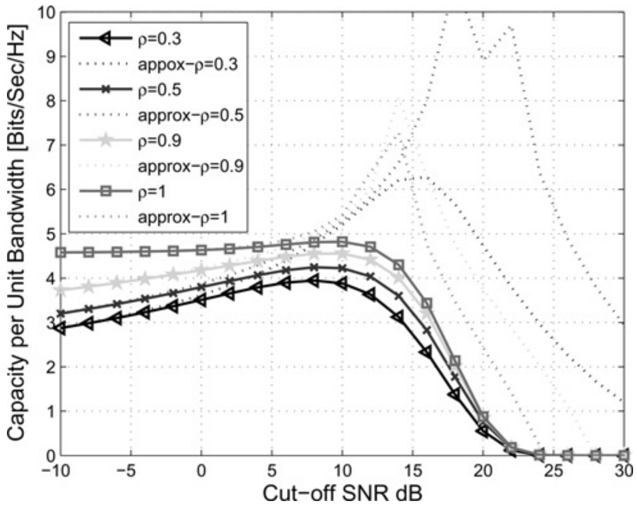

Figure 5 *Capacity per unit bandwidth for a Rayleigh fading with SCD diversity ($L = 3$) and various values of ρ against optimal cutoff SNR γ_0 with truncated channel inversion with $\gamma_t = 15$ dB*

seen that the tifr policy is very sensitive to the channel estimation errors whereas there is almost 7 dB difference in C_{tifr} between $\rho = 1$ and $\rho = 0.3$. Fig. 5 shows the dependence of C_{tifr} on the cutoff SNR, γ_0 for different values of ρ. All the curves show that the the capacity differs for different values of ρ. Also, expression (34) implies that the spectral efficiency is maximised for the optimal cutoff SNR γ_0. Fig. 6 shows the behaviour of the outage probability for different values of ρ. It can be observed that when there are no data to be transmitted because of the outage event, the tifr policy suffers an outage probability that is larger than the outage probability suffered by the opra policy.

Fig. 7 depicts the PDF curves for different values of ρ considering an average SNR of $\gamma_t = 15$ dB and $L = 3$.

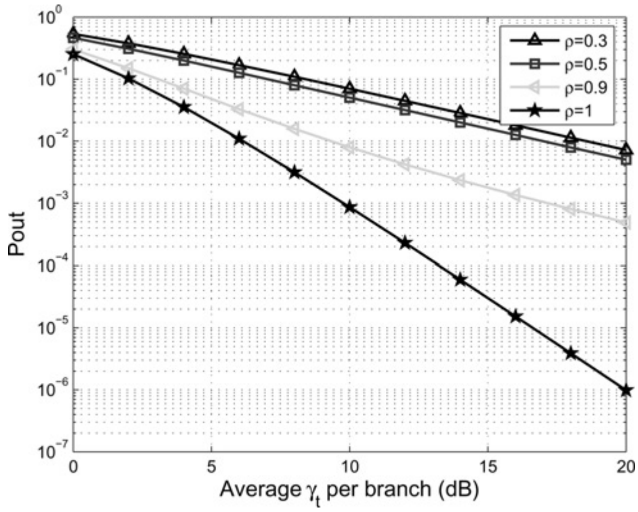

Figure 6 *Probability of outage for a Rayleigh fading with $L = 3$ and various values of ρ with $\gamma_t = 15$ dB*

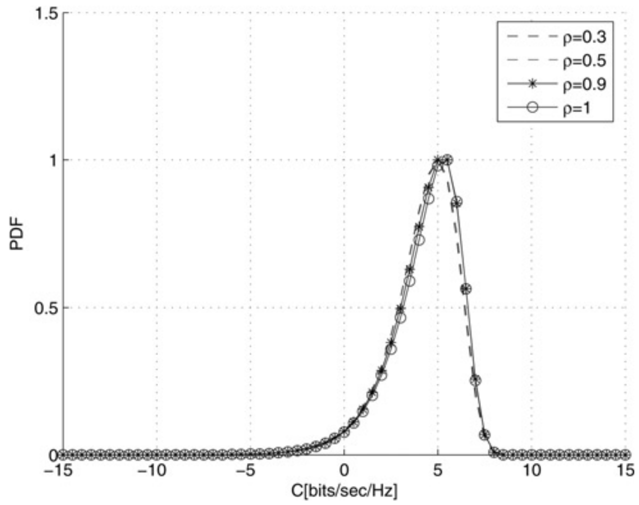

Figure 7 *Probability density function $P_C(C)$ for an SCD with $L = 3$ at $\gamma_t = 15$ dB and different values of ρ*

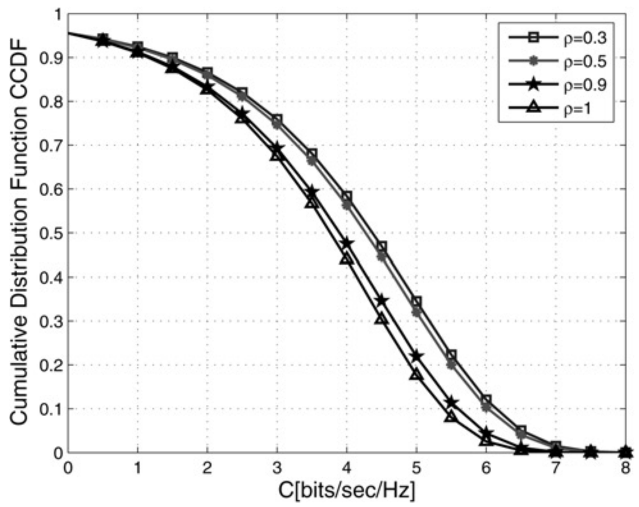

Figure 8 *Cumulative distribution function $F_C(C)$ for an SCD with $L = 3$ at $\gamma_t = 15$ dB and different values of ρ*

This figure shows that the capacity distribution has a Gaussian-like shape even in the presence of channel estimation errors. As expected, we see how the distribution of C shifts towards the left indicating a decreasing value of its mean as the value of ρ decreases Fig. 8 considers the same setting in Fig. 7 and depicts the CCDF curves for different values of ρ, with very similar observations.

6 Conclusions

The channel capacity for unit bandwidth for three different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for SCD with estimation error is discussed. Closed-form expressions for three adaptation policies are derived for an L-selection combiner. Our numerical results showed that for the same

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bandwidth, the capacity increases with increase of the diversity order L and increase of the average γ_t per branch. Also, the result showed that C_{opra} outperforms C_{ora} and C_{tifr}, however, C_{ora} is less sensitive to the estimation error than the other policies. Furthermore, we provided analytical results for the PDF and CDF as well as the MGF.

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