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Research Article

Analysis of Coded FHSS Systems with Multiple Access Interference over Generalized Fading Channels

Salam A. Zummo

Department of Electrical Engineering, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabia

Correspondence should be addressed to Salam A. Zummo, zummo@kfupm.edu.sa

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We study the effect of interference on the performance of coded FHSS systems. This is achieved by modeling the physical channel in these systems as a block fading channel. In the derivation of the bit error probability over Nakagami fading channels, we use the exact statistics of the multiple access interference (MAI) in FHSS systems. Due to the mathematically intractable expression of the Rician distribution, we use the Gaussian approximation to derive the error probability of coded FHSS over Rician fading channel. The effect of pilot-aided channel estimation is studied for Rician fading channels using the Gaussian approximation. From this, the optimal hopping rate in coded FHSS is approximated. Results show that the performance loss due to interference increases as the hopping rate decreases.

1. INTRODUCTION

A serious challenge to having good communication quality in wireless networks is the time-varying multipath fading environments, which causes the received signal-to-noise ratio (SNR) to vary randomly. One solution to fading is the use of spread spectrum (SS) techniques, which randomizes the fading effect over a wide frequency band. The main types of SS are the direct sequence SS (DSSS) and the frequency hopping SS (FHSS). FHSS is the transmission technique in Bluetooth, GSM, and the IEEE802.11 standard.

In FHSS, each user starts transmitting his data over a narrow band during a time slot (called dwell time), and then hops to other bands in the subsequent time slots according to a pseudorandom (PN) code (sequence) assigned to the user [1]. Thus, the transmission in FHSS takes place over the wideband sequentially in time. The main advantage of FHSS is the robust performance under multipath fading, interference, and jamming conditions. In addition, FHSS posses inherent frequency diversity, which improves the system performance significantly over fading channels [1]. Furthermore, data sent over a deeply faded frequency band can be easily corrected by employing error correcting codes with FHSS systems [1]. In particular, convolutional codes are considered to be practical for short-delay applications because the performance is not affected significantly by the frame size.

In cellular networks, multiple access interference (MAI) may arise when more than one user transmit over the same frequency band at the same time in the uplink. This happens when users in closely located cells are assigned PN codes that are not perfectly orthogonal. In this case, a *collision* occurs when two users transmit over the same frequency band simultaneously, which degrades the performance of both users significantly. Also, MAI may be due to the lack of synchronization between users transmitting in the same cell [2–4]. In this case, the borders of time slots used by different users to hop between frequency bands are not aligned, that is, a user hops before or after other users. This case is referred to as asynchronous FHSS. The performance of channel coding with fast FHSS and partial-band interference is well studied in the literature as in [5–8]. However, not much work was done to investigate the performance of coding with slow FHSS and partial-band interference.

In this paper, we derive a new union bound on the bit error probability of coded FHSS systems with MAI. We consider FHSS systems with perfect channel estimation and pilot-aided channel estimation over Rician and Nakagami fading channels. The derivation is based on modeling the FHSS effective channel as a block interference channel

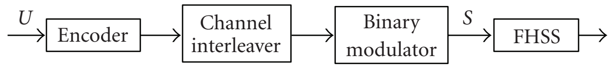

Figure 1: Block diagram of a coded FHSS transmitter.

[9]. Then, the pairwise error probability (PEP) is derived by conditioning over the number of interfering users in the network and then by averaging over this number. In modelling the MAI, we consider the exact statistics in the case of perfect channel state information (CSI) and Nakagami fading, as well as the Gaussian approximation in the case of imperfect CSI and Rician fading. We investigate that the tradeoff between channel estimation and diversity in FHSS systems is studied in order to approximate the optimal hopping rate in FHSS systems with MAI, defined as the hope rate at which the performance of the FHSS system is the best compared to its performance using different hopping rates.

The outline of this paper is as follows. The coded FHSS system model is described in Section 2. In Section 3, a union bound on the bit error probability for coded FHSS systems is derived for different fading statistics and channel estimation assumptions. Results are discussed in Section 4 and conclusions are presented in Section 5.

2. SYSTEM MODEL

The general block diagram of a coded FHSS transmitter is shown in Figure 1. The transmitter consists of a binary encoder (e.g., convolutional or turbo), an interleaver, a modulator, and an FHSS block. Time is divided into frames of duration NT, where T is the *transmission interval* of a bit. Each frame is encoded using a rate R_c encoder, and each coded bit is modulated using BPSK. Then, each frame is transmitted using FHSS, where the transmitter hops J times during the transmission of a frame. Thus, the frame undergoes J independent fading realizations, where blocks of $m = \lceil N/J \rceil$ bits undergo the same fading. In the FHSS context, the transmission duration of m bits represents the dwell time of the system. Effectively, each packet undergoes a block fading channel [9]. Note that the frame is bit-interleaved prior to the FHSS transmission in order to spread burst errors in the decoder.

We consider a multiple-access FHSS network of K users. The frequency band is divided into Q bands and users transmit their data by hopping randomly from one band to another. When more than one user transmit over the same band simultaneously, a hit (or collision) occurs. Throughout this paper, we assume synchronous transmission with a hit probability given by $p_h = 1/Q$. Given that only k users (among the total of K users operating in the network) interfere with the user of interest, the matched filter sampled output at time l in the jth hop is given by

$$y_{j,l} = \sqrt{E_s} h_j s_{j,l} + z_{j,l} + \sum_{f=1}^{k} \sqrt{E_I} h_{f,j} s_{f,j,l}, \tag{1}$$

where E_s is the average received energy, $s_{j,l} = (-1)^{c_{j,l}}$: $c_{j,l}$ is the corresponding coded bit out of the channel encoder, and $z_{j,l}$ is a noise sample modeled as independent zero-mean Gaussian random variable with a variance of $N_0/2$. The coefficient h_j is the channel gain in hop $j$which can be written as $h_j = a_j \exp(j\theta_j)$, where θ_j is uniformly distributed in $[0, 2\pi)$ and a_j is the channel amplitude.

If a line-of-site (LOS) exists between the transmitter and the receiver, the channel amplitude is modeled as a Rician random variable [10]. In this model, the received signal consists of a specular component due to the LOS and a diffuse component due to multipath. Hence, the channel gain in each hop is modeled as $\mathcal{CN}(b, 1)$, where b represents the specular component. Thus, the SNR pdf of a Rician fading is given by

$$f_\gamma(x) = \frac{(1+\kappa)}{\Omega} \exp\left[-\kappa - \frac{(1+\kappa)x}{\Omega}\right] \times I_0\left(2\sqrt{\frac{\kappa(1+\kappa)x}{\Omega}}\right), \quad x \geq 0, \tag{2}$$

where $\kappa = b^2$ is the energy of the specular component and $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. In this context, κ denotes the specular-to-diffuse component ratio. Another fading distribution is the Nakagami distribution, which was shown to fit a large variety of channel measurements. In Nakagami fading channels, the pdf of the received SNR [11] is given by

$$f_\gamma(x) = \left(\frac{\mu}{\Omega}\right)^{\mu} \frac{x^{\mu-1}}{\Gamma(\mu)} \exp\left(-\frac{\mu x}{\Omega}\right), \quad x > 0, \ \mu > 0.5, \tag{3}$$

where $\Gamma(\cdot)$ is the Gamma function and $\mu = \Omega^2/\mathrm{Var}[\sqrt{\gamma}]$ is the Nakagami parameter that indicates the fading severity.

The term E_I in (1) is the average received energy for each of interfering user and $s_{f,j,l}$ is the signal of the fth interfering user in the jth hop. The term $h_{f,j}$ denotes the channel gain affecting the fth interfering user in hop j and modeled as $\mathcal{CN}(0, 1)$. We define the signal-to-interference ratio (SIR) as the ratio $\Delta = E_s/E_I$. The SIR indicates the relative received energy of each of the interfering signals to the received energy of the desired signal. The average signal-to-interference-and-noise ratio (SINR) given k interfering users is defined as

$$\Lambda(k) = \frac{E_s}{N_0/2 + kE_I} = \frac{R_c \gamma_b}{1/2 + k(\gamma_b/\Delta)}, \tag{4}$$

where $\gamma_b = E_s/R_c N_0$ is the SNR per information bit.

The receiver employs maximum likelihood (ML) sequence decoding which is optimal for minimizing the frame error probability. If perfect CSI is available at the receiver, the decoder chooses the codeword $\mathbf{S} = \{s_{j,l},\ j = 1, \ldots, J,\ l = 1, \ldots, m\}$ that maximizes the metric:

$$\mathbf{m}(\mathbf{Y}, \mathbf{S} \mid \mathbf{H}) = \sum_{j=1}^{J} \sum_{l=1}^{m} \mathrm{Re}\{y_{j,l}^{*} h_j s_{j,l}\}, \tag{5}$$

where $\mathbf{Y} = \{y_{j,l},\ j = 1, \ldots, J,\ l = 1, \ldots, m\}$. The metric used in the case of imperfect CSI is presented in Section 3.2 in details.

3. BIT ERROR PROBABILITY

For linear convolutional codes with r input bits, the bit error probability is upper bounded [12] as

$$P_b \le \frac{1}{r}\sum_{d=d_{\min}}^{N} w_d P_e(d), \tag{6}$$

where r is the number of input bits to the encoder in each time interval, $d_{\min}$ is the minimum distance of the code, and $P_e(d)$ is the PEP defined as the probability of decoding a received sequence as a weight-d codeword given that the all-zero codeword is transmitted. In (6), w_d is the number of codewords with output weight d obtained from the weight enumerator of the code [12].

In FHSS systems, the PEP in (6) is a function of the distribution of the d nonzero bits over the J hops. This distribution is quantified assuming uniform channel interleaving of the coded bits over the hops [13]. Denote the number of hops with weight v by j_v and define $w = \min(m, d)$, then the hops are distributed according to the pattern $\mathbf{j} = \{j_v\}_{v=0}^{w}$ if

$$J = \sum_{v=0}^{w} j_v, \qquad d = \sum_{v=1}^{w} v j_v. \tag{7}$$

Denote by $L = J - j_0$ the number of hops with nonzero weights. Then, $P_e(d)$ is determined by averaging over all possible hop patterns as

$$P_e(d) = \sum_{L=\lceil d/m\rceil}^{\min(d,J)} \sum_{j_1=0}^{L_1} \sum_{j_2=0}^{L_2} \cdots \sum_{j_w=0}^{L_w} P_e(d \mid \mathbf{j}) p(\mathbf{j} \mid d), \tag{8}$$

where $P_e(d \mid \mathbf{j})$ is the PEP given the hop pattern $\mathbf{j}$ occurred, $p(\mathbf{j} \mid d)$ is the probability of the hop pattern $\mathbf{j}$ to occur when the number of errors is d, and

$$L_v = \min\left\{L - \sum_{r=1}^{v-1} j_r, \left\lfloor \frac{d - \sum_{r=1}^{v-1} r j_r}{v} \right\rfloor\right\}, \quad 1 \le v \le w. \tag{9}$$

The probability of a hop pattern $\mathbf{j}$ for a weight-d codeword is computed using combinatorics as

$$p(\mathbf{j} \mid d) = \frac{\binom{m}{1}^{j_1}\binom{m}{2}^{j_2}\cdots\binom{m}{w}^{j_w}}{\binom{mJ}{d}} \cdot \frac{J!}{j_0! j_1! \ldots j_w!}. \tag{10}$$

Substituting (8)–(10) in (6) results in the union bound on the bit error probability of convolutional coded FHSS systems.

It should be noted that carefully designed interleavers may outperform the uniform interleaver. However, analyzing coded systems with specific interleavers is much more complicated. Note that the number of summations involved in computing $P_e(d)$ in (8) increases as the hop length increases. A good approximation to the union bound is obtained by truncating (6) to a small value of $d_{\max} < N$. However, it is well known that the low-weight terms in the union bound dominate the performance at high SNR values, where the bound is more useful. Therefore, our bound approximation becomes more accurate at high SNR, where the bound is more useful.

The PEP conditioned on the channel fading gains and the hop pattern $\mathbf{j}$ is given by

$$P_e(d \mid \mathbf{H}, \mathbf{j}) = \Pr\big(\mathbf{m}(\mathbf{Y}, \mathbf{S} \mid \mathbf{H}) - \mathbf{m}(\mathbf{Y}, \hat{\mathbf{S}} \mid \mathbf{H}) < 0 \mid d, \mathbf{S}, \mathbf{H}, \mathbf{j}\big), \tag{11}$$

where $\mathbf{H} = \{h_j\}_{j=1}^{J}$. The PEP is found by substituting the decoding metric for a given receiver in (11) and then averaging over the fading gains as discussed below.

3.1. Perfect CSI

Conditioning on the number of interfering users and substituting the metric (5) in (11), the PEP for BPSK with perfect CSI is given by

$$P_e(d \mid \mathbf{H}, \mathbf{j}, k) = \Pr\left(\sum_{j=1}^{J}\sum_{l=1}^{m} \operatorname{Re}\{y_{j,l}^{*} h_j\} < 0 \mid d, \mathbf{S}, \mathbf{H}, \mathbf{j}, k\right). \tag{12}$$

3.1.1. Exact analysis

Given that k users are interfering with the user of interest, and conditioned on the fading amplitudes affecting the jth hop, the short-term SINR in the jth hop is written as in [14–18]:

$$\beta_j = \frac{a_j^2 \gamma_b}{1/2 + (\gamma_b/\Delta)\sum_{f=1}^{k} a_f^2}, \tag{13}$$

where a_f is the fading gain of the signal arriving from the fth interfering user. In (13), we assumed that the desired and interfering signals have different average-received energies related by $\Delta = E_s/E_I$. In order to find (12), the statistics of the SINR defined in (13) have to be found.

The PEP for coherent BPSK conditioned on the fading amplitudes and number of interfering users is given by

$$\begin{aligned} P_e(d \mid \mathbf{H}, \mathbf{j}, k) &= Q\left(\sqrt{\frac{R_c \gamma_b \sum_{v=1}^{w} v \sum_{i=1}^{j_v} a_i^2}{1/2 + (\gamma_b/\Delta)\sum_{f=1}^{k} a_f^2}}\right) \\ &= Q\left(\sqrt{\sum_{v=1}^{w} v \sum_{i=1}^{j_v} \beta_i}\right). \end{aligned} \tag{14}$$

Using the integral expression [19] of the Q-function, $Q(x) = (1/\pi)\int_0^{\pi/2} e^{(-x^2/2\sin^2\theta)} d\theta$, an exact expression of the PEP is found as

$$\begin{aligned} P_e(d \mid \mathbf{j}, k) &= \frac{1}{\pi}\int_0^{\pi/2} \mathrm{E}_{\{\beta\}}\left[\exp\left(-\alpha_\theta \sum_{v=1}^{w} v \sum_{i=1}^{j_v} \beta_i\right)\right] d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} \prod_{v=1}^{w} \left[\Phi_\beta(v\alpha_\theta)\right]^{j_v} d\theta, \end{aligned} \tag{15}$$

where $\alpha_\theta = 1/(2\sin^2\theta)$ and

$$\Phi_\beta(s) = \mathrm{E}_\beta[e^{-s\beta}] \tag{16}$$

is the moment generating function (MGF) of the random variable β. Note that the product in (15) results from the independence of the fading variables affecting different hops in a frame.

In order to find the MGF of β, we need to derive its pdf which is a function of the number of interfering users. The conditional pdf of the SINR, given that the number of interfering users is k for integer values of the Nakagami parameter μ [14], is found to be

$$f_{\beta|k}(x) = \frac{\mu^{\mu(1+k)}x^{\mu-1}e^{-\mu x}}{\Gamma(\mu)\Gamma(k\mu)}\sum_{h=0}^{\mu}\binom{\mu}{h}\frac{\Gamma(k\mu+h)}{(\mu x+\mu)^{k\mu+h}}, \quad x>0. \tag{17}$$

Since users collide with probability p_h and the total number of users is K, the number of interfering users is a binomial random variable with parameters p_h and K. Hence, the pdf of the SINR is found by averaging (17) over the statistics of the number of interfering users as follows:

$$f_\beta(x) = \sum_{k=0}^{K}\binom{K}{k}p_h^k(1-p_h)^{K-k}f_{\beta|k}(x). \tag{18}$$

Therefore, the MGF of the SINR, β is given by

$$\Phi_\beta(s) = \sum_{k=0}^{K}\binom{K}{k}p_h^k(1-p_h)^{K-k}\Phi_{\beta|k}(s), \tag{19}$$

where $\Phi_{\beta|k}(s)$ is the conditional MGF of the SINR, β. For integer Nakagami parameters [14], it is given by

$$\begin{aligned}\Phi_{\beta|k}(s) &= \frac{\mu^\mu}{\Gamma(k\mu)}\sum_{h=0}^{\mu}\binom{\mu}{h}\frac{\Gamma(k\mu+h)}{\mu^h}\\ &\quad\times U\left(\mu;\mu(1-k)-h+1;1+\frac{\mu}{s}\right),\end{aligned} \tag{20}$$

where $U(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function of the second kind defined in [20]. The MGF required to evaluate (15) is found by substituting (20) in (19) and expressing $U(\cdot;\cdot;\cdot)$ as

$$\begin{aligned}U(a;b;x) &= \frac{\pi}{\sin(\pi b)}\left[\frac{{}_1F_1(a,b;x)}{\Gamma(a-b+1)\Gamma(b)} - \frac{x^{1-b}}{\Gamma(a)\Gamma(2-b)}\right.\\ &\qquad\left.\times\ {}_1F_1(a-b+1,2-b;x)\right],\end{aligned} \tag{21}$$

where ${}_1F_1(\cdot,\cdot;\cdot)$ is the confluent hypergeometric function that is available in any numerical package such as Mathcad. Once the MGF is evaluated, the PEP is evaluated by substituting (19) in (15). Since the integral in (15) is definite, its computation is straightforward using standard numerical integration packages.

3.1.2. Gaussian approximation

The performance analysis of coded FHSS using the exact statistics of the SINR defined in (13) is not always a straightforward task, especially the cases of for Rician fading and imperfect CSI. To overcome this problem, the interference term is approximated by a Gaussian random variable [21]. According to [21], if the number of interfering users exceeds 5, the interference term in (1) can be safely approximated to be a Gaussian random variable with zero-mean and a variance of kE_I. In this paper, we will use this approximation for the cases of Rician fading and imperfect CSI.

Using the Gaussian approximation to simplify (12), the distribution of $\mathrm{Re}\{y_{j,l}\}$ conditioned on a_j is Gaussian with a mean $\sqrt{E_s}a_j s_{j,l}$ and a variance $N_0/2+kE_I$. In order to simplify, the PEP becomes

$$\begin{aligned}P_e(d\mid\mathbf{H},\mathbf{j},k) &= \mathrm{Q}\left(\sqrt{\frac{R_c\gamma_b\sum_{v=1}^{w}v\sum_{i=1}^{j_v}a_i^2}{1/2+k\gamma_I}}\right)\\ &= \mathrm{Q}\left(\sqrt{\Lambda(k)\sum_{v=1}^{w}v\sum_{i=1}^{j_v}a_i^2}\right),\end{aligned} \tag{22}$$

where $\Lambda(k)$ is the SINR defined in (4). For FHSS systems, the PEP is found by averaging (22) over the number of interfering users k as

$$\begin{aligned}P_e(d\mid\mathbf{H},\mathbf{j}) &= \sum_{k=0}^{K-1}\binom{K-1}{k}p_h^k(1-p_h)^{K-1-k}\\ &\quad\times\mathrm{Q}\left(\sqrt{\Lambda(k)\sum_{v=1}^{w}v\sum_{i=1}^{j_v}a_i^2}\right).\end{aligned} \tag{23}$$

One issue to be noted in (23) is that the Gaussian approximation of interference may not result in a good approximation for the terms with small number of interfering users k. Thus, we expect that our performance analysis will result in an optimistic result compared to the real case. However, for the sake of a preliminary system dimensioning, such an approximation will be enough.

In order to find the PEP, (23) is averaged over the statistics of the Rician fading amplitudes in (2) resulting in

$$\begin{aligned}P_e(d\mid\mathbf{j}) &= \frac{1}{\pi}\int_0^{\pi/2}\sum_{k=0}^{K-1}\binom{K-1}{k}p_h^k(1-p_h)^{K-1-k}\\ &\quad\times\exp\left(-\frac{\kappa v j_v\alpha_\theta\Lambda(k)}{1+\kappa+v\alpha_\theta\Lambda(k)}\right)\\ &\quad\times\prod_{v=1}^{w}\left[\frac{1+\kappa}{1+\kappa+v\alpha_\theta\Lambda(k)}\right]^{j_v}d\theta.\end{aligned} \tag{24}$$

3.2. Imperfect CSI

In FHSS systems, channel estimation is often achieved by transmitting a pilot signal with energy E_p in each hop. The corresponding received signal conditioned on k interfering users is given by

$$y_{j,p} = \sqrt{E_p}h_j + z_{j,p} + \sum_{f=1}^{k}\sqrt{E_I}h_f. \tag{25}$$

The ML estimator for h_j is given by $\widehat{h}_j = y_{j,p}/\sqrt{E_p} = h_j + e_j$, where e_j is the estimation error given by

$$e_j = \frac{z_{j,p} + \sum_{f=1}^{k}\sqrt{E_I}h_f}{\sqrt{E_p}}. \tag{26}$$

If the number of interfering users is large enough, the interference term can be approximated by a Gaussian distribution with zero-mean and variance of kE_I. Therefore, the distribution of e_j is $\mathcal{CN}(0, \sigma_e^2)$, where $\sigma_e^2 = (N_0+kE_I)/E_p$.

In an ML sequence decoding rule, it is desired to find the codeword that maximizes the likelihood function $p(\mathbf{Y}, \widehat{\mathbf{H}} \mid \mathbf{S})$. In [13], this ML rule was shown to be difficult-to-implement in a Viterbi receiver. Therefore, the following suboptimal decoding metric that maximizes the likelihood function $p(\mathbf{Y} \mid \widehat{\mathbf{H}}, \mathbf{S})$ is employed:

$$\mathbf{m}(\mathbf{Y}, \mathbf{S} \mid \widehat{\mathbf{H}}) = \sum_{j=1}^{J}\sum_{l=1}^{m}\text{Re}\{y_{j,l}^{*}\widehat{h}_j s_{j,l}\}. \tag{27}$$

Substituting the decoding metric (27) in (11), the PEP for the suboptimal decoder becomes

$$P_e(d \mid \widehat{\mathbf{H}}, \mathbf{j}, k) = \Pr\left(\sum_{j=1}^{J}\sum_{l=1}^{m}\text{Re}\{y_{j,l}^{*}\widehat{h}_j\} < 0 \mid d, \mathbf{S}, \widehat{\mathbf{H}}, \mathbf{j}, k\right). \tag{28}$$

Using the Gaussian approximation to simplify (28), we observe that the distribution of $y_{j,l}$ conditioned on $\widehat{h}_j$ is a complex Gaussian random variable with a mean $\sqrt{E_s}s_{j,l}\text{E}[h_j \mid \widehat{h}_j]$ and a variance $N_0 + kE_I + (1-\rho^2)E_s$, where $\text{E}[h_j \mid \widehat{h}_j] = (\rho/\sigma)(\widehat{h}_j - b) + b$, $\sigma = \text{Var}(\widehat{h}_j) = 1 + \sigma_e^2$, $b = \sqrt{\kappa}$, and

$$\rho = \frac{\text{E}[(h_j - b)(\widehat{h}_j - b)^*]}{\sqrt{\text{Var}(h_j)\text{Var}(\widehat{h}_j)}} = \frac{1}{\sqrt{1+\sigma_e^2}} \tag{29}$$

is the correlation coefficient between the actual channel gain and its estimate. Thus, the PEP for the suboptimal decoder is given by

$$P_e(d \mid \widehat{\mathbf{H}}, \mathbf{j}, k) = Q\left(\sqrt{\frac{E_s\sum_{j=1}^{J}d_j\left|(\rho/\sigma)(\widehat{h}_j - b) + b\right|^2}{N_0/2 + kE_I + (1-\rho^2)E_s}}\right), \tag{30}$$

where d_j is the number of nonzero error bits in hop j and we have assumed that $E_p = E_s$, that is, the energy used for pilot signals is equal to the signal energy. Define the normalized complex Gaussian random variable $\zeta_j = (\widehat{h}_j - b)/\sigma + b/\rho$ with distribution $\mathcal{CN}(b/\rho, 1)$. Then, the PEP simplifies to

$$P_e(d \mid \widehat{\mathbf{H}}, \mathbf{j}, k) = Q\left(\sqrt{\frac{\rho^2 R_c\gamma_b\sum_{v=1}^{w}v\sum_{i=1}^{j_v}\left|\zeta_i\right|^2}{1/2 + k\gamma_I + R_c\gamma_b(1-\rho^2)}}\right). \tag{31}$$

Define the SINR for imperfect CSI given k interfering users as

$$\begin{aligned}\widehat{\Lambda}(k) &= \frac{E_s}{N_0/2 + kE_I + (1-\rho^2)E_s}\\ &= \frac{R_c\gamma_b}{1/2 + k\gamma_b/\Delta + R_c\gamma_b(1-\rho^2)}.\end{aligned} \tag{32}$$

Hence, the PEP becomes

$$P_e(d \mid \widehat{\mathbf{H}}, \mathbf{j}, k) = Q\left(\sqrt{\widehat{\Lambda}(k)\sum_{v=1}^{w}v\sum_{i=1}^{j_v}\left|\zeta_i\right|^2}\right). \tag{33}$$

Averaging (33) over the fading amplitudes and the number of interfering users, the PEP of coded FHSS systems over Rician fading channels with imperfect CSI is given by (24) with $\Lambda(k)$ being replaced with $\widehat{\Lambda}(k)$.

4. RESULTS AND DISCUSSION

To illustrate the results, we consider coded FHSS systems employing a rate-1/2 convolutional code with a frame size of $N = 2 \times 512$ coded bits. The union bound is truncated to a distance $d_{\max} \leq 15$ in order to reduce the computational complexity. Throughout the results, we assume that $Q = 79$ as in the Bluetooth technology. For the case of perfect CSI over Nakagami fading, only the exact analysis is employed, whereas the Gaussian approximation is used in the cases of Rician fading and imperfect CSI.

Figure 2 shows the performance of an FHSS network with 10 users and perfect CSI for different hop lengths. We observe that the obtained analytical results closely approximate the simulation results. Thus, the proposed analytical approach provides an accurate measure of the performance of coded FHSS systems with MAI. In the rest of this paper, only analytical results are shown in order to make the presentation of the results clear.

Figure 3 shows the performance of a coded FHSS system over Nakagami fading with perfect CSI for different number of users and hop lengths of $m = 1$ and $m = 64$. Comparing the sets of curves corresponding to the cases of $m = 1$ and $m = 16$, we observe that the performance loss due to interference increases as the hop length increases (or in other words as the number of hops decreases). For example, for the case of $m = 1$, 20-user system is worse than the one-user system by almost 0.5 dB, whereas this difference is almost 1 dB for the case of $m = 64$. This is also clear in Figure 4, which shows the performance of the coded FHSS system over Nakagami fading with perfect CSI for different hop lengths m and for 1 and 40 users. The reason behind this phenomenon is that increasing the hop length decreases the diversity order

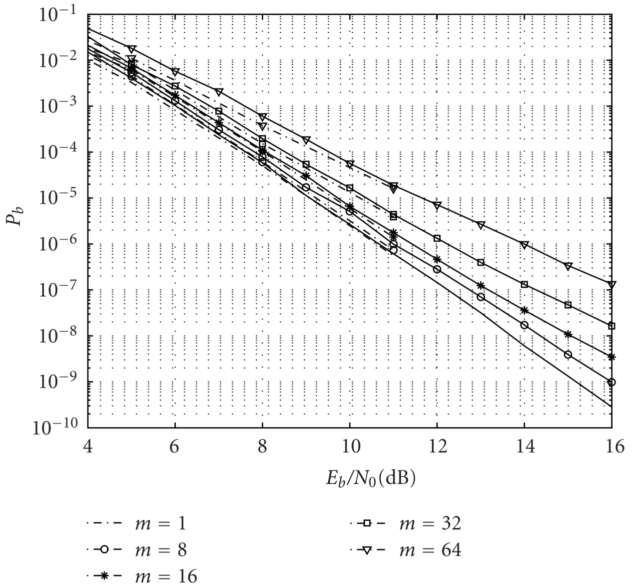

Figure 2: Performance of a rate-1/2 convolutionally coded FHSS system with perfect CSI for 10 users ($K = 10$) and different hop lengths $m = 1, 8, 16, 32, 64$ (solid: approximation using the union bound, dash: simulation).

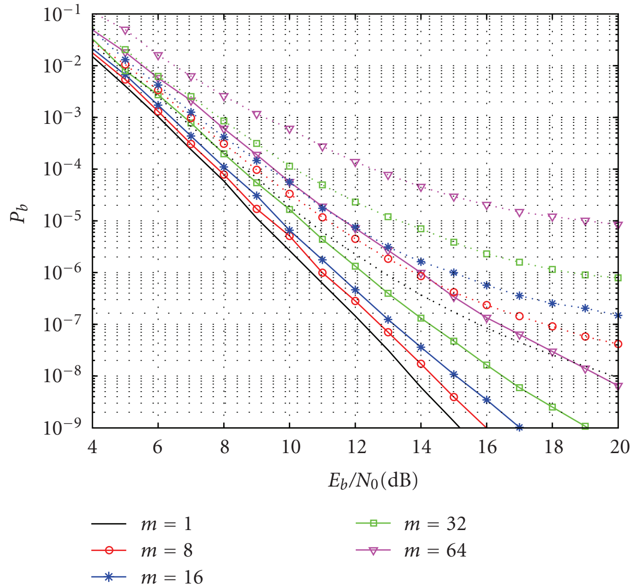

Figure 4: Performance of a convolutionally coded FHSS system over Nakagami fading with perfect CSI for different hop lengths m and SIR = 5 dB, (solid: $K = 1$, dashed: $K = 40$).

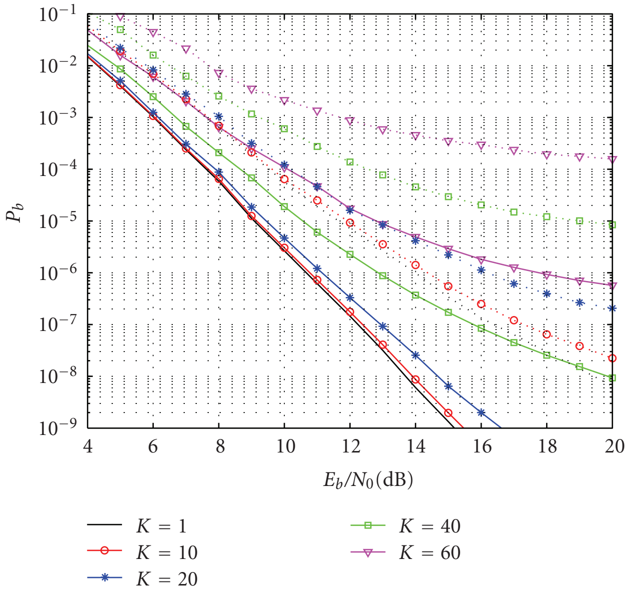

Figure 3: Performance of a convolutionally coded FHSS system over Nakagami fading with perfect CSI for different number of users K and SIR = 5 dB, (solid: $m = 1$, dashed: $m = 64$).

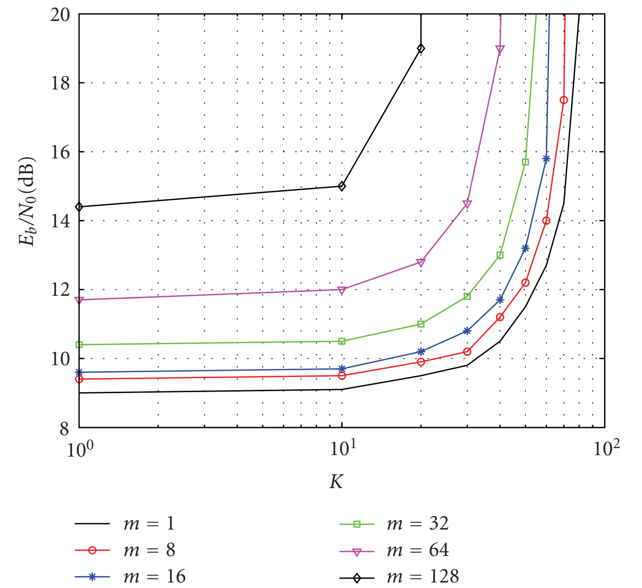

Figure 5: SNR required for a convolutionally coded FHSS system to achieve $P_b = 10^{-5}$ over Nakagami fading versus the number of users K with perfect CSI for $m = 1, 8, 16, 32, 64, 128$ and SIR = 5 dB.

provided to the coded system, which increases the impact of interference on the performance of the system.

Figure 5 shows the SINR required for the coded FHSS system to achieve $P_b = 10^{-5}$ over Rayleigh fading versus the number of users K with perfect CSI for different hop lengths. In the figure, we observe that as the hop length increases, the required SNR increases up to a maximum number of users beyond which the required performance cannot be achieved. For example, a coded FHSS system with $m = 64$ can achieve a $P_b = 10^{-5}$ with an SNR of 12 dB when only 10 users exist in the system. However, it cannot achieve the same performance whatsoever if the number of users in the system exceeds 40 users. Therefore, if more than 40 users need to be supported at a $P_b = 10^{-5}$, then the hop length has to be decreased, that is, the number of hops per frame has to be increased to increase the diversity order in the coded system.

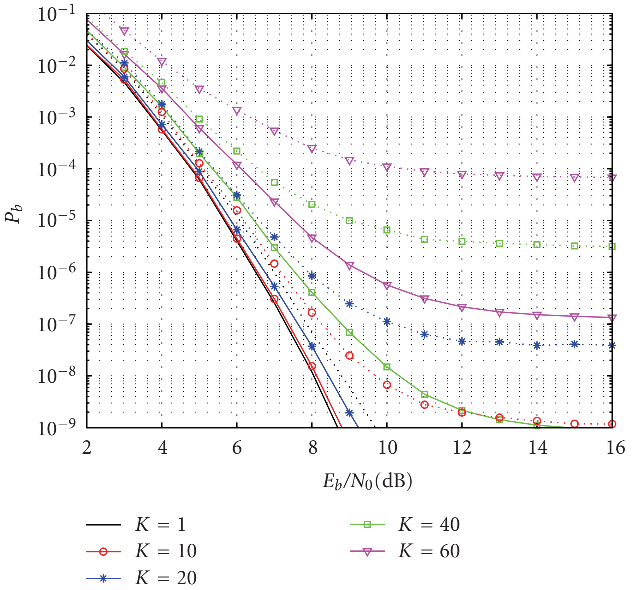

FIGURE 6: Performance of a convolutionally coded FHSS system over a Rician fading with $\kappa = 10$ dB and perfect CSI for different number of users K and SIR = 5 dB, (solid: $m = 1$, dashed: $m = 64$).

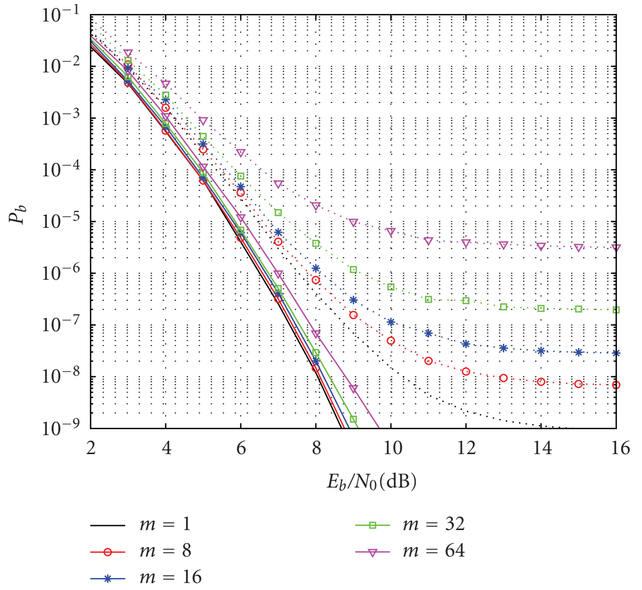

FIGURE 7: Performance of a convolutionally coded FHSS system over a Rician fading with $\kappa = 10$ dB and perfect CSI for different hop lengths m and SIR = 5 dB, (solid: $K = 1$, dashed: $K = 40$).

The performance of the coded FHSS system over Rician fading with $\kappa = 10$ dB and perfect CSI is shown in Figure 6 for different number of users and hop lengths of $m = 1$ and $m = 64$. Comparing with the results for Nakagami fading, we observe that the performance loss due to increasing the hop length decreases as the fading becomes less severe. Similar to the case of Nakagami fading, the performance loss due to interference increases as the hop length increases. Note that the error floor resulting from the interference is lower in the case of Rician compared to that in the case of Nakagami fading. In Figure 7, the performance of the coded FHSS system over a Rician fading with $\kappa = 10$ dB and perfect CSI is shown for different hop lengths and number of users of 1 and 40.

The results of imperfect CSI are obtained using only pilot estimation (OPE) with $E_p = E_s$. In this case, the estimation error variance of $\sigma_e^2 = (N_0+kE_I)/E_s$. In simulating systems with OPE, one coded bit is punctured every m coded bits to account for the rate reduction resulting from inserting a pilot signal in each hop. This affects the whole distance distribution of the resulting code and may reduce the minimum distance of the code. The resultant code rate after puncturing is given by

$$\widetilde{R}_c = \frac{mR_c}{m-1}. \tag{34}$$

Table 1 shows the code rates and the minimum distances of the punctured codes for different hop lengths. According to the table, we conclude that systems with short hop are expected to have more channel diversity at the cost of lower minimum distance and worse channel estimation quality.

In Figure 8, we show the performance of the coded FHSS system over Rayleigh and Rician fading channels with OPE for a number of users $K = 20$ and different hop lengths. We can observe that as the fading becomes more severe (Rayleigh compared to Rician), the optimal hop length decreases because the diversity becomes more crucial to the performance as the fading becomes more severe. In addition, the optimal hop length decreases as the SINR increases since diversity becomes more important at high SINR.

TABLE 1: Rates and minimum distances of the punctured rate-1/2 convolutional codes.

m	Code Rate $\widetilde{R}_c$	$d_{\min}$
4	0.667	4
8	0.571	5
16	0.533	6
32	0.516	6
64	0.508	6

Figure 9 shows the SINR required for the coded FHSS system to achieve $P_b = 10^{-4}$ over Rayleigh fading versus the number of users K with an OPE receiver for different hop lengths. We observe that as the hop length increases, the required SINR increases up to a maximum number of users beyond which the required performance cannot be achieved. This is similar to the observation made in the perfect CSI case. A more interesting observation is that short hop lengths start to outperform long hop lengths as the number of user increases. This is very clear in the behavior of the cases of $m = 16$ and $m = 64$, where the latter outperforms the former for small number of users, and the converse occurs as the number of users increases. This agrees with the observation made in Figures 3 and 4, where it was concluded that the performance loss due to interference increases with increasing the hop length of the coded system.

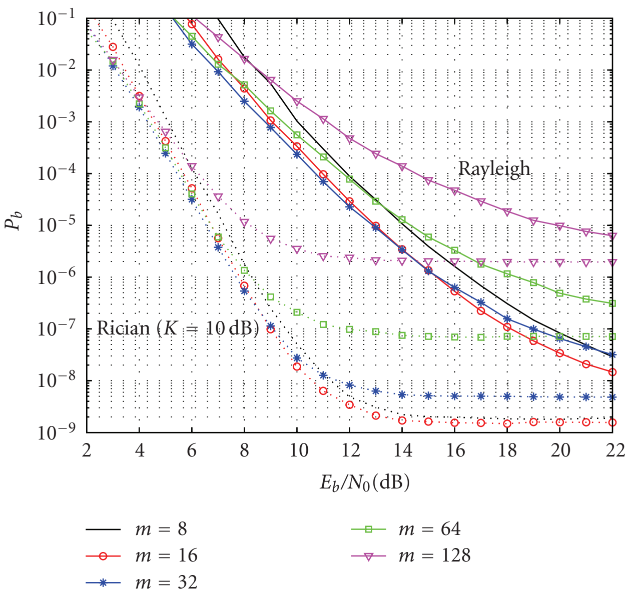

Figure 8: Performance of a convolutionally coded FHSS system over Rayleigh and Rician fading channels with an OPE receiver for number of users $K = 20$, SIR $= 5$ dB, and different hop lengths $m = 8, 16, 32, 64, 128$, (solid: Rayleigh, dashed: Rician with $\kappa = 10$ dB).

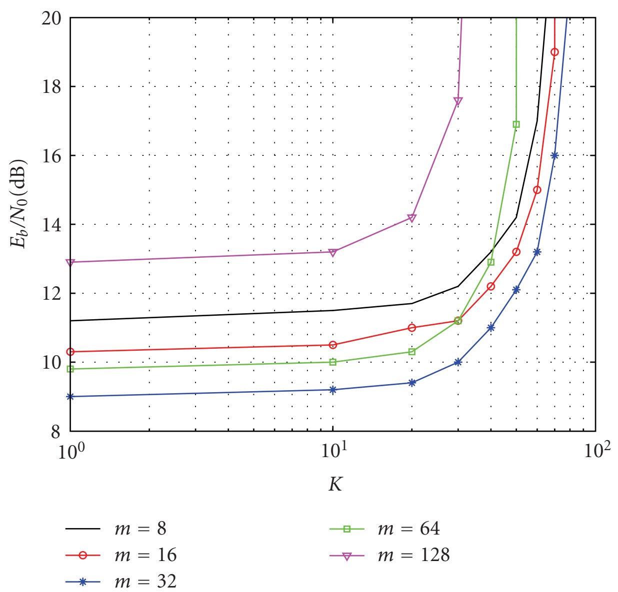

Figure 9: SNR required for a convolutionally coded FHSS system achieve $P_b = 10^{-4}$ over Rayleigh fading versus the number of users K with an OPE receiver for SIR $= 5$ dB and $m = 8, 16, 32, 64, 128$.

In Figure 9, we observe that the optimal hop for the coded FHSS system over Rayleigh fading channel is $m = 32$, for all the number of users. In Figure 10, the same information shown in Figure 9 is shown for Rician fading channel with $\kappa = 10$ dB, where we observe that the optimal hop increases as the channel become less severe (i.e., as the Rician factor increases) since diversity becomes less important. In particular, for Rician fading channels with

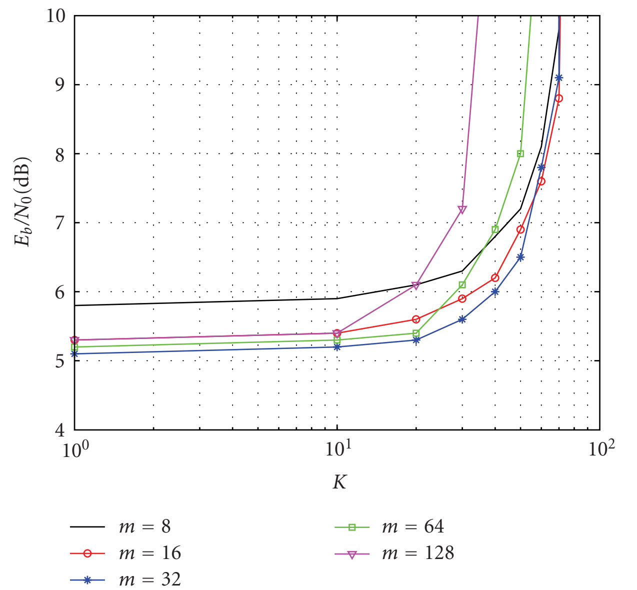

Figure 10: SNR required for a convolutionally coded FHSS system to achieve $P_b = 10^{-4}$ over Rician fading with $\kappa = 10$ dB versus the number of users K with an OPE receiver for SIR $= 5$ dB and $m = 8, 16, 32, 64, 128$.

$\kappa = 10$, the cases of $m = 16$ and $m = 32$ compete for the optimal hop length, and the former wins as the number of users increases.

5. CONCLUSIONS

In this paper, we derived a union bound for coded FHSS systems with MAI. Results show that the performance loss due to interference increases as the hop length increases (or in other words as the number of hops in FHSS systems decreases). This performance loss increases as the number of users increases. Furthermore, the tradeoff between channel diversity and channel estimation under interference conditions has been investigated analytically. It was found that as the fading becomes more severe (Rayleigh as compared to Rician), the optimal hop length decreases. In addition, the optimal hop length decreases as the SINR increases since diversity becomes more important at high SINR. Furthermore, the optimal hop length tends to increase as the SIR increases for the same reason. In the case of channel estimation, the proposed analytical approach can be safely applied to FHSS systems with large number of users.

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