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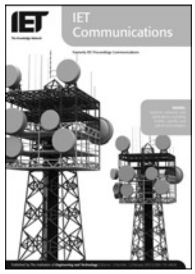

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New union bound on the error probability of bit-interleaved space–time codes with finite interleaver sizes

S.A. Zummo[1] P.-C. Yeh[2] W.E. Stark[3]

[1]Electrical Engineering Department, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabia
[2]Department of Electrical Engineering and Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan, Republic of China
[3]Department of Electrical Engineering and Computer Science, University of Michigan, MI, USA
E-mail: pcyeh@cc.ee.ntu.edu.tw

Abstract: A new union bound on the bit error probability of bit-interleaved space–time (BI-ST) coded systems is derived. Unlike existing performance analysis tools for BI-ST systems, the new bound provides a general framework for analysing the performance of BI-ST systems employing finite interleaver sizes. The derivation is based on the uniform interleaving assumption of the coded sequence prior to transmission over multiple antennas. The new bound is a function of the distance spectrum of the code, the signal constellation used and the space–time (ST) mapping scheme. The bound is derived for a general BI-ST coded system and applied to two specific examples, namely, the BI space–time coded modulation and the BI space–time block codes. Results show that the analysis provides a close approximation to the BI-ST performance for a wide range of signal-to-noise ratios. The analysis can also accurately characterise the performance differences between different interleaver sizes, which is a breakthrough in the analysis of BI-ST coded systems.

1 Introduction

Achieving high-rate reliable communications over a time-varying fading channel resulting from multi-path reception is a significant challenge. One standard approach to mitigate fading and achieve bandwidth efficiency is transmit diversity in which multiple antennas are used at the transmitter. Using the combination of multiple transmit antennas and error control coding is referred to as space–time (ST) coding [1, 2]. Simple and elegant space–time block codes (STBC) were proposed in [3] to provide diversity at the transmitter.

Coded modulation [4] is an efficient technique that provides high transmission rates at good quality by combining error control coding and modulation. The performance of coded modulation can be enhanced by interleaving the coded bits prior to mapping them onto the signal constellation [5, 6]. This method is referred to as bit-interleaved coded modulation (BICM). BICM is applied to multi-input multi-output (MIMO) systems in [7], in which the coded bits are bit-interleaved and each group of bits is mapped onto signals that are transmitted over multiple transmit antennas. Two approaches to mapping the coded bits onto the signals are considered in this article, namely, the BI-ST block code (BI-STBC) [8] and the BI-ST coded modulation (BI-STCM) [9, 10].

Because of the interleaver used in the transmitter, each signal vector (transmitted over the multiple transmit antennas) is composed of coded bits that are randomly located in the coded sequence (from the decoder point of view). At the receiver, the random distribution of the error bits over different symbols causes the performance analysis to be difficult. For fast Rayleigh fading case, union bounds on the bit error probability of BICM systems under the assumption of infinite interleaver size are presented in [6, 11–13]. Although BI-ST coded systems can be regarded as special cases of BICM, the performance analysis of BI-ST is not a trivial extension of BICM analysis owing to the more complex structures of the space–time codes. The bound analysis in [6, 11] is extended for BI-STBC in [8]. The expurgation method used in these articles is shown to be flawed in [14] which affects the validity of the bound analysis. Moreover, the bound analysis in these articles assumes that because of the infinite interleaving, every symbol error has only one bit error among the bits associated with the symbol. It is proved in this article that the assumption is only true when the interleaver size goes to infinity. As a result, the bound analysis in [6, 8, 11, 12] cannot characterise the performance of an interleaver with a specific size.

We derive a new union bound on the bit error probability of BI-ST coded systems over block fading channels with finite interleaver sizes, which is considered as a breakthrough in the analysis of BI-ST systems. As the analysis of BI-STBC significantly differs from that of BI-STCM, the error probability analysis is derived for both systems here. The new bound is based on the uniform (random) interleaving of the coded bits prior to mapping them onto modulation symbols that are transmitted over transmit antennas. The distribution of the error bits in a received vector is derived and the corresponding pairwise error probability is evaluated. Flat multipath fading channels following Rician and Nakagami distributions are considered for BI-STBC, whereas only Rician fading is considered for BI-STCM as the analysis under Nakagami fading is not tractable because of the nature of the random variables involved. Although no expurgation is done, simulation results show that the proposed bound is tight for different Gray-mapped constellations, ST coding schemes and channel models.

The outline of the article is as follows. The model for BI-ST coded system is described in Section 2. In Section 3, the proposed union bound is derived. The characteristic function required to evaluate the union bound is derived for the BI-STCM and BI-STBC systems in Sections 3 and 4, respectively. The asymptotic effect of the interleaver size is analysed in Section 5. Analytical and simulation results are presented in Section 6. Conclusions are discussed in Section 7.

2 System model

Consider the BI-ST coded system shown in Fig. 1. The encoder receives an information block $\boldsymbol{u}$ of K bits and generates an N-bit codeword $\boldsymbol{c}$ resulting in a code rate $R_c = K/N$. After encoding, the codeword $\boldsymbol{c}$ is bit interleaved to generate the interleaved codeword $\pi(\boldsymbol{c}) = (\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_L)$ that consists of L blocks each of q m bits. Each of the L blocks is referred to as an ST block (STB). Note that $N = qmL$. The STB $\boldsymbol{b}_l$ is mapped onto q symbols $(s_{l,1}, s_{l,2}, \ldots, s_{l,q})$ by an ST mapper. Each of the q symbols are drawn from an M-ary complex signal constellation that consists of $M = 2^m$ signal points with average symbol energy equal to mE_b, where E_b denotes the energy per information bit. Every q symbol is mapped by the ST encoder into p column vectors of length n_T for transmission by n_T transmit antennas.

The ST code is characterised by an $n_T \times p$ transmission matrix, where $p = 1$ or an integer that satisfies $p \geq n_T$. In the case of $p = 1$, the system is a multiplexing system which is given the name of STCM [1, 9, 10], whereas the case of $p \geq n_T$ results in the well-known STBC [3]. The rows of the transmission matrix consists of entries that are linear combinations of $s_{l,1}, s_{l,2}, \ldots, s_{l,q}$ and $s^*_{l,1}, s^*_{l,2}, \ldots, s^*_{l,q}$. Denote the transmission matrix by $\boldsymbol{x}_l = [\boldsymbol{x}_{l,1}{}^T, \boldsymbol{x}_{l,2}{}^T, \ldots, \boldsymbol{x}_{l,p}{}^T]$, where $\{\boldsymbol{x}_{l,t}{}^T\}$ are column vectors of dimension $n_T \times 1$. The ST encoder maps the vector $(s_{l,1}, s_{l,2}, \ldots, s_{l,q})$ onto the column vectors $\boldsymbol{x}_{l,1}{}^T, \boldsymbol{x}_{l,2}{}^T, \ldots, \boldsymbol{x}_{l,p}{}^T$, and the vectors $\{\boldsymbol{x}_{l,t}{}^T\}$ are transmitted by n_T antennas one at a time over p transmission intervals. The p transmission intervals constitute one STB. The code rate of the ST encoder is $R_s = qm/p$, and the overall rate of the BI-ST coded system is $R = R_s R_c = qmK/pN$. In the case of STBC, that is $p \geq n_T$, the rows of transmission matrix are constructed to be orthogonal in order to enable a linear complexity receiver [3]. Specific examples of ST codes are presented below.

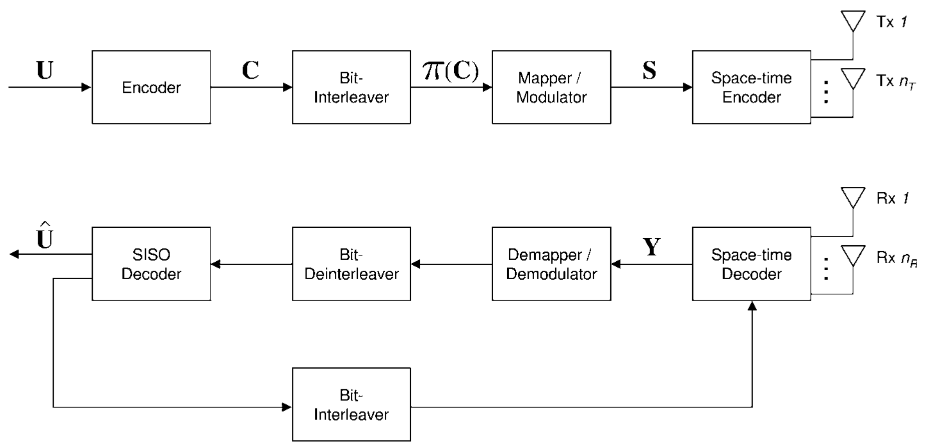

Figure 1 *Block diagram of the general BI-ST coded system*

Example 1: When $p = 1$ and $q = n_T$, the resulting system is an STCM system. In STCM, every n_T symbol is transmitted over n_T transmit antennas during one symbol duration. In this case, the fading processes affecting consecutive symbols are assumed to be independent.

Example 2: When $p = q = n_T = 2$, the resulting system is the Alamouti STBC. The Alamouti STBC is characterised by the 2×2 complex matrix $\boldsymbol{x}_l$ presented in [3]. In this case, the fading process should stay constant for at least two symbols to enable simple detection. This is a full-rate STBC.

Example 3: When $n_T < 2, q = p < n_T$, the resulting system is an STBC derived from orthogonal designs [2]. In this case, the matrix $\boldsymbol{x}_l$ is one of the $n_T \times p$ complex matrices presented in [2]. This is an STBC with a rate less than unity.

We assume n_R receive antennas, but the derivation is shown for one receive antenna case only which can be easily extended to the case of multiple receive antennas. The received signal vector corresponding to a codeword $\boldsymbol{c}$ is denoted as $\boldsymbol{r} = (\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_L)$, where $\boldsymbol{r}_l = (r_{l,1}, r_{l,2}, \ldots, r_{l,p})$ and

$$r_{l,t} = \boldsymbol{h}_{l,t} \cdot \boldsymbol{x}_{l,t}{}^{\mathrm{T}} + n_{l,t} \tag{1}$$

where $\boldsymbol{n}_{l,t}$ is a length-p AWGN vector at the receive antenna during the tth transmission period in the lth STB modelled as $\mathcal{CN}(\mathbf{0}_p, N_0\boldsymbol{I}_p)$, where $\boldsymbol{I}_p$ denotes the $p \times p$ identity matrix and $\mathbf{0}_p$ is the $1 \times p$ zero matrix. The fading vector $\boldsymbol{h}_{l,t}$ consists of n_T independent and identically distributed (i.i.d.) random variables $\{h^i_{l,t}\}^{n_T}_{i=1}$, where $h^i_{l,t}$ denotes the fading attenuation of the channel from the ith transmit antenna to the receive antenna during the tth transmission period in the lth STB. The fading channel is assumed to be constant during one STB to enable low-complexity receivers for the STBC case [3]. The magnitudes of the fading random variables are assumed to follow either Rician or Nakagami distribution. The receiver is assumed to have perfect channel state information (CSI) and the decoding is done by minimising the decision metric

$$\sum_{l=1}^{L} \sum_{t=1}^{p} \| r_{l,t} - \boldsymbol{h}_{l,t}\boldsymbol{x}_{l,t}{}^{\mathrm{T}} \|^2 \tag{2}$$

which can be closely achieved via iterative ST detection and decoding [15].

3 Union bound

In this section, we derive a union bound on the bit error probability of the BI-ST coded system described in Section 2. Here, only BI-ST coded systems employing convolutional codes are considered. However, the performance analysis applies equally well to any coding scheme with a known distance spectrum such as turbo codes [16–19], product codes [20] or block codes. The bit error probability for a convolutional code is upper bounded [21] by

$$P_b \lesssim \sum_{d=d_{\min}}^{N} \sum_{j=1}^{K} \frac{j}{K} w_{j,d} P_u(d) \tag{3}$$

where $d_{\min}$ is the minimum Hamming distance of the convolutional code, $w_{j,d}$ denotes the number of convolutional codewords with input Hamming weight j and total weight d, and $P_u(d)$ is the pairwise error probability defined as the probability of decoding a received sequence as a codeword $\hat{\boldsymbol{c}}$ of error weight d given that the codeword $\boldsymbol{c}$ was transmitted, i.e. the Hamming distance $d_H(\boldsymbol{c}, \hat{\boldsymbol{c}}) = d$. Note that the pairwise error probability also depends on the signal-to-noise ratio (SNR) and the signal constellation used. We express it using the abbreviated notation $P_u(d)$ to emphasise its dependence on d which motivates the analysis afterwards. Throughout the article, for any variable defined for $\boldsymbol{c}$, the corresponding variable defined for $\hat{\boldsymbol{c}}$ is denoted by using '^'. The subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. Clearly, $P_u(d)$ depends on the squared Euclidean distance $d^2_E \triangleq [d_E(\boldsymbol{c}, \hat{\boldsymbol{c}})]^2$ between the received sequences corresponding to the codewords $\boldsymbol{c}$ and $\hat{\boldsymbol{c}}$, which is a function of the distribution of the d non-zero bits over the L STBs in the codeword. Using the integral expression [22] of the Q function, $Q(x) = 1/\pi \int_0^{\pi/2} e^{(-x^2/2\sin^2\theta)} d\theta$, we have

$$\begin{aligned} P_u(d) &= E_{d^2_E|d}\left[Q\left(\sqrt{\frac{R\gamma_b}{2} \cdot d^2_E} \right) \right] \\ &= \frac{1}{\pi} \int_0^{\pi/2} \Psi_{d^2_E|d}\left(\frac{R\gamma_b}{4\sin^2\theta} \right) d\theta \end{aligned} \tag{4}$$

where $\gamma_b = E_b/N_0$ is the SNR per information bit and $\Psi_{d^2_E|d}(z) \triangleq E_{d^2_E|d}[e^{-zd^2_E}]$ is the conditional characteristic function of the random variable d^2_E given d.

As the combination of the signal constellation mapping with the ST encoding may not have a symmetric structure for all codewords, the Euclidean distance $d_E(\boldsymbol{c}, \hat{\boldsymbol{c}})$ may not be the same for different choices of $\boldsymbol{c}$ and $\hat{\boldsymbol{c}}$ even if the Hamming distance $d_H(\boldsymbol{c}, \hat{\boldsymbol{c}})$ is fixed at d. Hence we have to take the expectation in (4) with respect to the distribution of d^2_E given d. Thus, the task is to find the conditional distribution of d^2_E given d. Denote the error vector between two codewords $\pi(\boldsymbol{c})$ and $\pi(\hat{\boldsymbol{c}})$ by $\boldsymbol{e}(\boldsymbol{c}, \hat{\boldsymbol{c}}) = (\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_L)$, where $\boldsymbol{e}_l = (\boldsymbol{e}_{l,1}, \boldsymbol{e}_{l,2}, \ldots, \boldsymbol{e}_{l,p})$ and $\boldsymbol{e}_{l,t} = \boldsymbol{x}_{l,t} - \hat{\boldsymbol{x}}_{l,t}$. The squared Euclidean distance d^2_E can be expressed as

$$d^2_E = \sum_{l=1}^{L} \sum_{t=1}^{p} \| \boldsymbol{h}_{l,t}\boldsymbol{e}_{l,t}{}^{\mathrm{T}} \|^2 = \sum_{l=1}^{L} d^2_l \tag{5}$$

where $d^2_l = \sum_{t=1}^{p} \| \boldsymbol{h}_{l,t}\boldsymbol{e}_{l,t}{}^{\mathrm{T}} \|^2$ is the squared Euclidean distance between the received signal vectors associated

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with the lth STB, $\boldsymbol{b}_l$. As the total number of bit errors in the codeword is d, the distribution of d_l^2 depends on how many bit errors exist in the STB $\boldsymbol{b}_l$. Thus, it is necessary to find the conditional distribution of d_l^2 given f_l, where f_l denotes the number of bit errors in $\boldsymbol{b}_l$. Because of the uniform interleaving and the independent fading assumptions, the conditional distributions of $\{d_l^2|f_l\}$ are identical and the characteristic function of d_{E}^2 given d can be obtained as

$$\begin{aligned}\Psi_{d_{\mathrm{E}}^2|d}(z) &= E_{f_1,\ldots,f_L}\left[\prod_{l=1}^{L}\Psi_{d_l^2|f_l}(z)\right]\\ &= E_{j_1,\ldots,j_w}\left[\prod_{v=1}^{w}[\phi_v(z)]^{j_v}\right]\end{aligned} \tag{6}$$

where j_v denotes the number of STB's with v bit errors, $w = \min\{d, qm\}$ and $\phi_v(z)$ is given by

$$\begin{aligned}\phi_v(z) &\triangleq \Psi_{d_l^2|f_l}(z|f_l = v)\\ &= E_{d_l^2|f_l}\left[\mathrm{e}^{-zd_l^2}|f_l = v\right], \quad v = 1,\ldots,w\end{aligned} \tag{7}$$

Clearly, the form of $\phi_v(z)$ depends on the fading distribution, which will be derived in Section 4. As $d_{\mathrm{H}}(\boldsymbol{c}, \hat{\boldsymbol{c}}) = d$, the components of the vector $\boldsymbol{j} = (j_0, j_1, \ldots, j_w)$ are constrained by the conditions

$$L = \sum_{v=0}^{w} j_v, \quad d = \sum_{v=1}^{w} v j_v \tag{8}$$

The joint pdf of $\boldsymbol{j}$ given d can be derived using combinatorics as

$$\begin{aligned}p(\boldsymbol{j}|d) = &\frac{\binom{qm}{1}^{j_1}\binom{qm}{2}^{j_2}\cdots\binom{qm}{w}^{j_w}}{\binom{N}{d}}\\ &\cdot\frac{L!}{j_1! j_2! \ldots j_w!(L - \sum_{v=1}^{w} j_v)!}\end{aligned} \tag{9}$$

The left factor of $p(\boldsymbol{j}|d)$ in (9) is the probability of distributing d non-zero bits over L error vectors with j_v error vectors having v bits for possible values of v. The right term of $p(\boldsymbol{j}|d)$ is the number of combinations of $\boldsymbol{j} = \{j_v\}_{v=0}^{w}$ among the L error vectors. The expectation in (6) is computed as

$$\Psi_{d_{\mathrm{E}}^2|d}(z) = \sum_{j_w=0}^{L_w}\sum_{j_{w-1}=0}^{L_{w-1}}\cdots\sum_{j_1=0}^{L_1}\left(p(\boldsymbol{j}|d)\prod_{v=1}^{w}[\phi_v(z)]^{j_v}\right) \tag{10}$$

where

$$L_v = \max\left\{0, \left\lfloor\frac{d - \sum_{r=v+1}^{v} r j_r}{v}\right\rfloor\right\} \quad 1 \le v \le w \tag{11}$$

Substituting (7)–(11) into (4) results in the final form of the unconditional pairwise error probability. The rest of the article is devoted to deriving expressions of the characteristic function $\phi_v(z)$ for BI-STCM ($p = 1$) and BI-STBC ($p \ge n_{\mathrm{T}}$) systems with different fading distributions.

4 Characteristic function

4.1 BI-STCM

In BI-STCM systems, $p = 1$ and $q = n_{\mathrm{T}}$, and thus we use the notations $\boldsymbol{e}_l = \boldsymbol{e}_{l,1}$ and $\boldsymbol{h}_l = \boldsymbol{h}_{l,1}$. In this case, the distance is given by $d_l^2 = |\boldsymbol{h}_l\boldsymbol{e}_l^{\mathrm{T}}|^2$. Going through the derivation in [1], the distance d_l^2 simplifies to

$$d_l^2 = \|\boldsymbol{e}_l\|^2 \cdot |\beta_l(\boldsymbol{e}_l)|^2 \tag{12}$$

where

$$\beta_l(\boldsymbol{e}) \triangleq \frac{\boldsymbol{h}_l\boldsymbol{e}^T}{\|\boldsymbol{e}\|} \tag{13}$$

is a random variable whose distribution depends on the fading distribution which will be derived later. This implies that $\{\beta_l(\boldsymbol{e}_l)\}$ are independent random variables. Given a realisation of the error vector $\boldsymbol{e}_l$, the conditional characteristic function of d_l^2 given $\boldsymbol{e}_l$ becomes

$$\Psi_{d_l^2\|\boldsymbol{e}_l}(z) = \Psi_{|\beta_l(\boldsymbol{e}_l)|^2}(z\|\boldsymbol{e}_l\|^2) \tag{14}$$

To find $\phi_v(z) = \Psi_{d_l^2|f_l}(z)$, we first consider all $\binom{qm}{2}$ possible STB combinations of $\boldsymbol{b}_l$ and $\hat{\boldsymbol{b}}_l$. For each pair, we feed them to the STBC encoder to find the corresponding $\boldsymbol{x}_l$, $\hat{\boldsymbol{x}}_l$ and the error vector $\boldsymbol{e}_l$. Classify these STB pairs into groups according to $d_{\mathrm{H}}(\boldsymbol{b}_l, \hat{\boldsymbol{b}}_l)$. Suppose in the group of $d_{\mathrm{H}}(\boldsymbol{b}_l, \hat{\boldsymbol{b}}_l) = v$ bits, the STB pairs of the group generates error vectors $\boldsymbol{e}_{v,1}, \boldsymbol{e}_{v,2}, \ldots$ each with multiplicity $\mu_{v,1}, \mu_{v,2}, \ldots$, respectively. Then the conditional joint pdf of $\boldsymbol{e}_l$ given f_l can be written as

$$p_{\boldsymbol{e}_l|f_l}(\boldsymbol{e}|v) = \sum_k \chi_{v,k}\Delta(\boldsymbol{e} - \boldsymbol{e}_{v,k}) \tag{15}$$

where $\chi_{v,k} = \mu_{v,k}/\sum_k \mu_{v,k}$ is the probability for an error vector $\boldsymbol{e}_{v,k}$ to occur, and $\Delta(\boldsymbol{e}) \triangleq 1$ if $\boldsymbol{e} = 0$, and 0 otherwise. By (14) and (15), we have

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \chi_{v,k}\Psi_{|\beta_l(\boldsymbol{e}_{v,k})|^2}(z\|\boldsymbol{e}_{v,k}\|^2) \tag{16}$$

Clearly, the form of $\Psi_{|\beta_l(\boldsymbol{e}_{v,k})|^2}(z)$ is a function of the fading distribution. If a line of site (LOS) exists between the transmitter and the receiver, the amplitude of the channel gain can be modelled as a Rician random variable [23]. In this model, $\{h_{l,t}^i\}$ are complex Gaussian random variables with a mean of $\sqrt{\kappa/(1+\kappa)}$ and a variance of $1/(1+\kappa)$ per dimension, where κ denotes the ratio of the specular component energy to the diffuse component energy of each fading channel. Therefore, $\beta_l(\boldsymbol{e}_{v,k})$ is a complex Gaussian random variable with mean $\sqrt{\kappa/(1+\kappa)} \cdot \zeta(\boldsymbol{e}_{v,k})$

and variance $1/(1+\kappa)$ per dimension, where $\zeta(\boldsymbol{e})$ is defined as the sum of all elements of a vector $\boldsymbol{e}$. In this case, the pdf of the random variable $|h|$ [24] is given by

$$p_{|h|}(a) = 2a(1+\kappa)\exp\left[-\kappa - a^2(1+\kappa)\right] \cdot I_0\left(2a\sqrt{\kappa(1+\kappa)}\right), \quad a \geq 0 \tag{17}$$

Hence, $|\beta_l(\boldsymbol{e}_{v,k})|^2$ is a non-central chi-square distributed with a characteristic function [25] given by

$$\Psi_{|\beta_l(\boldsymbol{e}_{v,k})|^2}(z) = \frac{1+\kappa}{1+\kappa+z} \cdot \exp\left(\frac{-z\kappa|\zeta(\boldsymbol{e}_{v,k})|^2}{1+\kappa+z}\right) \tag{18}$$

which yields

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \chi_{v,k}\left[\frac{1+\kappa}{1+\kappa+z||\boldsymbol{e}_{v,k}||^2} \cdot \exp\left(\frac{-z\kappa|\zeta(\boldsymbol{e}_{v,k})|^2 \cdot ||\boldsymbol{e}_{v,k}||^2}{1+\kappa+z||\boldsymbol{e}_{v,k}||^2}\right)\right] \tag{19}$$

4.2 BI-STBC

In BI-STBC systems, the fading gain of each channel remains constant during each STB, i.e. $\boldsymbol{h}_{l,1} = \boldsymbol{h}_{l,2} = \cdots = \boldsymbol{h}_{l,p} = \boldsymbol{h}_l = \{h_l^i\}$. Recall that $\boldsymbol{e}_{l,t}$ is a vector of dimension $1 \times n_\mathrm{T}$ and denoted by $\boldsymbol{e}_{l,t} = (e_{l,t}^1, e_{l,t}^2, \ldots, e_{l,t}^{n_\mathrm{T}})$. Owing to the orthogonality of the row vectors of STBC transmission matrix, we have

$$d_l^2 = \sum_{t=1}^{p} \|\boldsymbol{h}_l \boldsymbol{e}_{l,t}{}^\mathrm{T}\|^2 = \sum_{i=1}^{n_T} |h_l^i|^2 \cdot \xi_l^i$$

where $\xi_l^i = \sum_{t=1}^{p} |e_{l,t}^i|^2$. As $\{h_l^i\}$ are i.i.d. random variables, the random variables $\{|h_l^i|^2\}$ are also i.i.d. with characteristic function denoted as $\Psi_{|h|^2}(z)$. As $\{|h_l^i|^2\}$ are independent, we can obtain the characteristic function of d_l^2 given $\boldsymbol{e}_l'$ given a realization of $\boldsymbol{e}_l' = (\xi_l^1, \xi_l^2, \ldots, \xi_l^{n_T})$ as

$$\Psi_{d_l^2|\boldsymbol{e}_l'}(z) = \prod_{i=1}^{n_T} \Psi_{|h|^2}\left(z\xi_l^i\right) \tag{20}$$

Using a similar approach of finding (15) in Section 4.1, we feed all $\binom{qm}{2}$ possible STB pairs of $\boldsymbol{b}_l$ and $\hat{\boldsymbol{b}}_l$ to the ST encoder to obtain the conditional joint pdf of $\boldsymbol{e}_l'$ given f_l

$$p(\boldsymbol{e}_l'|f_l = v) = \sum_k \chi'_{v,k}\Delta(\boldsymbol{e} - \boldsymbol{e}'_{v,k}) \tag{21}$$

Denote the ith component of $\boldsymbol{e}'_{v,k}$ by $\boldsymbol{e}'_{v,k}(i)$. By (20) and (21), we have

$$\phi_v(z) = \Psi_{d_l^2|f_l}(z) = \sum_k \chi'_{v,k}\prod_{i=1}^{n_T}\Psi_{|h|^2}\left(z\boldsymbol{e}'_{v,k}(i)\right) \tag{22}$$

Clearly, $\Psi_{|h|^2}(z)$ depends on the fading distribution of the channel. In the following, two different fading distributions are considered, namely, Rician and Nakagami.

1. *Rician fading*: If $|h|$ is a Rician random variable, then the characteristic function is given by

$$\phi_v(z) = \sum_v \chi'_{v,k}\left[\prod_{i=1}^{n_T}\frac{1+\kappa}{1+\kappa+z\boldsymbol{e}'_{v,k}(i)} \cdot \exp\left(-\frac{z\kappa\boldsymbol{e}'_{v,k}(i)}{1+\kappa+z\boldsymbol{e}'_{v,k}(i)}\right)\right] \tag{23}$$

2. *Nakagami fading*: Nakagami distribution is shown to fit a large variety of channel measurements [26]. Under Nakagami distribution, the pdf of $|h|$ is given by

$$p_{|h|}(a) = \frac{2\mu^\mu}{\Gamma(\mu)\Omega^\mu}a^{2\mu-1}\exp\left(-\frac{\mu a^2}{\Omega}\right) \quad a \geq 0, \quad \mu \geq 0.5 \tag{24}$$

where $\Omega = E[|h|^2] = 1$, $\mu = \Omega^2/\mathrm{Var}[|h|]$ is the fading parameter and $\Gamma(.)$ is the gamma function. In this case, the characteristic function can be written as

$$\phi_v(z) = \sum_v \chi'_{v,k}\left[\prod_{i=1}^{n_T}\frac{1}{1+(z/\mu)\boldsymbol{e}'_{v,k}(i)}\right]^\mu \tag{25}$$

5 Asymptotic effect of interleaver size

As the interleaver size increases, the interleaver provides higher time diversity and thus lowers the BER. To analyse the best possible performance from the BI-ST, one needs to explore the asymptotic effect of the interleaver size on the BER. Recall that the effect of an interleaver on BER is characterised by the term $p(\boldsymbol{j}|d)$in (9), which represents the joint pdf of $\boldsymbol{j}$ given d. As the interleaver size $N = qmL$ goes to ∞, the error pattern $\boldsymbol{j}_d = (d, 0, 0, \ldots, 0)$ dominates the performance. Note that this error pattern indicates that there are exactly d STB's with 1 bit error. To prove this, let us consider the limiting behaviour of (9) as $N \to \infty$ with $\boldsymbol{j}$ substituted by $\boldsymbol{j}_d = (d, 0, 0, \ldots, 0)$. In this case, we have

$$\lim_{N\to\infty} p(\boldsymbol{j}_d|d) = \lim_{L\to\infty}\frac{\binom{qm}{1}^d}{\binom{qmL}{d}} \cdot \frac{L!}{d!(L-d)!} = \lim_{L\to\infty}\frac{(qm)^d}{(qmL)^d/d!} \cdot \frac{L^d}{d!} = 1 \tag{26}$$

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In deriving (26), we have used the observations that $\binom{qml}{d} \to (qmL)^d/d!$ and $L!/(L-d)! \to L^d$ as $L \to \infty$. As the joint pdf values of all possible $\boldsymbol{j}$ sum up to 1, we have

$$\lim_{N\to\infty} p(\boldsymbol{j}|d) = \delta_{j_1,d} \tag{27}$$

where $\delta_{i,j}$ is the Kronecker delta. One can conclude from (27) that as the interleaver size goes to infinity, the probability of having more than one bit error in each STB diminishes. In other words, given that there are d bit errors with a very large interleaver size, it is highly likely to see d STB's each with one bit error. This gives the best possible performance of the BI-ST. The asymptotic union bound can be obtained simply by substituting (27) into (10) and then (4), (3).

6 Numerical results

As illustrative examples, we use BI-ST coded systems employing a rate- 1/2 (5, 7) convolutional code with two transmit antennas, that is $n_{\rm T} = 2$. The modulation techniques used are Gray-mapped quadrature-phase shift keying (QPSK) and Gray-mapped quadrature-amplitude modulation 16-QAM. Note that the derivation above applies to any signal constellation. The throughputs of the BI-STBC and BI-STCM systems are $mR_{\rm c}$ and $mn_{\rm T}R_{\rm c}$ bits/s/Hz respectively. Unless otherwise stated, the interleaver size is set to be $N = 1024$ coded bits.

The performance of BI-STBC and BI-STCM under Rayleigh fading is shown in Fig. 2. We observe that the bound is tight to simulation curves at medium-to-high SNR values. Note that the union bound becomes loose for SNR values lower than the cutoff rate of the system [21]. We observe that the performance of BI-STBC is better

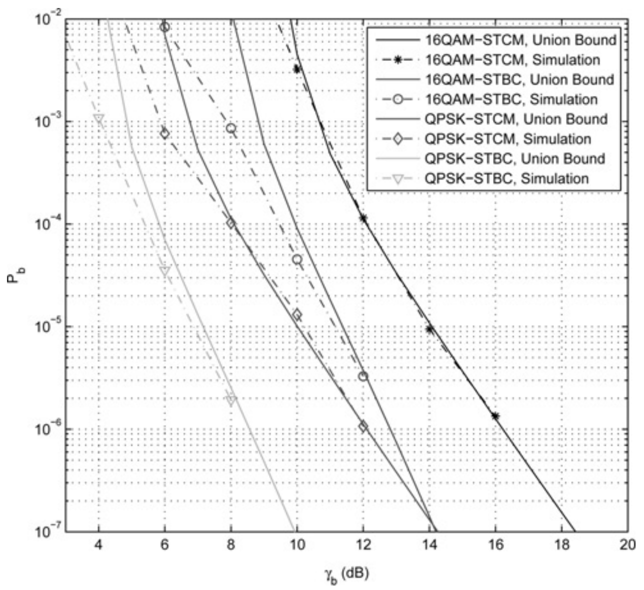

Figure 2 *Bit error probability of convolutionally encoded BI-ST systems using $n_T = 2$ under Rayleigh fading*

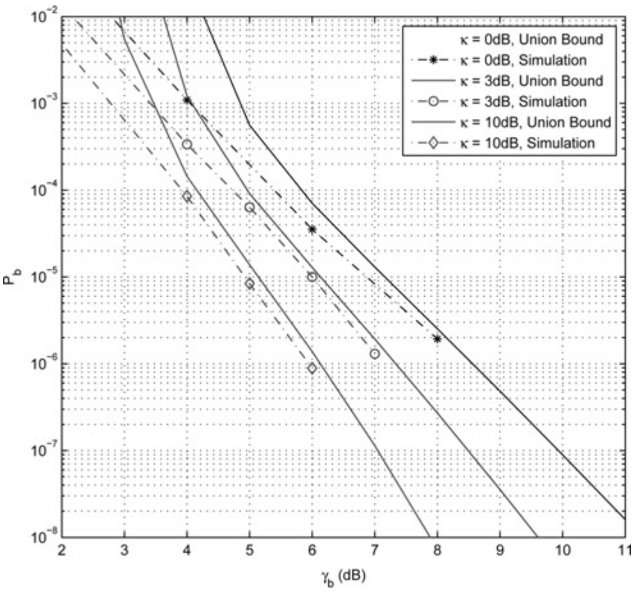

Figure 3 *Bit error probability of a convolutionally encoded BI-STBC using $n_T = 2$ over Rician fading channels with different κ values*

than that of the BI-STCM. This is the tradeoff of the higher throughput of BI-STCM that is $n_{\rm T}$ times larger than that of BI-STBC. Furthermore, in BI-STBC there are $n_{\rm T}$ observations available to detect the transmitted $n_{\rm T}$ signals as opposed to only one observation in BI-STCM. Note that the slope of the error probability curves achieved by BI-STBC is larger which indicates that the time diversity of BI-STBC is larger than that of BI-STCM.

Figs. 3 and 4 show the performance of BI-STBC and BI-STCM, respectively, using QPSK over Rician fading channels with different Rician parameters κ. Again, the

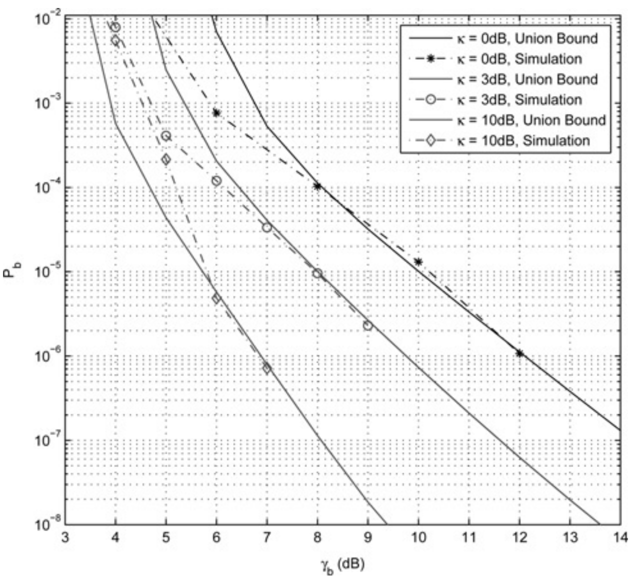

Figure 4 *Bit error probability of a convolutionally encoded BI-STCM using $n_T = 2$ over Rician fading channels with different κ values*

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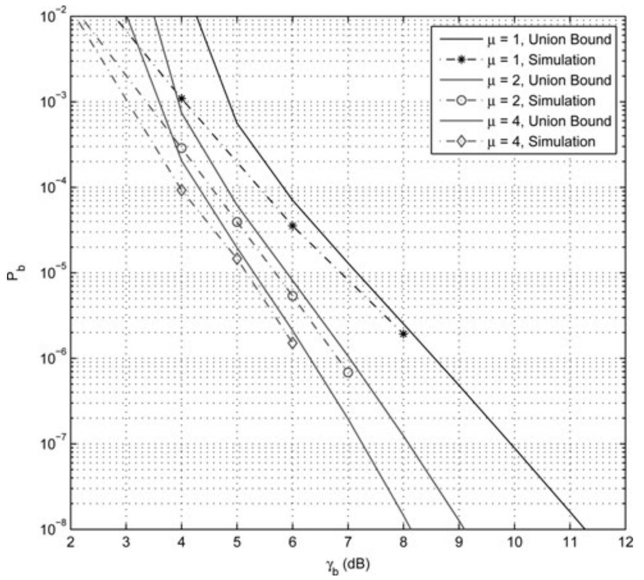

Figure 5 *Bit error probability of a convolutionally encoded BI-STBC using $n_T = 2$ over Nakagami fading channels with different μ values*

bound is shown to be tight to the simulation results for a wide range of the LOS energy of the channel. As the LOS energy increases, the channel becomes more dominated by the LOS component, and hence the performance of the BI-ST coded systems improves. In Fig. 5, we show the performance of BI-STBC over Nakagami fading channels with different Nakagami parameters μ. As the fading parameter μ of the channel increases, the channel becomes less faded and hence the performance of the BI-STBC system improves accordingly.

To show the effectiveness of the proposed bound on capturing the effect of the interleaver size, the BER curves

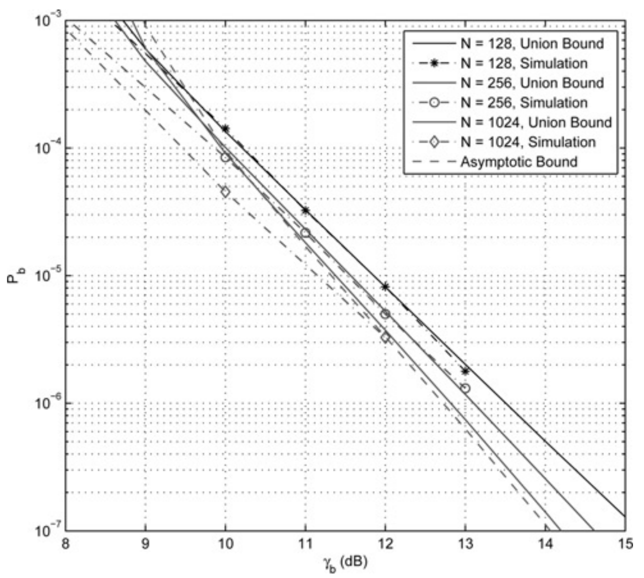

Figure 6 *Bit error probability of a convolutionally encoded BI-STBC using $n_T = 2$ under Rayleigh fading with different interleaver sizes*

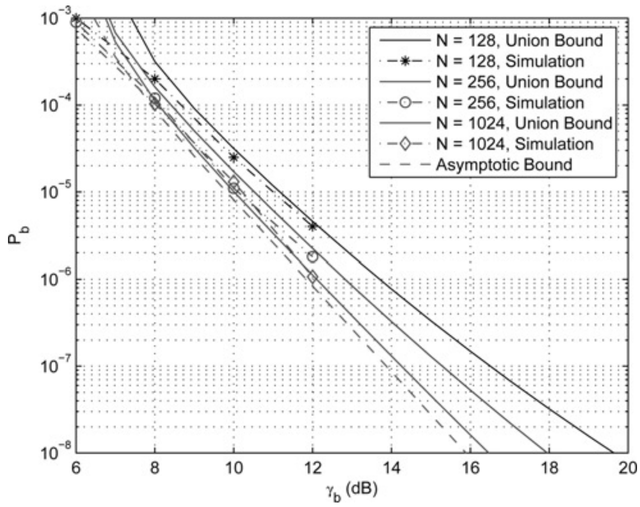

Figure 7 *Bit error probability of a convolutionally encoded BI-STCM using $n_T = 2$ under Rayleigh fading with different interleaver sizes*

of BI-STBC and BI-STCM systems with different interleaver sizes under Rayleigh fading are shown in Figs. 6 and 7. We can see that the bound accurately characterises the performance of different interleaver sizes. The figures imply that there is not much gain left for interleaver size over 1024 bits. Without our bound analysis, one generally chooses a large interleaver size to ensure low BER, which causes large delay to the data transmission. Our bound is useful for determining the interleaver size of BI-ST system by exploring the tradeoff of BER against interleaver size.

In Fig. 8, the SNR required to achieve a BER of 10^{-6} against interleaver size is shown for BI-STBC and BI-STCM systems

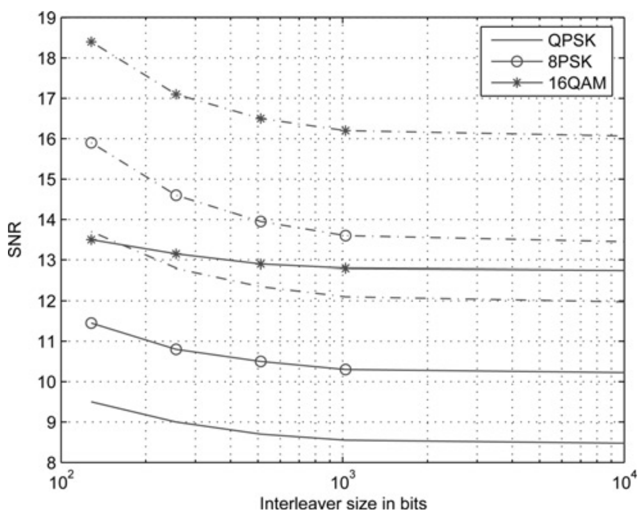

Figure 8 *SNR required to achieve a BER of 10^{-6} against the interleaver size for convolutionally encoded BI-STBC and BI-STCM systems employing $n_T = 2$ with different modulation schemes under Rayleigh fading (solid: BI-STBC, dashed: BI-STCM)*

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employing $n_T = 2$ with different modulation schemes under Rayleigh fading. We can see that the effect of the interleaver size is more pronounced in BI-STCM systems than in BI-STBC systems. This is intuitive as the diversity order of BI-STCM is smaller and thus is more in need of the time diversity provided by bit interleaving, which increases with the increasing interleaver size. Therefore the required SNR of BI-STCM has more dramatic reduction as the interleaver size increases.

7 Conclusions

In this article, we propose a new union bound on the bit error probability of BI-ST coded systems employing finite interleaver sizes over block fading channels. The derivation is based on the uniform interleaving of coded bits prior to ST mapping. Tight BER bounds have been derived for BI-STCM and BI-STBC systems under different fading distributions. The proposed BER analysis also provides a novel analytical framework to evaluate the effect of the interleaver size on the performance of BI-ST coded systems. Results show that the proposed bound is tight in medium to high SNR regions. The bound also accurately characterises the performance of different interleaver sizes which is a major breakthrough in the analysis of BI-ST coded systems.

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