PERFORMANCE ANALYSIS OF BINARY CODED SYSTEMS OVER RICIAN BLOCK FADING CHANNELS

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ABSTRACT

This paper considers the performance analysis of binary coded systems over Rician block fading channels. In the transmitter, the coded bits are interleaved prior to transmission to spread burst errors resulting from deeply faded blocks. The union bound on the bit error probability (BEP) is derived assuming uniform interleaving prior to transmission over the channel and using the weight enumerator of the code. To compute the union bound, the pairwise error probability (PEP) is derived for coherent detection with perfect and imperfect channel side information (SI) at the receiver. The bound is evaluated for convolutional codes with different values of specular power and results show that the bound predicts the performance degradation due to channel memory. Channel estimation/diversity tradeoff is investigated assuming imperfect SI.

I. INTRODUCTION

Radio communications suffer mainly from multipath fading. The fading distribution varies according to the environment. Rician fading distribution [1] arises if a line-of-site exists between the transmitter and receiver in addition to the multipath reception. In Rician channels the received signal is composed of two signal-dependent components; namely the specular and diffuse components. The specular component is due to the line-of-site reception and the diffuse component results from multipath reception. As the ratio of specular-to-diffuse component energy increases, the channel approaches the Gaussian channel, i.e., no fading.

Error correcting codes and diversity techniques are standard approaches to mitigating the fading. We define the *channel diversity* as the number of independent fading realizations available for a codeword. The performance of coded systems over infinitely interleaved fading channels is commonly analyzed using the union bound as in [2], [3]. The performance of diversity reception over Rician channels with noncoherent detection was derived by Jacobs [4]. In delay-sensitive applications infinite interleaving becomes an impractical assumption. Therefore, channel models that exhibit memory are needed to model wireless systems. The *block fading* channel [5] provides an acceptable model for many wireless communication systems including frequency-hopped spread-spectrum (FH-SS) [6], time-division multiplexing (TDM) and orthogonal-frequency division multiplexed (OFDM) systems. In this model, a frame undergoes a number of independent fading realizations, each affecting a number of consecutive signals. Recently in [7], a union bound was derived for binary coded systems over Rayleigh block fading channels with different receivers.

In this paper we consider the performance analysis of binary codes over Rician block fading channels. If a coherent receiver is used, the phase of the fading process is needed for demodulation. In general, *channel side information* (SI) is defined as the phase and amplitude of the fading process. If the receiver knows the channel SI perfectly, large channel diversity improves the system performance resulting in an optimal channel memory of unity. On the other hand, if the receiver must estimate the channel, long channel memory provides more observations for each fading realization which permits a better channel estimation. Therefore, longer channel memory improves the performance if the frame size is infinite. However, if the frame size is finite, there exists a fundamental tradeoff between the channel diversity and channel estimation [8]. As the channel memory length increases, the channel diversity is reduced but the channel estimation becomes easier since more observations of each fading realization is available. On the converse, short channel memory increases the number of independent fading realizations available to

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the decoder, and hence it is able to average out the channel behavior at the cost of less accurate channel estimation.

In this paper, the union bound for binary coded systems over Rician block fading channels is derived. Expressions for PEP are derived for coherent detection with perfect and imperfect channel SI at the receiver. Moreover, the effect of channel estimation and channel diversity is investigated for different Rician channels. The paper starts with describing the system model in Section II. The union bound is reviewed in Section III and the PEP is derived in Section IV. Conclusions are presented in Section V.

II. System Model

The block diagram of the system is shown in Figure 1. In each transmission interval of length kT, the encoder maps k information bits into n coded bits, resulting in a code rate of $R_c = \frac{k}{n}$. Each coded bit is modulated to a signal from a binary unit-energy constellation. The frame of signals S of length N is transmitted over a block fading channel with F fading blocks, where a block of length $m = \lceil \frac{N}{F} \rceil$ signals undergoes the same fading realization. Due to deeply faded blocks bursts of low instantaneous signal-to-noise ratio (SNR) occur at the demodulator output. Therefore the coded bits are interleaved to spread the bursts in the decoder. The demodulator output at time l in the f^{th} fading block is

$$y_{f,l} = \sqrt{E_s} h_f s_{f,l} + z_{f,l}, \tag{1}$$

where E_s is the average received energy and $z_{f,l}$ is an additive noise modeled as independent zero-mean complex Gaussian random variables with variance N_0, i.e., $\mathcal{CN}(0, N_0)$. The variable h_f is the channel gain in block f and is modeled as complex Gaussian with $\mathcal{CN}(b, 1)$, where b represents the specular component of the channel. Hence, it can be written as $h_f = a_f \exp(j\theta_f)$, where $j = \sqrt{-1}$, the phase θ_f is uniformly distributed on $[0, 2\pi]$ and the amplitude a_f has a Rician distribution. The density function for a normalized Rician random variable is

$$f_a(a) = 2a(1+K)\exp\left[-K - a^2(1+K)\right] \times I_0\left(2a\sqrt{K(1+K)}\right), \qquad a \geq 0, \tag{2}$$

where $K = b^2$ is the energy of the specular component and $I_0(.)$ is the zero-order modified Bessel function of the first kind. In this context, K denotes the ratio of the specular component energy to the diffuse component energy.

The receiver employs maximum likelihood (ML) decoding for the coherent detection case. In this paper, a coherent receiver is assumed to have either perfect or imperfect channel SI. If perfect SI is available the decoder chooses the codeword S that maximizes the metric

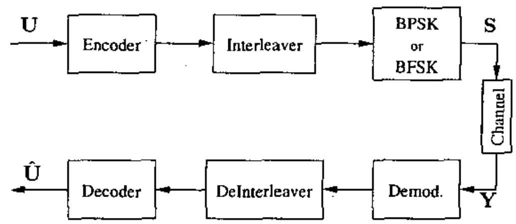

Fig. 1. The structure of the binary coded system.

$$\mathbf{m}(\mathbf{Y}, \mathbf{S}) = \sum_{f=1}^{F} \sum_{l=1}^{m} \mathrm{Re}\{y_{f,l}^* h_f s_{f,l}\}, \tag{3}$$

where Re{.} represents the real part of a complex number. The union bound for block fading channels is presented in the following.

III. The Union Bound

In this section, a union bound on the bit error probability of convolutional codes over block fading channels is derived. Throughout the paper, the subscripts c, u and b are used to denote conditional, unconditional and bit error probabilities, respectively. For linear convolutional codes with k input bits, the bit error probability is upper bounded [2] as

$$P_b \leq \frac{1}{k} \sum_{d=d_{\min}}^{N} w_d P_u(d), \tag{4}$$

where $d_{\min}$ is the minimum distance of the code, $P_u(d)$ is the unconditional pairwise error probability defined as the probability of decoding a received sequence as a weight-d codeword given that the all-zero codeword is transmitted. In (4), w_d is the number of codewords with output weight d, which is obtained from the weight enumerator of the code [2].

In block fading channels the pairwise error probability $P_u(d)$ is a function of the distribution of the d nonzero bits over the F fading blocks. This distribution is quantified assuming uniform channel interleaving of the coded bits over the fading blocks. Denote the number of fading blocks with weight v by f_v and define $w = \min(m, d)$, then the fading blocks are distributed according to the pattern $\mathbf{f} = \{f_v\}_{v=0}^{w}$ if the following conditions are satisfied

$$F = \sum_{v=0}^{w} f_v, \qquad d = \sum_{v=1}^{w} v f_v. \tag{5}$$

Denote by $L = F - f_0$ the number of fading blocks with nonzero weights. Then $P_u(d)$ averaged over all possible fading block patterns is given by

$$P_u(d) = \sum_{L=\lceil d/m \rceil}^{d} \sum_{f_1=0}^{L_1} \sum_{f_2=0}^{L_2} \cdots \sum_{f_w=0}^{L_w} P_u(d|\mathbf{f})p(\mathbf{f}), \quad (6)$$

where

$$L_v = \min\left\{ L - \sum_{r=1}^{v-1} f_r, \frac{d - \sum_{r=1}^{v-1} r f_r}{v} \right\}, \qquad 1 \le v \le w. \quad (7)$$

The probability of a fading block pattern $p(\mathbf{f})$ is computed using combinatorics as

$$p(\mathbf{f}) = \frac{\binom{m}{1}^{f_1}\binom{m}{2}^{f_2}\cdots\binom{m}{w}^{f_w}}{\binom{mF}{d}} \cdot \frac{F!}{f_0! f_1! \cdots f_w!}. \quad (8)$$

The left factor of $p(\mathbf{f})$ in (8) is the probability of distributing d nonzero bits over F blocks with f_v blocks having v bits, for possible values of v. The right term of $p(\mathbf{f})$ is the probability of having such combinations $\mathbf{f} = \{f_v\}_{v=0}^{w}$ among the F fading blocks. The union bound on the bit error probability of convolutional codes over block fading channels is found by substituting (6) in (4) and using (6)-(8). The number of summations involved in computing $P_u(d)$ in (6) increases as the channel memory length increases. A good approximation to the union bound is obtained by truncating it for a small value of $d < N$. This results in an approximation to the error probability rather than an upper bound. Expressions for $P_u(d|\mathbf{f})$ for coherent detection with different SI assumptions are derived in the next section.

IV. Pairwise Error Probability

In this section, we derive the PEP of binary codes over Rician block fading channels for coherent receivers. The PEP is defined as the probability of decoding a received sequence erroneously as a codeword $\hat{\mathbf{S}}$ given that the all-zero codeword $\mathbf{S}$ was transmitted. The PEP conditioned on the fading amplitudes and for a fading block pattern $\mathbf{f}$ is given by

$$P_c(d|\mathbf{f}) = \Pr\left(\mathbf{m}(\mathbf{Y},\mathbf{S}) - \mathbf{m}(\mathbf{Y},\hat{\mathbf{S}}) < 0 \middle| \mathbf{H},\mathbf{S}\right), \quad (9)$$

where $\hat{\mathbf{S}}$ is a codeword with weight d and $\mathbf{H} = \{h_f\}_{f=1}^{F}$. The cases of perfect and imperfect SI are considered in the following.

A. Perfect SI

Recall that the received signal over a block fading channel is given by (1) and the corresponding ML decoding rule is given by (3). Substituting the metric (3) in (9), the conditional pairwise error probability for coherent detection with perfect SI is given by

$$P_c(d|\mathbf{f}) = \Pr\left(\sum_{f=1}^{L} a_f \sum_{l=1}^{m} \mathrm{Re}\{y_{f,l}\} < 0 \middle| \mathbf{H},\mathbf{S} \right). \quad (10)$$

The distribution of $\mathrm{Re}\{y_{f,l}\}$ conditioned on h_f is complex Gaussian with mean $\sqrt{E_s} h_f s_{f,l}$ and variance N_0. The conditional pairwise error probability simplifies to

$$P_c(d|\mathbf{f}) = \mathrm{Q}\left(\sqrt{2R_c\gamma_b \sum_{v=1}^{w} v \sum_{i=1}^{f_v} a_i^2} \right), \quad (11)$$

where $\gamma_b = \frac{E_b}{N_0}$ is the SNR per information bit. Note that the average energy per bit is given by $E_b = R_c E_s$, where R_c is the encoder rate. To find $P_u(d|\mathbf{f})$, (11) is averaged over the fading amplitudes $\mathbf{A} = \{a_f\}_{f=1}^{F}$. An exact expression of the pairwise error probability can be found by using the integral expression of the Q-function, $\mathrm{Q}(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{(-x^2/2\sin^2\theta)} d\theta$ [9]

$$\begin{aligned} P_u(d|\mathbf{f}) &= \frac{1}{\pi}\mathrm{E}_{\mathbf{A}}\left[\int_0^{\frac{\pi}{2}} \exp\left(\frac{R_c\gamma_b}{\sin^2\theta} \sum_{v=1}^{w} v \sum_{i=1}^{f_v} a_i^2 \right) d\theta \right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left(\frac{1}{1 + vR_c\gamma_b/\sin^2\theta} \right)^{f_v} d\theta. \end{aligned} \quad (12)$$

The union bound was evaluated for a rate-$\frac{1}{2}$ convolutional code with generators (23,35) and a frame size of $N = 2 \times 512$ coded bits. Note that the convolutional code has 4 memory elements. As discussed in Section III, the union bound is truncated to reduce computation complexity. For the results in this paper the union bound was truncated after codewords with distances $d > 12$. According to [7], the bound was shown to be tight to the simulation results for Rayleigh fading channels. Hence, only analytical results are shown in this paper.

Figure 2 shows the SNR required to achieve bit error rate of $P_b = 10^{-4}$ plotted versus the specular-to-diffuse ratio K of the Rician channel. We observe that the SNR degradation due to longer channel memory increases as the specular-to-diffuse ratio decreases. Thus increasing the energy of the channel line-of-site component reduces the effect of the diversity provided by the independent fading blocks. This is expected since increasing K causes the energy of the specular component to increase and the channel approaches the "no fading" scenario, where diversity becomes less important.

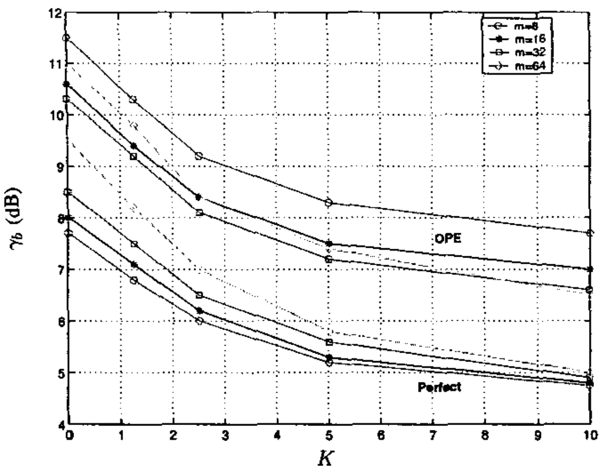

Fig. 2. SNR required for a (23,35) convolutional code to achieve $P_b = 10^{-4}$ versus the specular-to-diffuse ratio K (linear scale) for memory lengths $m = 8, 16, 32, 64$.

B. Imperfect SI

For coherent detection with imperfect SI it is necessary to estimate the channel SI. This is achieved by transmitting a pilot signal with energy E_p in each fading block. The corresponding received signal is given by

$$y_{f,p} = \sqrt{E_p} h_f + z_{f,p}. \tag{13}$$

The ML estimator for h_f is given by $\hat{h}_f = \frac{y_{f,p}}{\sqrt{E_p}} = h_f + e_f$, where $e_f = \frac{z_{f,p}}{\sqrt{E_p}}$ is the estimation error. The distribution of e_f is $\mathcal{CN}(0, \sigma_e^2)$ where $\sigma_e^2 = \frac{N_0}{E_p}$. The correlation coefficient between the true channel gain and its estimate is given by

$$\mu = \frac{\mathrm{E}[(h_f - b)(\hat{h}_f - b)^*]}{\sqrt{\mathrm{Var}(h_f)\mathrm{Var}(\hat{h}_f)}} = \frac{1}{\sqrt{1+\sigma_e^2}}. \tag{14}$$

In order implement a ML decoding rule, the likelihood function of the channel observations (received and pilot signals) conditioned on the transmitted codeword should be maximized. In [7] the ML decoding rule was shown to be difficult to implement in a Viterbi receiver. Therefore, a suboptimal decoding metric that maximizes the likelihood function $p(\mathbf{Y}|\hat{\mathbf{H}}, \mathbf{S})$ is used. It is given by

$$\mathbf{m}(\mathbf{Y}, \mathbf{S}) = \sum_{f=1}^{F} \sum_{l=1}^{m} \mathrm{Re}\{y_{f,l}^* \hat{h}_f s_{f,l}\}. \tag{15}$$

TABLE I

RATES, MINIMUM DISTANCES AND PUNCTURING PATTERNS OF THE PUNCTURED RATE-$\frac{1}{2}$ CODES.

m	Code Rate R_c	Puncturing Location	$d_{\min}$
4	0.667	3	4
8	0.571	7	5
16	0.533	15	6
32	0.516	31	6
64	0.508	63	6

The conditional pairwise error probability for the suboptimal decoder is given by

$$P_c(d|\mathbf{f}) = \Pr\left(\sum_{f=1}^{L} \sum_{l=1}^{m} \mathrm{Re}\{y_{f,l}^* \hat{h}_f\} < 0 \middle| \hat{\mathbf{H}}, \mathbf{S}\right). \tag{16}$$

The received signal $y_{f,l}$ conditioned on $\hat{h}_f$ is a complex Gaussian random variable with mean $\sqrt{E_s} s_{f,l} \mathrm{E}[h|\hat{h}]$ and variance $N_0 + (1-\mu^2)E_s$, where $\mathrm{E}[h|\hat{h}] = \frac{\mu}{\sigma}(\hat{h}_f - b) + b$. Thus the conditional pairwise error probability for the suboptimal decoder is given by

$$P_c(d|\mathbf{f}) = \mathrm{Q}\left(\sqrt{\frac{2E_s \sum_{f=1}^{F} d_f \left|\frac{\mu}{\sigma}(\hat{h}_f - b) + b\right|^2}{N_0 + (1-\mu^2)E_s}}\right), \tag{17}$$

where d_f is the number of nonzero error bits in fading block f. Define the normalized complex Gaussian random variable $\zeta_f = \frac{\hat{h}_f - b}{\sigma} + \frac{b}{\mu}$ with distribution $\mathcal{CN}(\frac{b}{\mu}, 1)$. Then, the conditional pairwise error probability becomes

$$P_c(d|\mathbf{f}) = \mathrm{Q}\left(\sqrt{\frac{2\mu^2 R_c \gamma_b \sum_{v=1}^{w} v \sum_{i=1}^{f_v} |\zeta_i|^2}{1 + R_c \gamma_b (1-\mu^2)}}\right), \tag{18}$$

where ζ_f is as defined above. Therefore, the pairwise error probability for the case of imperfect SI is the same as that of perfect SI, by replacing γ_b by $\hat{\gamma}_b = \frac{\mu^2 \gamma_b}{1 + R_c \gamma_b (1-\mu^2)}$ and K by $\frac{K}{\mu^2}$.

For systems employing pilot-aided channel estimation, the energy of the pilot signal is taken into account in the SNR axis. Furthermore, one coded bit is punctured to maintain the same transmission rate for systems with

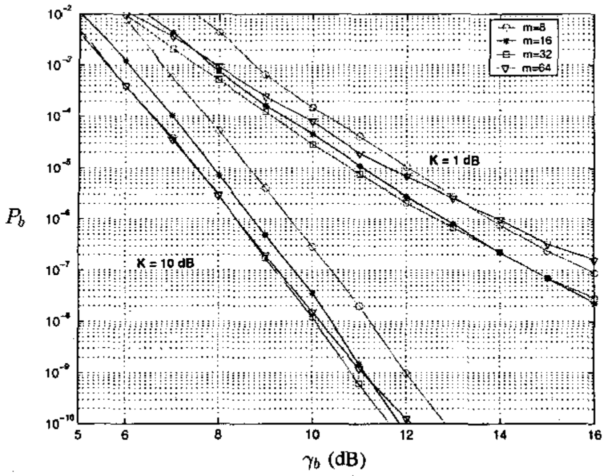

Fig. 3. Approximation of the bit error probability of a (23,35) convolutional code over a Rician fading channel with $K = 1, 10$ dB, imperfect SI (OPE receiver) and a frame size $N = 1024$ for memory lengths $m = 8, 16, 32, 64$.

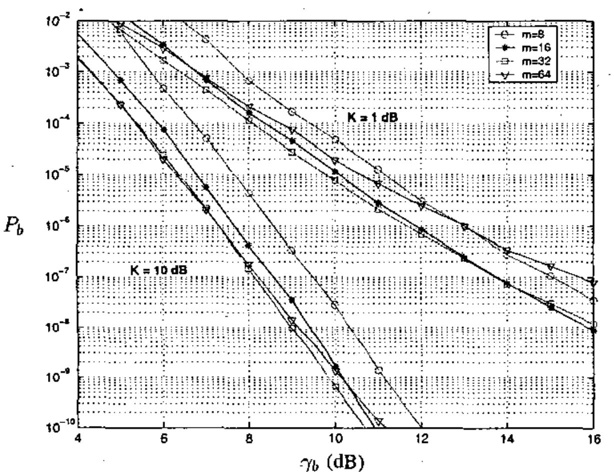

Fig. 4. Approximation of the bit error probability of a (23,35) convolutional code over a Rician fading channel with $K = 1, 10$ dB, imperfect SI (CDE assumption) with $E_p = E_s$ and a frame size $N = 1024$ for memory lengths $m = 8, 16, 32, 64$.

different channel memory. The resultant code rate after puncturing one bit every fading block is given by

$$\bar{R}_c = \frac{mR_c}{m-1}. \tag{19}$$

In Table I we show the code rates and the minimum distances of the punctured codes for different channel memory lengths. Also, the location of the punctured coded bit in a m-length fading block. According to the table, the code rate increases with reducing the channel memory length, which decreases the error correcting capabilities of the code. Thus systems with short channel memory are expected to have more channel diversity at the cost of lower minimum distance and worse channel estimation quality. On the other hand, longer channel memory results in more a powerful code as well as better channel estimation at the cost of less channel diversity.

Two scenarios can be considered for channel estimation using pilots with $E_p = E_s$. The first one results from only pilot estimation (OPE) with an estimation error variance of $\sigma_e^2 = \frac{N_0}{E_s}$. The second case considers a lower bound on the performance of receivers employing iterative joint decoding and channel estimation. In such receivers the decoding results are used to improve the channel estimates, which are used to improve the decoding results. This process is repeated iteratively. In general, the more reliable the decoding results, the more accurate is channel estimation. A lower bound on the performance of iterative receivers is obtained if the signals in each fading block are known with probability one. In this case they can be considered as pilots resulting in an estimation error variance of $\sigma_e^2 = \frac{N_0}{mE_s}$. This case is referred to as correct data estimation (CDE).

From Figure 2, the optimal channel memory value for an OPE receiver with $E_p = E_s$ is $m = 32$ for a Rayleigh fading channel, i.e. $K = 0$, whereas it is $m = 64$ for a Rician channel with $K = 10$. Also, note that the case of $m = 8$ outperforms the case of $m = 64$ when the channel is more fading, where the reverse occurs for channels that are less faded, i.e., larger values of K. Figures 3 and 4 show the results of imperfect SI with an OPE receiver and the CDE assumptions, respectively. The gain loss in SNR due to the channel memory is less compared to the case of perfect SI. Also, we observe that systems with long channel memory perform better as the energy of the specular component of the channel increases. This is because as K increases the channel becomes less fading which reduces the need for the decoder to average over the statistics of the channel. Therefore, the channel diversity becomes less crucial causing in systems with long channel memory to outperform systems with short memory. Another reason for this is the larger energy fraction spent on pilot signals in systems with short channel memory lengths than in systems with long memory. This is obvious for the case of $K = 10$ dB, where the performance of $m = 64$ is nearly optimal for most of the SNR values. On the other hand, the case of $m = 8$ is the worst every where when $K = 10$ dB, where it outperforms the case of $m = 64$ when $K = 1$ dB.

Figure 5 shows a comparison of systems with channel memory lengths $m = 8$ and $m = 32$. The SNR degradation due to channel estimation reduces as the channel memory increases. Moreover, the SNR loss in OPE receivers with long channel memory increases with increased energy of

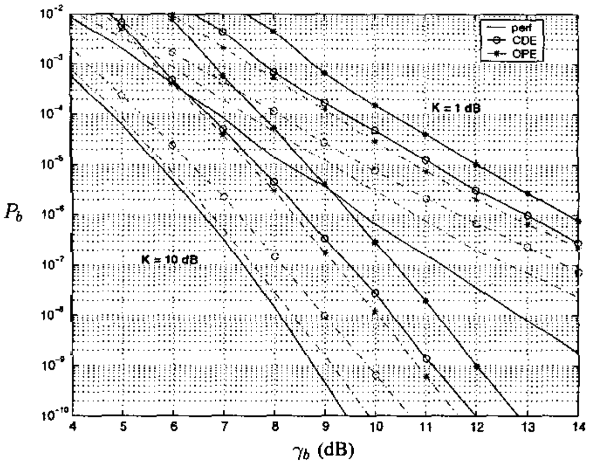

Fig. 5. Approximation of the bit error probability of a rate-$\frac{1}{2}$ (23,35) convolutional code over a Rician fading channel with $K = 1, 10$ dB, frame size $N = 1024$ and memory lengths $m = 8, 32$ using perfect and imperfect SI with $E_p = E_s$ (solid: $m = 8$, dash: $m = 32$).

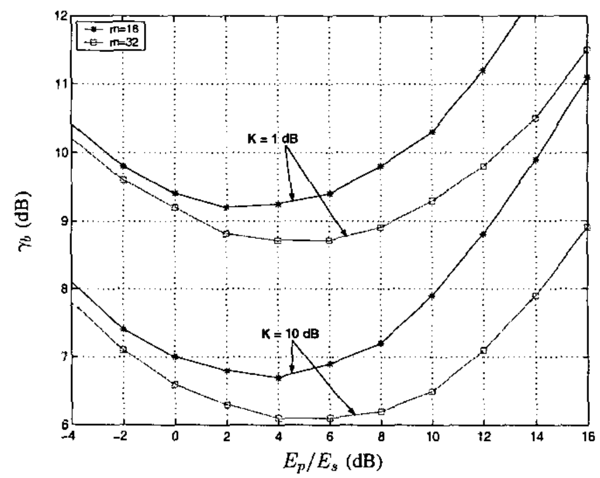

Fig. 6. SNR required for a rate-$\frac{1}{2}$ (23,35) convolutional code to achieve $P_b = 10^{-4}$ versus E_p/E_s for the OPE receiver with memory lengths $m = 16, 32$.

the specular component of the channel. When the channel is estimated using a pilot signal, the channel estimation error adds a fading component to the channel gain at the decoder. The effect of this new fading component increases as the energy of the specular component increases of the channel, which degrades the performance of OPE receivers more as K increases.

The energy allocated for the pilot signal is optimized as shown in Figure 6. We observe that the optimal pilot energy allocation is almost independent of the fading nature of the channel, i.e., independent of the energy of the specular component K of the channel. This is because the amount of energy available in each fading block, which can be used in estimating the channel, is the controlling factor of the optimal pilot energy allocation. Clearly, this energy amount is a function of the channel memory length only. Also, the SNR gain resulting from optimizing the pilot energy is almost independent of the channel fading behaviour.

V. Conclusions

In this paper, the union bound for binary coded systems over Rician block fading channels was derived. Coherent detection was considered with perfect and imperfect SI at the receiver, and the corresponding PEP were derived. The bound was evaluated for convolutional codes and results show that the bound predicts the trend of the code performance with channel memory. Also, it was observed that the effect of channel memory reduces with increasing the power of the channel specular component. The tradeoff between channel estimation and effective diversity was investigated, where it was shown that the optimal channel memory increases as the channel becomes less fading.

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