Performance of Coded Systems with Noncoherent Detection over Block Fading Channels

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***Abstract*— This paper considers the performance analysis of binary coded systems over block fading (BF) channels with noncoherent detection. In the transmitter, the coded bits are interleaved to spread burst errors resulting from deeply faded blocks. The union bound is derived assuming uniform interleaving and using the weight enumerator of the code. The proposed bound is evaluated for convolutional and turbo codes and results show that the bound provides insight on the gain loss due to channel memory.**

I. Introduction

Radio communications suffer mainly from fading of the received signal. Diversity is a techniques to mitigate this problem. Error control codes provide a form of time diversity. In BF channels [1], a frame of length N is affects by F independent fading channels, where each block of $m = \lfloor \frac{N}{F} \rfloor$ bits undergo the same fading realization. The performance of coded systems over independent fading channels is commonly analyzed using the union bound and the weight enumerator of the code as in [2]. Similar bounds were derived in [3] for turbo codes with coherent and noncoherent detection. In [4], the union bound for coherent coded systems over BF channels was derived. In this paper, the work in [4] is extended to noncoherent detection with no side information. The paper starts with the system model in Section II. In Section III, the performance is analyzed and the results are discussed. Finally, conclusions are presented in Section IV.

II. System Model

The system considered in this paper is the binary coded system in [4] with noncoherent detection. In each transmission interval, the encoder maps k information bits into n coded bits, resulting in a code rate of $R_c = \frac{k}{n}$. Each coded bit is modulated using BFSK and the frame of signals $\{s_{f,1}, ..., s_{f,m}\}_{f=1}^{F}$ is transmitted over a BF channel. Due to deeply faded blocks, bursts errors occur and therefore the coded bits are interleaved after the encoder. The received signal at time l in the f^{th} fading block is

$$y_{f,l} = \sqrt{E_s}\alpha_f s_{f,l} + \eta_{f,l}, \tag{1}$$

where E_s is the average received energy, $\eta_{f,l}$ is an additive white Gaussian noise modeled as $\mathcal{CN}(0, \sigma_\eta^2)$, and α_f is the channel gain in the block f and is modeled as $\mathcal{CN}(0, 1)$. Hence, it can be written as $\alpha_f = a_f \exp(j\theta_f)$, where a_f has a Rayleigh distribution and θ_f is uniformly distributed on $[0, 2\pi]$. The receiver employs a square-law detector [5] whose output can be written as

$$\begin{aligned}
X_{f,l}^{(c,0)} &= \sqrt{E_s} a_f \delta(s_{f,l}, 0) \cos(\theta_f) + \eta_{f,l}^{(c,0)} \\
X_{f,l}^{(s,0)} &= \sqrt{E_s} a_f \delta(s_{f,l}, 0) \sin(\theta_f) + \eta_{f,l}^{(s,0)} \\
X_{f,l}^{(c,1)} &= \sqrt{E_s} a_f \delta(s_{f,l}, 1) \cos(\theta_f) + \eta_{f,l}^{(c,1)} \\
X_{f,l}^{(s,1)} &= \sqrt{E_s} a_f \delta(s_{f,l}, 1) \sin(\theta_f) + \eta_{f,l}^{(s,1)},
\end{aligned} \tag{2}$$

where $X_{f,l}^{(c,z)}$ and $X_{f,l}^{(s,z)}$ correspond to the case of $s_{f,l} = z$, where $z = 0$ or 1. In (2), θ_f is the unknown phase of the channel in block f, $\delta(x, y) = 1$ if $x = y$ and $\delta(x, y) = 0$ otherwise; and $\eta_{f,l}^{(c,0)}$, $\eta_{f,l}^{(s,0)}$ $\eta_{f,l}^{(c,1)}$ and $\eta_{f,l}^{(s,1)}$ are independent variables with $\mathcal{N}(0, \sigma_\eta^2)$ distribution. The decoder chooses the codeword $\mathbf{z}$ that maximizes

$$\mathbf{m}(\mathbf{y}, \mathbf{s}) = \sum_{f=1}^{F} \sum_{l=1}^{m} (X_{f,l}^{(c,z)})^2 + (X_{f,l}^{(s,z)})^2 \tag{3}$$

This decoder makes no use of the side information $\{\alpha_f\}_{f=1}^{F}$ in decoding. The union bound is derived in the next section.

III. Performance Analysis

A. The Union Bound

The bit error probability (BEP) of a linear convolutional code with k input bits is upper bounded [5] as

$$P_b \leq \frac{1}{k} \sum_{d=d_{free}}^{N} w_d P_u(d), \tag{4}$$

where d_{free} is the free distance of the code, $P_u(d)$ is the pairwise error probability (PEP), defined as the probability of decoding in favor of a codeword with weight d when the all-zero codeword is transmitted. For convolutional codes, w_d is the number of codewords with output weight d, which is obtained directly from the weight enumerator of the code [5]. For turbo codes with interleaver size $\tilde{N} = R_c N$, the BEP is upper bounded as in [6]

$$P_b \leq \sum_{i=1}^{\tilde{N}} \frac{i}{\tilde{N}} \binom{\tilde{N}}{i} \sum_{d=d_{free}}^{\tilde{N}} p(i, d) P_u(d), \tag{5}$$

where $p(i, d)$ is the probability of having an encoder input sequence with weight i and an output codeword with weight d. For a turbo code with two component codes, $p(i, d)$ is given by

$$p(i, d) = \sum_{d_0, d_1, d_2 : d_0 + d_1 + d_2 = d} p_0(i, d_0) p_1(i, d_1) p_2(i, d_2), \tag{6}$$

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where $p_0(i,d_0) = \delta(i,d_0)$ represents the systematic bit; and $p_j(i,d_j) = A_{i,d_j}/\binom{\tilde{N}}{i}$ for $j=1,2$, accounts for the interleaver.

For BF channels, $P_u(d)$ is a function of the distribution of the d nonzero bits among the F fading blocks, which can be quantified by assuming uniform interleaving. Denote the number of fading blocks with weight v by f_v and $w = \min(m,d)$, then the pattern $\mathbf{f} = \{f_v\}_{v=0}^{w}$ occurs if

$$F = \sum_{v=0}^{w} f_v, \qquad d = \sum_{v=1}^{w} v f_v. \tag{7}$$

By averaging over possible patterns,

$$P_u(d) = \sum_{f_1=1}^{F} \sum_{f_2=1}^{F/2} ... \sum_{f_w=1}^{F/w} P_u(d|\mathbf{f}) p(\mathbf{f}). \tag{8}$$

Here, $p(\mathbf{f}) = \frac{n(\mathbf{f})}{\sum_{\mathbf{f}} n(\mathbf{f})}$, where $n(\mathbf{f})$ is the number of occurrence of the pattern $\mathbf{f}$, which depends on the number of combinations of $\{f_v\}_{v=0}^{w}$ among the F blocks, and the number of ways the bits can be ordered in the pattern. Using combinatorics,

$$n(\mathbf{f}) = \frac{F!}{f_0! f_1! ... f_w!} \cdot \frac{d!}{(2!)^{f_2} (3!)^{f_3} ... (w!)^{f_w}}. \tag{9}$$

The probability $P_u(d|\mathbf{f})$ for noncoherent detection is derived in the following.

B. Pairwise Error Probability (PEP)

The conditional PEP for decoding in favor of a codeword $\mathbf{z}$ when the all-zero codeword is transmitted [5] is

$$P_c(d) = P\Big(\sum_{f=1}^{F} d_f \big(|\eta_f^{(1)}|^2 - |2E_s\alpha_f + \eta_f^{(0)}|^2\big) > 0\Big). \tag{10}$$

Define $g = \sum_{f=1}^{F} d_f (|\eta_f^{(1)}|^2 - |2E_s\alpha_f + \eta_f^{(0)}|^2) = \sum_{v=1}^{w} v \sum_{i=1}^{f_v} (|\eta_i^{(1)}|^2 - |2E_s\alpha_i + \eta_i^{(0)}|^2)$. Then, the unconditional PEP is found by averaging over the pdf of g as

$$P_u(d|\mathbf{f}) = \int_0^\infty f_g(g) dg \le E\left[e^{\lambda g}\right], \tag{11}$$

where the Chernoff bound was invoked in the last step. Substituting for g results in

$$P_u(d|\mathbf{f}) \le \prod_{v=1}^{w} E\left[e^{\lambda v |\eta_v^{(1)}|^2}\right]^{f_v} E\left[e^{-\lambda v |2E_s\alpha_v + \eta_v^{(0)}|^2}\right]^{f_v}. \tag{12}$$

As in [5], the Chernoof parameter λ can be optimized. The resulting Chernoff bound for the PEP of noncoherent detection over BF channels is

$$P_u(d|\mathbf{f}) \le \prod_{v=1}^{w} \left[4D_v(1-D_v)\right]^{f_v}, \tag{13}$$

where $D_v = \frac{1}{2+vR_c\gamma_b}$. The proposed union bound was evaluated for the (5,7) convolutional code with a frame size $N=200$, and the (1,5/7,5/7) turbo code with an interleaver size $\tilde{N}=100$. In Figures 1 and 2, it is clear that the bound is tight to the simulation. Also, the bound shows the loss in performance due to the "unexploited" memory in the channel.

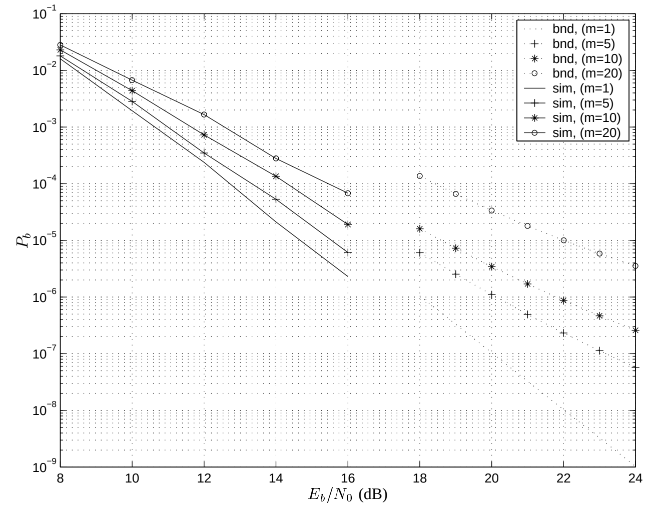

Fig. 1. Performance of the (5,7) convolutional code with $N = 200$.

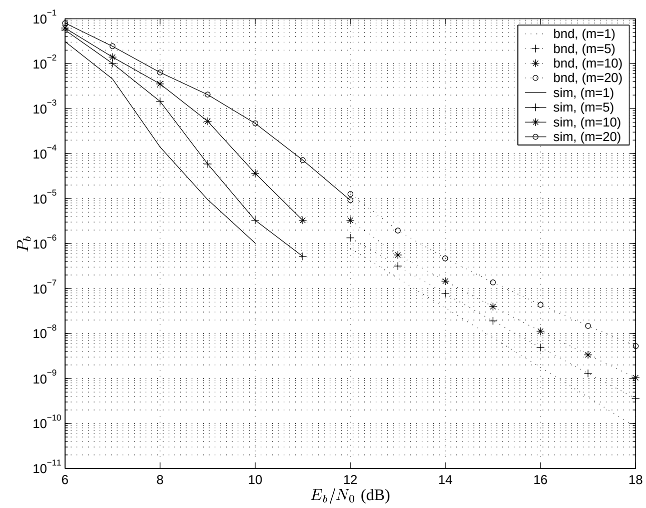

Fig. 2. Performance of the (1,5/7,5/7) turbo code with $\tilde{N} = 100$.

IV. CONCLUSIONS

In this paper, the union bound of noncoherently detected binary coded systems over BF channels without channel information at the decoder was derived and the Chernoff bound on the PEP was derived. The proposed bound was evaluated for convolutional and turbo codes. Results show that the proposed bound provides the trend of the code performance with channel memory.

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