AN ALGORITHM FOR ITERATIVE DECODING AND CHANNEL ESTIMATION OF SPACE-TIME CODED FH SYSTEMS

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Abstract— This paper proposes an algorithm for joint decoding and channel estimation (JDE) of frequency-hopping (FH) system employing space-time (ST) codes. The channel is modeled as a block fading channel, where the fade is constant during each hop and independent from other hops. The algorithm is based on quantizing the fading process, such as done in Markov fading models. The algorithm is semiblind, where initial estimation of the channel is obtained using orthogonal pilot sequence insertion (OPSI). Unlike existing JDE algorithms, the algorithm exploits soft information from the decoder to improve the channel estimation iteratively. The performance of the algorithm approaches the performance of the system where all signals in the frame are assumed to be known with probability one, i.e. acting as pilots.

I. INTRODUCTION

Wireless links suffer mainly from multipath fading channels, which cause the transmitted signal to be attenuated randomly. If the coherence time of the channel is comparable to the packet length, the packet may fall in a deep fading interval causing the performance to degrade severely. In [1,2], it was shown that using multiple antennas at the transmitter and receiver improves the system capacity significantly in the presence of block fading channels. The performance and design criteria of ST codes were derived in [3] for fading channels with perfect channel state information (CSI). The case of imperfect CSI was considered in [4], were it was shown that the performance of ST codes is sensitive to estimation errors, especially when large number of transmit antennas are used.

In general, the code's effective diversity reduces with increasing the hop length, where the channel estimation improves. The tradeoff between the channel estimation and the code's performance was investigated in [5] for singleantenna systems. A semiblind channel estimation scheme for ST coded OFDM systems was considered in [6] using a least-square estimator and a decoder, with no information exchanged between the decoder and the estimator. In [7], a semiblind iterative JDE was proposed for quasi-static channels, where hard decisions from the decoder are used to update the channel estimates. A maximum likelihood (ML) sequence estimation employing expectation-maximization (EM) algorithm was proposed in [8] for correlated fading channels. In [9], a maximum aposteriori version of the EM algorithm, which is suitable for turbo-like codes, was derived for JDE of ST coded OFDM systems. This algorithm makes hard decisions in the demodulator, and use these decisions in updating the channel estimates. A different strategy to blind iterative JDE for turbo-coded FH systems was proposed in [10], where the fading process is quantized and probabilities of each quantization level is updated iteratively using soft information from the decoder. This algorithm can be extended for the case when the channel is modeled as a Markov process, which was applied in [11] to the phase process.

In this paper, the FH system employing ST codes is considered and a semiblind iterative JDE algorithm is proposed. The algorithm uses OPSI to get initial estimates of the channel, and then improves it by exploiting the soft information from the decoder. Also, the optimal hop length is investigated for different coding schemes in the literature. In Section II, the model of the FH ST coded system is described. Section III reviews the channel estimation in ST coded systems using OPSI. Then, the proposed iterative JDE algorithm is derived. In Section IV, simulation results are presented and conclusions are discussed in Section V.

II. SYSTEM MODEL

The block diagram of the FH system employing ST codes is shown in Figure 1. The transmitter consists of an encoder, interleaver, a ST modulator and n_t transmit antennas. The input to the transmitter is a sequence U of input bits, which are encoded and the coded sequence is interleaved bit-wise

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or symbol-wise depending on the code. In this paper, I-Q ST codes [12], and turbo ST codes [13] are used as illustrative examples. The ST modulator collects each n_t signals in a vector, resulting in a length N sequence S of signal vectors $\{s_l\}_{l=1}^N$. The i^{th} element in the signal vector s_l^i is drawn from a 2-D signal constellation, such as MPSK or M-QAM, and transmitted using the i^{th} transmit antenna in the time interval l.

Each frame is transmitted over F independent hops, where the independence is achieved if the frequencies of the hops are separated by an amount larger than the coherence bandwidth of the channel. The interleaver in the transmitter is used to break the burst errors resulting from deeply faded hops. The receiver consists of n_r receive antennas, a ST demodulator, a deinterleaver and a soft-input soft-output (SISO) decoder. To simplify the notation, the subscript for receive antennas is dropped. The received signal at time interval l in the f^{th} hop is

$$y_{f,l} = \sqrt{E_s} \sum_{i=1}^{n_t} \alpha_f^i s_{f,l}^i + \eta_{f,l},$$
(1)

where E_s is the average received energy of the constellation used at each transmit antenna and $\eta_{f,l}$ is an additive white noise modeled as independent zero-mean complex Gaussian random variables with variance N_0 , i.e. $\mathcal{CN}(0, N_0)$. Here, α_f^i is the channel gain from the *i*th transmit antenna in the hop f modeled as $\mathcal{CN}(0, 1)$. The channel gains from different transmit antennas are assumed to be uncorrelated, which is achieved by keeping the antennas apart by a distance greater than half the wavelength of the carrier [14].

The receiver employs an iterative demodulation decoding (IDD) algorithm. To summarize the IDD algorithm, the ST demodulator calculates the aposteriori probabilities of each signal involved in the received signal at each time. These probabilites are fed to the SISO decoder, which employs a MAP decoding that produces soft information about signals in the frame. This is implemented using the BCJR algorithm in [15]. The receiver exchanges the soft information between the SISO decoder and the ST demodulator for a number of iterations. The reader is referred to [16] for the details of the IDD of I-Q ST codes, and to [13] for the IDD of turbo coded ST systems. In the following, the channel estimation for ST codes using OPSI is reviewed and the proposed JDE algorithm is derived.

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III. SEMIBLIND ITERATIVE JDE

A. Orthogonal Pilot Sequence Insertion (OPSI)

In this scheme, a known pilot sequence $\underline{p}^i = \{p^i, ..., p^i\}_{l=1}^K$ of length $K \ge n_t$ is transmitted from the i^{th}

transmit antenna once in each hop. Denote by $\underline{\hat{p}}_{f} = {\{\hat{p}_{l}^{i}\}}_{l=1}^{K}$ the received row vector corresponding to the pilot sequences at each receive antenna in the f^{th} hop. It is given by

$$\underline{\hat{p}}_{f} = \sum_{i=1}^{n_{t}} \alpha_{f}^{i} \underline{p}^{i} + \underline{\eta}_{f}, \qquad 1 \le f \le F.$$
(2)

Let the complex conjugate transpose of a row vector be denoted by superscript (*). If the pilot sequences from different transmit antennas are orthogonal, i.e., $\underline{p}^i \underline{p}^{j*} = 0$ when $i \neq j$, then the ML estimator of the channel gain α_f^j is obtained by projecting $\underline{\hat{p}}_f$ on \underline{p}^i as

$$\underline{\hat{p}}_{f} \cdot \underline{p}^{i*} = \alpha_{f}^{i} (\underline{p}^{i} \cdot \underline{p}^{i*}) + \underline{\eta}_{f} \cdot \underline{p}^{i*}.$$
(3)

By doing this, the contribution of pilot sequences from other transmit antennas is cancelled. Hence, the estimate of α_{f}^{i} is

$$\hat{\alpha}_{f}^{i} = \frac{\underline{\hat{p}}_{f} \cdot \underline{p}^{i*}}{\|\underline{p}^{i}\|^{2}} - \frac{\underline{\eta}_{f} \cdot \underline{p}^{i*}}{\|\underline{p}^{i}\|^{2}} = \alpha_{f}^{i} + e_{f}^{i}, \qquad (4)$$

where $e_f^i = \left(\underline{\eta}_f \cdot \underline{p}^{i*} / \| \underline{p}^i \|^2\right)$ is the estimation error associated with the channel of the *i*th transmit branch in the *f*th hop with the distribution $\mathcal{CN}(0, \sigma_e^2)$, where $\sigma_e^2 = \frac{N_0}{K}$.

In [4], the MLD metric for decoding ST codes in the presence of a Gaussian estimation error was found to be

$$\left| y_{f,l} - \frac{\mu}{\sigma} \sqrt{E_s} \sum_{i=1}^{n_t} \hat{\alpha}_f^i s_{f,l}^i \right|^2, \tag{5}$$

where $\mu = 1/\sqrt{1 + \sigma_e^2}$ is the correlation coefficient between the true and estimated channel gain, and $\sigma^2 = Var(\hat{\alpha})$. From the expression of σ_e^2 , we see that a longer pilot sequence improves the channel estimation at the cost of reducing the effective energy of the coded system. We define the effective signal-to-noise ration (SNR) per bit as $\frac{E_b}{N_0} = \left(\frac{N-KF}{N}\right) \frac{E}{N_0}$, where $\frac{E}{N_0}$ is the SNR per bit, including pilots. If the decoding results are reliable, they can be used to improve the estimation of the channel as will be presented in the following.

B. Algorithm

The proposed iterative JDE algorithm uses OPSI scheme to get an initial estimate of the channel. In each hop, the shortest possible pilot sequence to preserve orthogonality, of length n_t symbols, is transmitted over each transmit antenna. As a result, initial estimates of channel gains at all transmit antennas are obtained from (4) with $\sigma_e^2 = \frac{N_0}{n_t}$. The block diagram of the proposed iterative JDE algorithm is shown in Figure 2. An additional block to update the CSI



Fig. 1. The structure of the FH ST coded system.

estimate was added to the diagram in Figure 1. The ST demodulator uses the initial estimate in the first iteration, which are denoted by $\hat{\Omega}_f = {\{\hat{\alpha}_f^i\}_{i=1}^{n_t}}$. These estimates are to be improved in the next iterations. After demodulation is finished, the SISO decoder computes the aposteriori probabilities of the coded bits in the frame using the information from the demodulator. These probabilities are fed to the CSI update block, which updates the channel estimates by using a quantized fading process as follows.

For the channel from the i^{th} transmit antenna in the f^{th} hop, the channel gain is written as $\alpha_f^i = a_f^i \exp(j\theta_f^i)$, where $j = \sqrt{-1}$, a_f^i and θ_f^i are the amplitude and the phase of α_f^i , which are modeled as Rayleigh and uniform random variables, respectively. The domain of the amplitude random variable of the channel at each transmit branch is quantized into L levels $\{A_j\}_{j=1}^L$. The same is done for the phase random variables, resulting in $\{\Theta_j\}_{j=1}^L$, and a total of L^{2n_t} possible fading intervals $\{\Phi_j\}_{j=1}^{L^{2n_t}}$ at each receive branch. This makes blind estimation complicated in this context. Moreover, because the phase of the received signal in (1) is the cumulative phase of all $s_{f,l}^i$ and α_f^i , it is impossible to distinguish a signal vector by averaging over all fading quantization levels. Thus, blind estimation is not possible in this context.

Denote by Y the vector containing the received signals of one frame. The CSI update block computes the probabilities $p(\alpha_f^i \in \Phi_j | \mathbf{Y})$ for channel gains of all transmit branches and hops in the frame. The quantization intervals, for which the probabilities are computed, lie within an ϵ interval Φ from the obtained estimate $\hat{\alpha}_f^i$, where $\Phi = \{\Phi_j : |\alpha - \hat{\alpha}_f^i| < \epsilon, \forall \alpha \in \Phi_j\}$. The width of the interval is 2ϵ , which is a design parameter that depends on the received SNR. The higher the value of ϵ , the wider the ϵ -interval and the larger the number of the fading levels for which the aposteriori probabilities are computed. This increases the computational complexity as well as the reliability of the algorithm. As a tradeoff between computational complexity and performance, a good choice for 2ϵ was found to be on the order of σ_e . Once the aposteriori probabilities are computed for all fading levels in the ϵ -interval, the estimates are updated according to

set
$$\hat{\alpha}_{f}^{i} = \alpha_{j}$$
, if
 $p(\alpha_{f}^{i} \in \Phi_{j} | \mathbf{Y}) \ge p(\alpha_{f}^{i} \in \Phi_{q} | \mathbf{Y}), \quad \forall \Phi_{q} \in \Phi,$ (6)

where α_j is the center of the quantization interval Φ_j . After getting the new estimates, they are fed to the ST demodulator to update the probabilities of the signal vectors in the frame. The process of demodulation, decoding and updating the channel estimates continues iteratively for a number of times.

An important issue in this algorithm is the computation of $p(\alpha_f^i \in \Phi_j | \mathbf{Y})$. Let $\tilde{\mathbf{Y}}_f$ be the vector containing the channel observations in all hops in the frame except the hop f. Also, let \mathbf{Y}_f be the vector containing the channel observations in the hop f. The aposteriori probabilities $p(\alpha_f^i \in \Phi_j | \mathbf{Y})$ are computed as

$$p(\alpha_{f}^{i} \in \Phi_{j} | \mathbf{Y}) = p(\mathbf{Y} | \alpha_{f}^{i} \in \Phi_{j}) \frac{p(\alpha_{f}^{i} \in \Phi_{j})}{p(\mathbf{Y})}$$
$$= p(\mathbf{Y}_{f} | \alpha_{f}^{i} \in \Phi_{j}, \tilde{\mathbf{Y}}_{f}) p(\tilde{\mathbf{Y}}_{f} | \alpha_{f}^{i} \in \Phi_{j}) \frac{p(\alpha_{f}^{i} \in \Phi_{j})}{p(\mathbf{Y})}$$
$$\approx Cp(\mathbf{Y}_{f} | \alpha_{f}^{i} \in \Phi_{j}) p(\alpha_{f}^{i} \in \Phi_{j}), \tag{7}$$

where $C = p(\mathbf{\hat{Y}}_f | \alpha_f^i \in \Phi_j) / p(\mathbf{Y})$ is a normalization constant and $p(\alpha_f^i \in \Phi_j)$ is the apriori probability of the fading gain. The approximation in (7) results from the fact that channel outputs in different hops are slightly correlated because of the interleaving used in the transmitter. Now, the probability $p(\mathbf{Y}_f | \alpha_f^i \in \Phi_j)$ is written as

$$p(\mathbf{Y}_{f}|\alpha_{f}^{i} \in \Phi_{j})$$

$$= \sum_{\mathbf{s}_{f,1},...,\mathbf{s}_{f,m}} p(\mathbf{Y}|\alpha_{f}^{i} \in \Phi_{j}, \tilde{\Omega}_{f}^{i}, \mathbf{s}_{f,1}, ..., \mathbf{s}_{f,m})$$

$$.p(\mathbf{s}_{f,1})...p(\mathbf{s}_{f,m}), \qquad (8)$$

where $\tilde{\Omega}_{f}^{i}$ is the row vector containing the estimates of the channel gains in the f^{th} fading block at all transmit antennas except the i^{th} one. If the size of the signal constellation used at each antenna is M, then computing (8) involves summing over M^{m} quantities, which is very complex. An iterative algorithm to calculate (8) efficiently was derived in [10] for blind estimation in single-antenna FH systems. A modified version of the algorithm that fits the semiblind JDE is given in the following



Fig. 2. The structure of the iterative JDE receiver.

1) Initialization:

$$p(\alpha_{f}^{i} \in \Phi_{j}|y_{f,1}) = p(y_{f,1}|\alpha_{f}^{i} \in \Phi_{j}) \frac{p(\alpha_{f}^{i} \in \Phi_{j})}{p(y_{f,1})}$$

$$p(y_{f,l}|\alpha_{f}^{i} \in \Phi_{j}) \approx \sum_{\forall \mathbf{s}} p(y_{f,l}|\alpha_{f}^{i} \in \Phi_{j}, \tilde{\Omega}_{f}^{i}, \mathbf{s}_{f,l}) p(\mathbf{s}_{f,l})$$

$$(10)$$

2) Recursion:

$$p(\alpha_{f}^{i} \in \Phi_{j} | y_{f,1}, ..., y_{f,l}) = \frac{p(y_{f,l} | \alpha_{f}^{i} \in \Phi_{j}, y_{f,1}, ..., y_{f,l-1})}{p(y_{f,l} | y_{f,1}, ..., y_{f,l-1})}$$

$$p(\alpha_{f}^{i} \in \Phi_{j} | y_{f,1}, ..., y_{f,l-1}) = \frac{p(y_{f,l} | \alpha_{f}^{i} \in \Phi_{j}) p(\alpha_{f}^{i} \in \Phi_{j} | y_{f,1}, ..., y_{f,l-1})}{p(y_{f,l} | y_{f,1}, ..., y_{f,l-1})}.$$
(11)

In (11), $p(y_{f,l}|\alpha_f^i \in \Phi_j)$ is computed as in (10) and the approximation in (11) is due to the small correlation between the channel outputs in different hops because of the interleaving used. The computation in (10) is modified from the original algorithm in [10], where the approximation is due to using $\tilde{\Omega}_f^i$ instead of averaging over the channel gains. Note that computing (10) requires summing over M^{n_t} signals. The aposteriori probabilities of the fading levels in the ϵ -interval are used to update the estimate in the next iteration as in (6). The results of the algorithm are presented in the following.

IV. SIMULATION RESULTS

In this section, the proposed JDE algorithm is tested for coded systems with two transmit and one receive antennas. The cases of perfect CSI, iterative JDE algorithm and no JDE are considered. In addition, we consider feeding the correct decisions to the CSI update block. This is equivalent to estimating the channel using OPSI where each pilot sequence has length K = m. This is the best the can be achieved using semiblind JDE, and is referred to as the best achievable performance. In all cases, OPSI is used with the shortest sequence length $K = n_t$. Throughout the simulation, the estimation interval width $2\epsilon = \sigma_e$, and each amplitude and phase processes are quantized into 64 levels each, using a Llyod-Max algorithm [17]. Also, 3 iterations are used to exchange information between the decoder and the CSI update module. It was observed that using more than $\hat{3}$ iterations provides small gains, and hence only 3 iterations were considered.

A. I-Q Codes

The I-Q ST code used in this paper is the 4-state trellis code in [12]. Each of the I and Q codes uses BPSK constellation, resulting in a QPSK 2-D constellation at each transmit antenna. The frame length in this case is N = 500. In Figure 3, the performance when the hop length m = 10is shown, which corresponds to 50 hops in the frame. It is clear that the proposed JDE gives around 1.5 dB better than the case of no JDE but still 1.5 dB away from the perfect case. However, the proposed JDE is less than 1 dB from the best case. The case of hop length m = 100, corresponding to 5 hops, is presented in Figure 4. Here, the JDE is less than 0.5 dB from the best that can be achieved. It is also closer to the perfect CSI case because the hop is longer in this case and hence it is easier to estimate the channel than in the case of m = 10. From the figure, we see that the proposed JDE gives appreciable gain (around 2.5 dB) when compared to the system without JDE for slightly more complexity. Comparing the case of m = 10 and m = 100, it is observed that reducing the hop length from 100 to 10 improves the performance for the case of perfect CSI by 0.5 dB. This is due to the increased diversity order in m = 10. However, the performance of the best case for m = 100 is slightly better than the best case for m = 10 because longer hops permit better channel estimation. The same happens to the curves of the JDE algorithm.

In Figure 5, the BER is plotted versus the hop length for effective SNR $E_b/N_0 = 10$ dB. We see that the optimal hop length in the case perfect CSI is around m = 10. For the case of JDE and the best achievable performance, the optimal hop length is around m = 50. This is expected since higher hop length increases the quality of estimating the channel. For hop lengths larger than 50, the performance of the JDE degrades because longer hops reduce the diversity order without adding much to the estimation accuracy.

B. Turbo Codes

The turbo-coded ST system uses a (1, 5/7, 5/7) turbo code, with an interleaver size of $32 \times 32 = 1024$ bits. The parity bits of the turbo encoder are punctured resulting in a rate- $\frac{1}{2}$ turbo code. The coded bits are bit-interleaved and each two bits are mapped to a QPSK constellation. The ST modulator collects each two QPSK signals and transmits them using the two transmit antennas. Thus, the resulting frame size is of length N = 512 signal vectors.

Figure 6 shows the performance for hop length m = 10. It observed that the JDE provides a gain of 2.2 dB over the case of no JDE and is worse than the best case by 0.7 dB at BER of 10^{-4} . Also, the performance of the best case is 1 dB from the perfect CSI case, because of hop length is short. The case of hop length m = 100 is presented in Figure 7. Here, the JDE is 0.2 dB from the best case at BER of 10^{-4} , which is much closer than in the case of the I-Q code. This is expected since the code here is stronger, and hence more sensitive to hop length. Also, the best case is closer to the perfect CSI case because the hop is longer and the code is stronger than the I-Q code. Furthermore, the proposed JDE gives a gain of 3.5 dB when compared to the system without JDE. In a turbo coded system, increasing the hop lengths from 10 to 100 degrades the performance by 3.3 dB and 1.7 dB for the cases of perfect CSI and JDE, respectively. When perfect CSI is available, the turbo code performance is affected much more than when JDE is employed. This becomes more clear when the performance is plotted versus the hop length as in Figure 8, where $E_b/N_0 = 9$ dB. From the figure, the optimal hop length in the case perfect CSI is between m = 5 and m = 10, whereas it is m = 20 in the best and the JDE cases. Note that the optimal hop length is shorter than the I-O code for the best case, which is expected since the code is more sensitive to hop length.

V. CONCLUSIONS

In this paper, FH system employing ST codes was considered and a simple iterative JDE algorithm was proposed. The algorithm is less than 0.5 dB from the best achievable performance. Also, the trend between the channel estimation and the code's performance was considered and the optimal hop length was found.

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Fig. 3. Simulation of FH ST system employing the 1-Q code for m = 10



Fig. 4. Simulation of FH ST system employing the I-Q code for m = 100



Fig. 5. BER vs hop length of FH ST system with 1-Q code for E_b/N_o =10 dB



Fig. 6. Simulation of FH ST turbo coded system for m = 10



Fig. 7. Simulation of FH ST turbo coded system for m = 100

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Fig. 8. BER vs hop length of FH ST turbo coded system for $E_b/N_0=9$ dB

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