A Robust LMS Adaptive Algorithm Over Distributed Networks

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***Abstract*—This work studies the effect of erroneous noise power estimates on the behavior of noise constrained diffusion-based adaptive algorithm for distributed adaptive networks. The noise constrained diffusion least mean square (NCDLMS) algorithm assumes knowledge of the noise variance is available at each node for good performance. Here, it is shown that the NCDLMS algorithm is robust to large variations in noise variance estimation. Moreover, the mean and steady-state analyses of the NCDLMS algorithm are carried out and simulation results are found to corroborate the theoretical findings. Great improvement in performance is obtained through the use of the proposed algorithm even when no information on the noise variance is available. The increased computational complexity of the NCDLMS algorithm is justified through the performance improvement it offers.**

Index Terms – Adaptive filters, Variable step-size least mean square, noise constrained least mean square, diffusion algorithm.

I. Introduction

Distributed adaptive sensor networks have found various applications in recent years ranging from industrial process monitoring, environment and habitat monitoring, target localization and battlefield surveillance systems, just to name a few. In these sensor networks, a collection of nodes collaborate to estimate a common parameter. The collaboration among the nodes is limited to neighbors in the immediate vicinity of each node, either due to physical limitations or computational considerations.

To address the above mentioned concerns, various adaptive collaboration strategies have been suggested in the literature in recent years [1]-[8]. These include incremental approach [1], diffusion LMS (DLMS) [2], [3], diffusion RLS (DRLS) [4], distributed least mean squares algorithm [5], DLMS with adaptive combiners [6] and variable step size diffusion least mean square (VSSDLMS) approach [7].

Recently, a noise constrained diffusion LMS (NCDLMS) algorithm was presented in [8] where the estimation process at each node was aided by the exact knowledge of the noise variance. Obtaining frequent and accurate noise power estimate might not be possible in all scenarios and it might be impossible to obtain under certain ones. This motivates us to study the behavior performance of the NCDLMS algorithm when faulty or no information is available of the noise power.

In this work, we investigate the behavior of the NCDLMS under various degree of mismatches between the actual and the estimated noise variance at each node. Also, the mean and steady-state analyses of the NCDLMS algorithm when no information about the noise variance is available to the nodes are presented. In the worst-case scenario, it turns out that the zero noise constrained diffusion LMS (ZNCDLMS) algorithm performs remarkably well when compared to other algorithms.

II. Problem Formulation

Consider an adaptive network consisting of N sensor nodes, as shown in Fig. 1, deployed over a geographical area to estimate an M dimensional unknown parameter vector $\mathbf{w}^o \in \mathbb{R}^M$. We denote the neighborhood of a node k by $\mathcal{N}_k$ and its cardinality by n_k. The neighborhood of a node k is a set of nodes in close vicinity such that they have a direct link with node k, i.e., for $l = 1, 2, \cdots, N$ and $l \neq k$, a link exists between nodes l and k iff $l \in \mathcal{N}_k$.

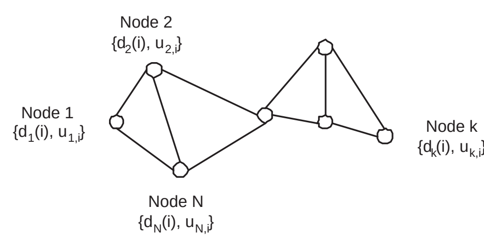

Fig. 1. Adaptive network of *N* nodes.

Each node k has access to a time realization of a known regressor row vector $\mathbf{u}_{k,i}$ of length M and a scalar measurement $d_k(i)$, related as

$$d_k(i) = \mathbf{u}_{k,i}\mathbf{w}^o + v_k(i), \tag{1}$$

where $k = 1, \ldots, N$, $v_k(i)$ is zero-mean spatially uncorrelated additive white Gaussian noise with variance $\sigma^2_{v_k}$, and i is the time index. At each node, the scalar measurement and the regressor vector are used to generate an estimate of the unknown vector $\mathbf{w^o}$. Let $\mathbf{\Psi}_{k,i}$ denote its estimate. We assume that each node cooperates only with its neighbors.

At every time instant i, each node k has access to its own estimate $\mathbf{\Psi}_{k,i}$, as well as to estimates $\mathbf{\Psi}_{l,i}$, from its $\mathcal{N}_k$ neighbor nodes. Each node diffuses the estimates received from its neighbors to update its local estimate. Two different schemes have been introduced in the literature for diffusion estimation. The adapt-then-combine (ATC) scheme [3],[6] first updates the local estimate using the adaptive algorithm and then the estimates from the neighbors are fused together. The combine-then-adapt (CTA) scheme [2] performs the operations of the ATC scheme in a reverse order. It has been shown that the ATC scheme outperforms the CTA scheme [3],[6], and therefore the ATC scheme has been adopted in this work.

The local cost function at each node takes the form

$$J_k(\mathbf{w}) = \mathrm{E}\left[|d_k - \mathbf{u}_k\mathbf{w}|^2\right], \tag{2}$$

where $\mathbf{w}$ is the estimate of the unknown vector. Completing the squares and observing that $\mathbf{R}_{\mathbf{u},k} = \mathrm{E}[\mathbf{u}_k^*\mathbf{u}_k]$, (2) becomes [3]

$$J_k(\mathbf{w}) = \|\mathbf{w} - \mathbf{w}_k\|^2_{\mathbf{R}_{\mathbf{u},k}} + \mathrm{MMSE}, \tag{3}$$

where "MMSE" stands for the minimum mean square error, $\mathbf{w}_k$ is a vector of local estimate at the k^{th} node, and $\|\mathbf{w} - \mathbf{w}_k\|^2_{\mathbf{R}_{\mathbf{u},k}} = (\mathbf{w} - \mathbf{w}_k)^T \mathbf{R}_{\mathbf{u},k}(\mathbf{w} - \mathbf{w}_k)$ (weighted Euclidean norm). For a fully connected network we have $n_k = N - 1$ for all k and the global cost function takes the form

$$J(\mathbf{w}) = \sum_{l\in\mathcal{N}_k} \mathrm{E}\left[|d_l - \mathbf{u}_l\mathbf{w}|^2\right] + \sum_{\substack{l=1\\ l\neq k}}^{N} \|\mathbf{w} - \mathbf{w}_l\|^2_{\mathbf{R}_{\mathbf{u},l}}. \tag{4}$$

Practically, however, the cardinality of a node k is much less then $N - 1$, i.e., $n_k << N - 1$ and the global cost function can be approximated by the following local cost function:

$$J_k(\mathbf{w}) = \sum_{l\in\mathcal{N}_k} \mathrm{E}\left[|d_l - \mathbf{u}_l\mathbf{w}|^2\right] + \sum_{l\in N_k/\{k\}} b_{lk}\|\mathbf{w} - \mathbf{\Psi}_l\|^2. \tag{5}$$

where $\mathbf{w}_l$ is replaced by its intermediate estimate at node l, denoted by $\mathbf{\Psi}_l$. Furthermore, the second term of the right hand side of (5) is no longer weighted by $\mathbf{R}_{\mathbf{u},l}$, instead it is replaced by a constant weighting factor b_{lk} [3]. Defining

$$J_k^1(\mathbf{w}) = \sum_{l\in\mathcal{N}_k} \mathrm{E}\left[|d_k - \mathbf{u}_k\mathbf{w}|^2\right], \tag{6}$$

we get

$$\min_{\mathbf{w}} J_k(\mathbf{w}) = J_k^1(\mathbf{w}) + \sum_{l\in N_k/\{k\}} b_{lk}\|\mathbf{w} - \mathbf{\Psi}_l\|^2. \tag{7}$$

Finally, solving (7) results in the diffusion LMS (DLMS) algorithm.

III. Adaptive Diffusion Algorithms

In this section we give an overview of the DLMS, the variable step size DLMS (VSSDLMS), and the noise constrained DLMS (NCDLMS) algorithms. Table I summarises these algorithms. Some clarifications for some of the parameters used in these algorithms are explained next.

In the DLMS algorithm [2], $\mathbf{\Psi}_{k,i}$ is the intermediate local estimate at node k at time instant i, μ_k is the step size associated with the k^{th} node and c_{lk} represents the combiner coefficient. Various choices for combiner coefficients are possible. In this work, we use the Metropolis combiner rule defined as follows [2]:

$$c_{lk} = \begin{cases} \frac{1}{\max(\mathcal{N}_k, N_l)}, & \text{nodes } k \text{ and } l \text{ are linked and } k \neq l \\ 0, & \text{nodes } k \text{ and } l \text{ are not linked} \\ 1 - \sum_{l\in\mathcal{N}_k/\{k\}} c_{kl}. & k = l \end{cases} \tag{8}$$

The error at the k^{th} node is given by

$$e_k(i) = d_k(i) - \mathbf{u}_{k,i}\mathbf{w}_{k,i-1}. \tag{9}$$

In the VSSDLMS algorithm [7], the step-size in the DLMS algorithm, μ_k, is replaced by $\mu_{k,i}$, with positive constants α_{VSS} and γ_{VSS}. The error is still given by (9).

In the NCDLMS algorithm [8], the step-size in the DLMS algorithm, μ_k is replaced by $\mu_{k,i}$, $\bar{\mu}_k$ is the fixed step-size, and $\beta_{k,i-1}$ is a Langrange multiplier with $0 < \alpha_{\text{NC}} < 1$.

Algorithm	Mathematical Formulation
DLMS [2]	$\mathbf{\Psi}_{k,i} = \mathbf{w}_{k,i-1} + \mu_k\mathbf{u}_{k,i}^T(d_k(i) - \mathbf{u}_{k,i}\mathbf{w}_{k,i-1})$ $\mathbf{w}_{k,i} = \sum_{l\epsilon\mathcal{N}_k} c_{lk}\mathbf{\Psi}_{l,i}$
VSSDLMS [7]	$\mathbf{\Psi}_{k,i} = \mathbf{w}_{k,i-1} + \mu_{k,i}\mathbf{u}_{k,i}^T(d_k(i) - \mathbf{u}_{k,i}\mathbf{w}_{k,i-1})$ $\mathbf{w}_{k,i} = \sum_{l\epsilon\mathcal{N}_k} c_{lk}\mathbf{\Psi}_{l,i}$ $\mu_{k,i+1} = \alpha_{\text{VSS}}\mu_{k,i} + \gamma_{\text{VSS}}e_k^2(i)$
NCDLMS [8]	$\mu_{k,i} = \bar{\mu}_k(1 + \gamma_{\text{NC}}\beta_{k,i-1})$ $\mathbf{\Psi}_{k,i} = \mathbf{w}_{k,i-1} + \mu_{k,i}\mathbf{u}_{k,i}^T(d_k(i) - \mathbf{u}_{k,i}\mathbf{w}_{k,i-1})$ $\mathbf{w}_{k,i} = \sum_{l\epsilon\mathcal{N}_k} c_{lk}\mathbf{\Psi}_{l,i}$ $\beta_{k,i} = (1 - \alpha_{\text{NC}})\beta_{k,i-1} + \frac{\alpha_{\text{NC}}}{2}\left(e_k^2(i-1) - \sigma_{v,k}^2\right)$

TABLE I
Summary of algorithms

IV. Mean Analysis of NCDLMS

In this section, the mean analysis of the NCDLMS algorithm is presented. Here, first we introduce new global variables

$$\begin{array}{cc} \mathbf{w}_i = col\{\mathbf{w}_{1,i}, ..., \mathbf{w}_{N,i}\} & \mathbf{\Psi}_i = col\{\mathbf{\Psi}_{1,i}, ..., \mathbf{\Psi}_{N,i}\} \\ \mathbf{U}_i = diag\{\mathbf{u}_{1,i}, ..., \mathbf{u}_{N,i}\} & \mathbf{D} = diag\{\mu_1\mathbf{I}_M, ..., \mu_N\mathbf{I}_M\} \\ \mathbf{d}_i = col\{d_1(i), ..., d_N(i)\} & \mathbf{v}_i = col\{v_1(i), ..., v_N(i)\} \end{array}$$

Hence, from the above newly defined variables, we can form a complete new set of equations representing the functionality of the network. Second, the measurements, $\mathbf{d}_i$, can be set as

$$\mathbf{d}_i = \mathbf{U}_i\mathbf{w}^{(o)} + \mathbf{v}_i \tag{10}$$

where $\mathbf{w}^{(o)} = \mathbf{Q}\mathbf{w}^o$ and $\mathbf{Q} = col\{\mathbf{I}_{\text{M}}, \mathbf{I}_{\text{M}}, ..., \mathbf{I}_{\text{M}}\}$ is a $MN \times M$ matrix. Similarly, the update equations can be remodeled to represent the entire network instead of just a single node as follows

$$\begin{aligned} \mathbf{e}_i &= \mathbf{d}_i - \mathbf{U}_i\mathbf{w}_{i-1} && (11)\\ \mathbf{\Psi}_i &= \mathbf{w}_{i-1} + \mathbf{D}(\mathrm{I}_{MN} + \gamma_{\text{NC}}\mathbf{B}_i)\mathbf{U}_i^T\mathbf{e}_i && (12)\\ \mathbf{w}_i &= \mathbf{G}\mathbf{\Psi}_i && (13)\\ \mathbf{B}_{i+1} &= (1 - \alpha_{\text{NC}})\mathbf{B}_i + \frac{\alpha_{\text{NC}}}{2}(\mathbf{E}_i - S) && (14) \end{aligned}$$

where $\mathbf{G}=\mathbf{C}\otimes\mathbf{I}_M$, $\mathbf{C}$ is the $N\times N$ weighting matrix, $\mathbf{B}_i = diag\left\{\beta_1\mathbf{I}_M,...,\beta_N\mathbf{I}_M\right\}$ is the diagonal update matrix for the Lagrange multipliers, $\mathbf{E}_i = diag\left\{e_1^2(i)\mathbf{I}_M,...,e_N^2(i)\mathbf{I}_M\right\}$ is the diagonal matrix for instantaneous error, $S = diag\left\{\hat{\sigma}_1^2\mathbf{I}_M,...,\hat{\sigma}_N^2\mathbf{I}_M\right\}$ is the diagonal matrix containing the estimated noise powers for all nodes and $\otimes$ is the Kronecker Product operator. Introducing the weight-error global vector $\tilde{\mathbf{w}}_i=\mathbf{w}^{(o)}-\mathbf{w}_i$ and noting that $\mathbf{G}\mathbf{w}^{(\mathbf{o})}=\mathbf{w}^{(\mathbf{o})}$, we rewrite equations (12) and (13) as

$$\begin{aligned}\tilde{\mathbf{\Psi}}_i &= \tilde{\mathbf{w}}_{i-1}-\mathbf{D}_i\mathbf{U}_i^T\left(\mathbf{U}_i\tilde{\mathbf{w}}_{i-1}+\mathbf{v}_i\right) &(15)\\ \tilde{\mathbf{w}}_i &= \mathbf{G}\tilde{\mathbf{\Psi}}_i\\ &= \mathbf{G}\tilde{\mathbf{w}}_{i-1}-\mathbf{G}\mathbf{D}_i\mathbf{U}_i^T\left(\mathbf{U}_i\tilde{\mathbf{w}}_{i-1}+\mathbf{v}_i\right) &(16)\end{aligned}$$

which can be further simplified to

$$\tilde{\mathbf{w}}_i=\mathbf{G}\left(\mathrm{I}-\mathbf{D}_i\mathbf{U}_i^T\mathbf{U}_i\right)\tilde{\mathbf{w}}_{i-1}-\mathbf{G}\mathbf{D}_i\mathbf{U}_i^T\mathbf{v}_i \tag{17}$$

where the step-size matrix, $\mathbf{D}_i$, is simply

$$\mathbf{D}_i=\mathbf{D}\left(\mathrm{I}+\gamma_{\mathrm{NC}}\mathbf{B}_i\right) \tag{18}$$

Now assuming that the noise variance is not estimated exactly, resulting in a mismatch, and taking the expectation on both sides of (14) gives

$$\begin{aligned}\mathrm{E}\left[\mathbf{B}_{i+1}\right] &= \left(1-\alpha_{\mathrm{NC}}\right)\mathrm{E}\left[\mathbf{B}_i\right]+\frac{\alpha_{\mathrm{NC}}}{2}\left(\mathrm{E}\left[\mathbf{E_i}\right]-\mathbf{S}\right)\\ &= \left(1-\alpha_{\mathrm{NC}}\right)\mathrm{E}\left[\mathbf{B}_i\right]+\frac{\alpha_{\mathrm{NC}}}{2}\left(\mathrm{EMSE}_i-\tilde{\mathbf{S}}\right)\end{aligned} \tag{19}$$

where EMSE_i is a diagonal matrix containing EMSE values of the entire network from the previous iteration, and $\tilde{\mathbf{S}}=\mathrm{diag}\left\{(\sigma_1^2-\hat{\sigma}_1^2)\mathbf{I}_M,(\sigma_2^2-\hat{\sigma}_2^2)\mathbf{I}_M,...,(\sigma_N^2-\hat{\sigma}_N^2)\mathbf{I}_M\right\}$ is a diagonal matrix containing the values for the mismatch in the estimation of the noise variance. For perfect estimation, this matrix disappears whereas for the ZNCDLMS algorithm, the matrix contains the actual noise variance values only.

Now taking the expectation on both sides of equation (18) results in

$$\mathrm{E}\left[\mathbf{D}_i\right]=\mathbf{D}\left(\mathrm{I}+\gamma_{\mathrm{NC}}\mathrm{E}\left[\mathbf{B}_i\right]\right) \tag{20}$$

and finally taking the expectation for the equation (17) leads to

$$\mathrm{E}\left[\tilde{\mathbf{w}}_i\right]=\mathbf{G}\left(\mathrm{I}-\mathrm{E}\left[\mathbf{D}_i\right]\mathbf{R_U}\right)\mathrm{E}\left[\tilde{\mathbf{w}}_{i-1}\right], \tag{21}$$

where $\mathbf{R_U}=\mathbf{U}_i^T\mathbf{U}_i$ is the auto-correlation matrix for the regressor vectors across the entire network and is block diagonal.

From (21) and the fact that the matrix $\mathbf{R_U}$ is block diagonal, it can be seen that the system is stable if, for each node

$$0<\mu_k<\frac{2}{\left(1+\gamma_{\mathrm{NC}}\mathrm{E}\left[\beta_{k,i}\right]\right)\lambda_{\max}\left(\mathbf{R}_{\mathbf{u},k}\right)},\quad k=1,...,N \tag{22}$$

where $\lambda_{\max}(.)$ denotes the maximum eigenvalue of $\mathbf{R}_{\mathbf{u},k}$. The values for the parameters are chosen arbitrarily and usually depend on the signal-to-noise ratio (SNR). The effect of the value of $\lambda_{\max}$ and γ_{NC} on μ_k will be further discussed in the simulations section.

V. Steady-State Analysis of NCDLMS

In this section we briefly look at the steady-state performance of the NCDLMS algorithm. From Table I we can note that the Lagrange multiplier update for any node k is independent of other nodes. Therefore, each node can be treated independently in order to obtain the steady-state misadjustment. Thus, the steady-state misadjustment for any node k is

$$\begin{aligned}\mathcal{M}_k = &\left(1+\frac{\gamma\sigma_{v,k}^2\left(1-a\right)}{2}+\frac{\gamma^2\alpha\sigma_{v,k}^2}{2\left(2-\alpha\right)\left(1+\gamma\sigma_{v,k}^2\left(1-a\right)/2\right)}\right)\\ &.\frac{\mu_k Tr\left\{\mathbf{R}_{\mathbf{u},k}\right\}}{2},\end{aligned} \tag{23}$$

where $a=\hat{\sigma}_{v,k}^2/\sigma_{v,k}^2$ is the ratio between the estimated and actual noise power at node k. For a perfect estimate, this would result in $a=1$, for a mismatch $0<a<1$, and $a=0$ for the case of ZNCDLMS. Now at steady-state the Lagrange multiplier update equation becomes

$$\beta_{k,ss}=\frac{\sigma_{v,k}^2}{2}\left(\mathcal{M}_k+1-a\right). \tag{24}$$

Similarly, we get an expression for $\beta_{k,ss}^2$

$$\begin{aligned}\beta_{k,ss}^2 = &\frac{\alpha}{\left(1-\alpha\right)}\beta_{k,ss}\sigma_{v,k}^2\left(\mathcal{M}_k+1-a\right)\\ &+\frac{\sigma_{v,k}^4}{4\left(1-\alpha\right)^2}\left(\mathcal{M}_k+1-a\right)^2\end{aligned} \tag{25}$$

Now solving for the steady-state mean-square value for the entire network, we get the weight-error equation [2]

$$\left\|\bar{\mathbf{w}}_{ss}\right\|_{\bar{\sigma}}^2=\mathbf{b}_{ss}^T\left[\mathrm{I}_{M^2N^2}-\mathbf{F}_{ss}\right]^{-1}\bar{\sigma}, \tag{26}$$

where $\mathbf{b}_{ss}=bvec\left\{\mathbf{R_v}\mathbf{D}_{ss}^2\mathbf{\Lambda}\right\}$, $\mathbf{\Lambda}$ is a block-diagonal matrix containing the eigenvalues for the entire network, $\mathbf{R_v}$ is the auto-correlation matrix for the noise vector at all nodes, and $\mathbf{F}_{ss}$ is the steady-state weighting matrix given by

$$\begin{aligned}\mathbf{F}_{ss} = &\left[\mathbf{I}_{M^2N^2}-\left(\mathbf{I}_{MN}\odot\mathbf{\Lambda}\mathbf{D_{ss}}\right)-\left(\mathbf{\Lambda}\mathbf{D_{ss}}\odot\mathbf{I_{MN}}\right)\right.\\ &\left.+\left(\mathbf{D}_{ss}\odot\mathbf{D}_{ss}\right)A\right]\left(\mathbf{G}^T\odot\mathbf{G}^T\right).\end{aligned} \tag{27}$$

Using $\bar{\sigma}=(1/N)\,bvec\left\{\mathbf{I}_{MN}\right\}=\mathbf{q}_\eta$ in (26), the mean-square deviation (MSD) is given by

$$\eta_{ss}=\mathbf{b}_{ss}^T\left[\mathrm{I}_{M^2N^2}-\mathbf{F}_{ss}\right]^{-1}\mathbf{q}_\eta. \tag{28}$$

VI. Simulation Results

Simulations have been carried out for adaptive networks with 20 nodes. The unknown vector length is fixed at 5 throughout the simulations. The noise at each node is assumed to be independent of the noise at other nodes.

All results have been obtained for an average SNR of 10 dB. The step size used for the DLMS algorithm is 0.01. For the VSSDLMS algorithm, $\alpha_{\mathrm{VSS}}=0.997$, $\gamma_{\mathrm{VSS}}=10^{-4}$ and the step size is initialized at 0.01. For the NCDLMS, $\mu_k=0.001$, $\gamma=180$ and $\beta=10^{-3}$.

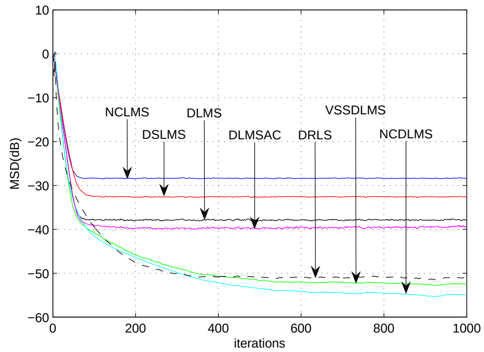

Fig. 2. MSD comparison of distributed LMS, DLMS, DLMS with adaptive combiners, VSSDLMS and NCDLMS for a network of 20 nodes.

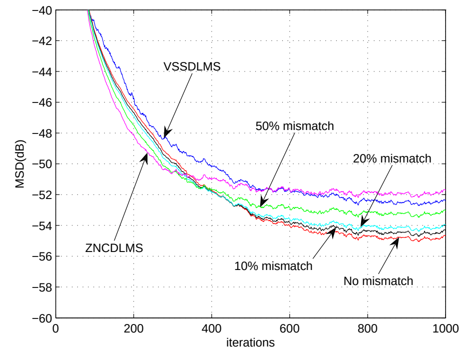

Fig. 3. MSD comparison of VSSDLMS, ZNCDLMS with NCDLMS with various degrees of noise power mismatch.

Figure 2 depicts the MSD comparison for various distributed adaptive algorithms. As can be seen from these this figure that the NCDLMS algorithm outperforms the rest of the algorithms.

Figure 3 reports the MSD comparison of NCDLMS with various degrees of noise variance estimate mismatch. A mismatch of 50% between the actual noise variance and its estimate results in just 1.5 dB performance degradation. Moreover, it is interesting to note that even for the ZNCDLMS algorithm, the performance degradation is about 3 dB. Also, as can be seen from this figure that the performance of the ZNCDLMS algorithm is comparable to that of the VSSDLMS algorithm.

Next we look at the stability analysis of the algorithm. Here, the auto-correlation matrix, $\mathbf{R}_{\mathbf{u},k}$, is taken to be an identity matrix. Table II gives results for steady-state MSD for the network when the value of μ_k is varied, for $k = 3$, $\gamma_{\rm NC} = 0.1$, $\alpha_{\rm NC} = 0.01$, and SNR = 20 dB. From this table, it can be seen that the simulations corroborate the theoretical finding for the steady-state MSD. Moreover, the bound in (22) holds true.

μ_k	$\gamma_{\rm NC}$	**SS-MSD (simulation)**	**SS-MSD (theory)**
1.9	0.1	-15.3	-15.7
1.75	0.1	-18.4	-18.6
1.5	0.1	-21.5	-21.5
1	0.1	-25.8	-26

TABLE II
COMPARISON OF MSD, FROM SIMULATIONS AND THEORY.

Finally, the computational complexity of the NCDLMS algorithm is two more multiplications and three more additions than the VSSDLMS algorithm in addition to the algorithm for the noise variance estimation at each node. The performance improvement is worth deploying this algorithm in real applications.

VII. CONCLUSION

In this work, the performance behavior of the NCDLMS algorithm is investigated and analyzed. First, the step-size range for each node is derived. Second, the steady-state analysis is carried out and a comparison is shown for the simulated and theoretical values for steady-state MSD. The comparison also verifies the step-size range. Third, a comparison scenario among the most popular algorithms, including the DLMS and DRLS algorithms, is carried out under the assumption that full knowledge of the noise variance is available. Finally, the robustness of the NCDLMS is fully investigated. Simulation results show that even with 50% mismatch in noise variance estimation, the algorithm performs better than the rest of the algorithms, even the VSSDLMS algorithm. Furthermore, if the noise variance estimate process is removed to get the ZNCDLMS algorithm, the performance and complexity both become almost similar to that of VSSDLMS algorithm.

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