Performance and Tradeoff Analysis of Layered Steered Space-Time Codes (LSSTC)

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Abstract—In this work we study a recently proposed multipleinput multiple-output (MIMO) system called the Layered Steered Space-Time Codes (LSSTC) that combines the benefits of vertical Bell Labs space-time (VBLAST) scheme, space-time block codes (STBC) and beamforming. The aim of this research is to investigate the analytical error performance of single user LSSTC. In addition, the tradeoff between several parameters of LSSTC is analyzed.

I. INTRODUCTION

Various techniques have been proposed to counter the problem of propagation conditions, and to achieve data rates that are very close to the Shannon limit. One of these techniques is using MIMO systems which uses antenna arrays at both the transmitter and the receiver. Wolniansky *et al.* has proposed in [1] the well-known MIMO scheme, known as VBLAST. In VBLAST architecture, parallel data streams are sent via the transmit antennas at the same carrier frequency. Given that the number of receive antennas is greater than or equal to the number of transmit antennas, the receiver employs a low complexity method based on successive interference cancellation (SIC) to detect the transmitted data streams. In this manner, VBLAST can achieve high spectral efficiencies without any need for increasing the system's bandwidth or transmitted power.

While MIMO systems as VBLAST can improve the system capacity greatly [2], it is difficult to implement antenna arrays on hand-held terminals due to size, cost and hardware limitation [3], also it has poor energy performance and does not fully exploit the available diversity. In order to overcome these problems, Alamouti has presented in [3] a new scheme called STBC with two transmit and one receive antennas that provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit and two receive antennas. This scheme can be generalized to two transmit antennas and M receive antennas to provide a diversity order of 2M. Similar work was considered in [4] where space time trellis codes (STTC) were used as the component codes. With the tempting advantages of VBLAST and STBC, many researchers has attempted to combine these two schemes to result in a multilayered architecture called multilayered space-time block codes (MLSTBC) [5] with each layer being composed of antennas that corresponds to a specific STBC. This combined scheme arises as a solution to jointly achieve spatial multiplexing and diversity gains simultaneously. With MLSTBC scheme, it is possible to increase the data rate while keeping a satisfactory link quality in terms of symbol error rate (SER) [6].

In [7] beamforming was combined with MLSTBC to produce a hybrid system called the layered steered space time codes(LSSTC). The addition of beamforming to MLSTBC further improves the performance of the system by focusing the energy towards one direction, where the antenna gain is increased in the direction of the desired user, while reducing the gain towards the interfering users.

The main contribution of this paper is to analyze to the error performance of LSSTC. The analytical results are compared to simulations. In addition, the tradeoffs between diversity gains, spatial multiplexing and beamforming are investigated. The remainder of this paper is organized as follows. Section II gives a description of the system model we consider. Section III presents the performance analysis of LSSTC, in which we derive a formula for the probability of error. In Section IV the tradeoff between several advantages of LSSTC is analyzed. Section V presents the simulation results conducted for evaluating LSSTC. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

Figure 1 shows the block diagram of a single-user LSSTC system proposed in [7], the system has N_T total transmitting antennas and N_R receiving antennas and is denoted by an (N_T, N_R) system. The antenna architecture employed in Figure 1 has M transmit adaptive antenna arrays (AAs) spaced sufficiently far apart in order to experience independent fading and hence achieve transmit diversity. Each of the AAs consists of L elements that are spaced at a distance of $d = \lambda/2$ to ensure achieving beamforming. A block of B input information bits is sent to the vector encoder of LSSTC and



Fig. 1. Block diagram of a single user LSSTC system.

serial-to-parallel converted to produce K streams (layers) of length B_1, B_2, \ldots, B_K , where $B_1+B_2+\cdots+B_K = B$. Each group of B_k bits, $k \in [1, K]$, is then encoded by a component space-time code STC_k associated with m_k transmit AAs, where $m_1 + m_2 + \cdots + m_K = M$. The output of the k^{th} STC encoder is a $m_K \times l$ codeword, \mathbf{c}_i , that is sent over ltime intervals. The space-time coded symbols from all layers can be written as $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]^T$, where \mathbf{C} is an $M \times l$ matrix, and superscript 'T' denotes the matrix transposition.

The coded symbols from **C** are then processed by the corresponding beamformers, and then transmitted simultaneously. The transmit antennas of all groups are synchronized and allocated equal power, moreover, the total transmission power is fixed, where the transmitted symbols have an average power of $P_T = 1$, where the average is taken across all codewords over both spatial and temporal components. For the LSSTC system to operate properly, the number of receive antennas N_R should be at least equal to the number of layers K.

Denote the *L*-dimensional channel impulse response (CIR) vector spanning the m^{th} AA, $m \in [1, \ldots, M]$ and the n^{th} receiver antenna, $n \in [1, \ldots, N_R]$ as $\mathbf{h}_{n,m}(t)$. Over flat fading channels $\mathbf{h}_{n,m}$ can be expressed as [8]

$$\mathbf{h}_{n,m}(t) = [\mathbf{d}_{n,m}]^T \cdot \alpha_{n,m}(t), \tag{1}$$

where $\alpha_{n,m}$ is the rayleigh faded coefficient coupling the m^{th} AA to the n^{th} receiver antenna, and $\mathbf{d}_{n,m}$ is the adaptive antenna array response corresponding to the m^{th} AA and the n^{th} receiver antenna, defined as [8]

$$\mathbf{d}_{n,m} = [1, e^{-j2\pi d(m)\sin(\Psi_{n,m})/\lambda}, \\ \dots, e^{-j2\pi (L-1)d(m)\sin(\Psi_{n,m})/\lambda}]^T$$
(2)

Where d(m) is the distance between the elements of the m^{th} AA, $\Psi_{n,m}$ is the nm^{th} link's direction of arrival (DOA). and superscript 'T' denotes the matrix transposition. We assume independent Rayleigh coefficients. The system model also

assumes that the receiver has perfect channel state information(CSI), whereas the transmitter has the DOA data sent from the receiver.

This system model can be described in matrix notation where the received baseband data matrix \mathbf{Y} can be expressed as

$$\mathbf{Y} = \mathbf{HWC} + \mathbf{N},\tag{3}$$

where Y is the received signal over l time intervals and has a dimension of $N_R \times l$, and **H** is an $N_R \times M$ matrix whose entries are $\mathbf{h}_{n,m}$ defined in (1), and **N** is an $N_R \times l$ matrix that characterizes the Additive White Gaussian Noise (AWGN). Furthermore, **W** is an $M \times M$ diagonal weight matrix, whose diagonal entry $\mathbf{w}_{m,m}$ is the L-dimensional beamforming weight vector for the m^{th} beamformer AA and the n^{th} receive antenna, and can be written as $\mathbf{w}_{m,m} = [b_{m1}, \cdots, b_{mL}]$, where b_{mi} , $i \in [1, \ldots, L]$, is the i^{th} weighting gain of the m^{th} AA.. The beamforming vector \mathbf{w}_{mm} can be found by [8] $\mathbf{w}_{m,m} = \mathbf{d}_{n,m}^*$, where the superscript * represents the hermitian conjugate operator. Now, we can define a modified channel matrix as $\hat{\mathbf{H}} = \mathbf{HW}$ according to [7] it was shown that the channel coefficient of the n^{th} row and the m^{th} column, $\hat{\mathbf{H}}(n,m)$, can be expressed as

$$\hat{\mathbf{H}}(n,m) = L \cdot \alpha_{n,m}.$$
(4)

Therefore the received signal can be expressed as in [7]:

$$\mathbf{Y} = L\mathbf{H}\mathbf{C} + \mathbf{N},\tag{5}$$

where $\tilde{\mathbf{H}}$ is an $(N_R \times M)$ matrix whose entries are $\alpha_{n,m}$. Looking at (5), the effect of beamforming can be clearly seen, which is a direct SNR gain.

Throughout this paper, whenever the phrase "sub-stream" is mentioned it refers to the data stream of each AA, which we denote as x, whereas, the term "layer", denoted by s, represents the data stream to be encoded by STBC. In the case of Alamouti's $2T_x - 1R_x$ scheme, $\mathbf{s} = [x_1, x_2]^T$ where x_i is the symbol of the i^{th} AA substream.

III. PERFORMANCE ANALYSIS OF LSSTC

In this section we derive a nearly exact error probability analysis for the LSSTC with Serial Group Interference Cancelation (SGIC) receiver employing Binary Phase-Shift Keying (BPSK) modulation. In the analysis we will take the effect of error propagation into account. We will analyze the system assuming that the power is equally split among the AAs. Our analysis will give recursive expressions of error probability of each symbol which is evaluated using a recursive procedure [9]. In our analysis we assumed using Alamouti's encoding matrix where two symbols are transmitted over two time slots from each layer.

For the purpose of finding the probability of error we will use the virtual MIMO model proposed in [10], where the system will be equivalent to an M branch VBLAST system. For convenience we rewrite (5) as:

$$\mathbf{y}_{(N_R*l)\times 1} = L \underbrace{\mathbf{H}_v}_{(N_R*l)\times M} \underbrace{\mathbf{s}}_{M\times 1} + \underbrace{\mathbf{n}}_{(N_R*l)\times 1}, \tag{6}$$

where \mathbf{H}_v can be partitioned into groups corresponding to each sub-stream as $\tilde{\mathbf{H}}_v = [\mathbf{h}_1, \dots, \mathbf{h}_M]$. Now, that we have the received signal, the detector will perform SGIC, and here we used the non-ordered scheme since the post-ordered scheme will make our analysis more complicated, also in [11] it is also shown that the post-ordering does not result in increased diversity order, but only in a fixed SNR gain [9]. The detector will apply the algorithm discussed in [5], [12], where at the end of each stage after subtracting the contribution of $\{s_1, \dots, s_k\}$ we can write the updated received signal as

$$\mathbf{y}^{k} = \mathbf{y} - \sum_{j=1}^{k} \mathbf{h}_{j} \hat{s}_{j}$$

$$= \sum_{\substack{j=k+1\\\text{faded target signal\\\text{with interference}}}^{M} \mathbf{h}_{j} s_{j} + \underbrace{\left(\mathbf{n} + \sum_{j=1}^{k} \mathbf{h}_{j} \cdot (s_{j} - \hat{s}_{j})\right)}_{\text{equivalent noise}},$$
(7)

where it can be seen that y^k is composed of three parts: the yet to be detected symbols, the noise vector and the potential error propagation signal. We refer to the last two terms of (7) as the equivalent noise. Assuming a total transmit power of P_t , each AA will have (P_t/M) of transmit power, and since each AA will result in one dimensional $h_{i,j}$ after multiplying by the weight matrix W, then we treat each AA as one antenna for the purpose of calculating transmit power and received SNR. According to [13], if a system has M independent one-dimensional sub-channels, the exact probability of bit error on the k^{th} symbol using BPSK modulation can be expressed as

$$P_{e_k} = \left[\frac{1}{2}(1-\mu)\right]^{D_k} \sum_{t=0}^{D_k-1} \binom{D_k-1+t}{t} \left[\frac{1}{2}(1+\mu)\right]^t = P_e(D_k,\rho),$$
(8)

where $\mu = \sqrt{\frac{\rho}{1+\rho}}$, ρ is the sub-stream SNR, $\rho = \frac{(L^2 P_t/M)}{N_0}$, and D_k is the diversity order of the layer $\Gamma(k)$ from which the k^{th} symbol is transmitted, because all the sub-streams associated with the same layer have the same diversity order, for instance if we used alamouti's STBC, the first and second sub-streams will have the same diversity order. The diversity order of the k^{th} sub-stream is given by $D_k = m_{\Gamma(k)} (N_R - K + \Gamma(k))$. As shown in [3] the probability of error for any symbol using STBC is equal, as they have the same diversity order and is equal to the average probability of

error, therefore all the sub streams of the same layer has the same probability of error. when considering the presence of error propagation, the probability of error can be expressed as

$$P_{e_{k}} = \operatorname{Prob}\{S_{k} \neq \hat{S}_{k}\} \\ = \sum_{i=0}^{k-1} \operatorname{Prob}\{S_{k} \neq \hat{S}_{k} \mid A_{k-1}^{i}\} \operatorname{Prob}\{A_{k-1}^{i}\},$$
(9)

where A_{k-1}^i defines the event of having *i* errors in the symbols $\hat{S}_1 \sim \hat{S}_{k-1}$. Now in order to find P_{e_k} , we need to find $\operatorname{Prob}\{S_k \neq \hat{S}_k \mid A_{k-1}^i\}$ and $\operatorname{Prob}\{A_{k-1}^i\}$ first, which will be discussed below. According to the methodology presented in [9] the equivalent noise ,denoted by $\mathbf{N}^{i,k}$, can be assumed to follow a white Gaussian distribution. We calculate the mean, and covariance matrix of $\mathbf{N}^{i,k}$ in [14] as

$$\mathbf{E}\left[\mathbf{N}^{i,k}\right] = E[\mathbf{n}] = 0 \tag{10}$$

$$\operatorname{Cov}\left[\mathbf{N}_{m}^{i,k},\mathbf{N}_{n}^{i,k}\right] = \left[N_{0} + \frac{4P_{t}iL^{2}}{M}\right]I_{N_{R}\times N_{R}},\qquad(11)$$

and thus $\operatorname{Prob}\{S_k \neq \hat{S}_k \mid A_{k-1}^i\}$ in (9) can be expressed as

$$\operatorname{Prob}\{S_{k} \neq \hat{S}_{k} \mid A_{k-1}^{i}\} = P_{e}\left(m_{k}(N_{R} - K + k), \frac{P_{t}L^{2}}{MN_{0} + 4P_{t}iL^{2}}\right).$$
(12)

Next, we will summarize the formulas used to find $\operatorname{Prob}\{A_{k-1}^i\}$ for three cases using the approach adopted in [9]. The details of the derivation can be reviewed from [14].

$$\operatorname{Prob}\{A_{k-1}^{i}\} = \left\{ \begin{bmatrix} 1 - P_{e}\left(m_{k-1}(N_{R} - K + k), \frac{P_{t}L^{2}}{MN_{0}}\right) \end{bmatrix} \\ \times \operatorname{Prob}\{A_{k-2}^{0}\}, \quad i = 0 \\ P_{e}\left(m_{k-1}(N_{R} - K + k), \frac{P_{t}L^{2}}{MN_{0} + 4P_{t}(k-2)L^{2}}\right) \\ \times \operatorname{Prob}\{A_{k-2}^{k-2}\}, \quad i = k-1 \\ \operatorname{Prob}\{S_{k-1} \neq \hat{S}_{k-1} \mid A_{k-2}^{i-1}\}\operatorname{Prob}\{A_{k-2}^{i-1}\} \\ + \left[1 - \operatorname{Prob}\{S_{k-1} \neq \hat{S}_{k-1} \mid A_{k-2}^{i}\}\right] \operatorname{Prob}\{A_{k-2}^{i}\}, \quad 0 < i < k-1 \end{cases}$$
(13)

At this stage, the probability of error on the k^{th} layer denoted as P_{e_k} can be evaluated directly using (9), and from that we can find the probability of error of the individual sub streams by

$$\operatorname{Prob}\{x_m \neq \hat{x}_m\} = P_{e_{\Gamma(m)}},\tag{14}$$

where $\Gamma(m)$ is the layer from which the m^{th} sub-stream (x_m) is sent. x_m is the m^{th} substream, and the average probability of error over all M sub-streams can be found by averaging over all sub-streams

IV. DIVERSITY, MULTIPLEXING, AND BEAMFORMING TRADEOFF IN LSSTC

In [15] the authors have found the tradeoff curve for a MIMO system that has the capability of providing both diversity and multiplexing advantage. In this work, we add to that the beamforming advantage of LSSTC by providing a comparison among the LSSTC system configurations. A system is said to have a diversity gain of d if the error probability decays as $(SNR)^{-d}$ [15], and a spatial multiplexing gain of r if the rate of the scheme is $(r \log SNR)$.

In an LSSTC system with N_T transmit and N_R receive antennas, assuming the path gains between individual antenna pairs are i.i.d. Rayleigh faded, the maximum diversity gain ignoring the antennas assigned for beamforming is $\left(\frac{N_TN_R}{L}\right)$, which is the total number of fading realizations over which system performance is averaged.

The tradeoff curve shows the diversity advantage achievable by the LSSTC system for each multiplexing gain r, and beamforming gain which we define as the number of beamforming elements (L). Clearly, L cannot exceed the total number of transmit antennas N_T . On the other hand, r cannot exceed the total number of degrees of freedom provided by the channel $min\left(\frac{N_TN_R}{L}, N_R\right)$; and d(r, L) cannot exceed the maximum diversity gain of the channel $\left(\frac{N_TN_R}{L}\right)$. The tradeoff curve links between these three extreme limits. The tradeoff curve is found in a similar manner to [15], and is given by the piecewise-linear function connecting the points $(r, d(r, L)), r = 0, 1, \ldots, \min\left\{\frac{N_TN_R}{L}, N_R\right\}$. For each possible value of L, the diversity gain d(r) is given by $d(r, L) = \left(\frac{N_T}{L} - r\right) \left(N_R - r\right)$.

V. NUMERICAL RESULTS

In all the simulations conducted in this work, the STBC encoders used Alamouti's STBC matrix with unity rate.



Fig. 2. SER of LSSTC employing non-ordered SGIC at 4 bps/Hz and different modulation schemes with $N_T = 8 \& N_R = 4$ (comparing VBLAST to LSSTC fairly).

A fair comparison between LSSTC and VBLAST is conducted. This fairness is achieved by structure and spectral efficiency fairness, that means that the total number of antennas at the transmitter N_T and the number of symbols sent every time slot are the same for both systems. Figure 2 shows a comparison between LSSTC and VBLAST in terms of the symbol error rate. The two systems use a total number of transmit antennas, $N_T = 8$, and the receiver is equipped with 4 antennas. In this comparison we have also compared many transmitter configurations, in each a different modulation scheme is used such that the spectral efficiency would be the same for all of them, which is set to 4 bps/Hz. From Figure 2 it can be clearly seen that VBLAST outperforms LSSTC in the low range of SNR, whereas for values of SNR that exceed 9 dB, the LSSTC outperforms VBLAST because it has a higher diversity order resulting from using STBC, which drives the SER to decay sharply.

Table I lists the proposed transmitter configuration, and modulation scheme depending on the SNR level in the system. For example if the SNR in some wireless system ranges in the second range($6.6 \ dB - 9.2 \ dB$), then the performance will be better if VBLAST scheme with 16-QAM modulation is used, while if it lies in the last range(> 9.2 \ dB) then the system will perform better if we used LSSTC scheme with 16-QAM modulation. One might say, why do not we

 TABLE I

 PROPOSED TRANSMITTER CONFIGURATION AND MODULATION SCHEMES.

SNR level (dB)	Transmitter configuration	Modulation scheme
< 6.6	VBLAST	QPSK
6.6 - 9.2	VBLAST	16-QAM
> 9.2	LSSTC	16-QAM

design an adaptive system that chooses between VBLAST and LSSTC? This can be done using an antenna array with the capability of electronically activating specific antenna elements and deactivating the remaining ones. This is done to meet the antenna separation conditions of each mode in the multiconfiguration system. In LSSTC, there are two conditions for the antenna element separation. (1) The AAs should be sufficiently far apart in order to experience independent fading. (2) Beamforming elements within each AA should be spaced at small distance that is less than $\lambda/2$ for the sake of achieving beamforming. On the other hand, VBLAST requires all the antennas to be spaced sufficiently far from each other. Figure 3 shows the layers' SER of 16×2 LSSTC using SGIC detector without ordering employing BPSK modulation with K = 2 and L = 4 obtained from both the simulation and the analysis. The Figure compares the results obtained from the LSSTC scheme to those obtained from the simulation results with equal power allocation. It is clear that the Monte Carlo simulation makes a nearly perfect match to the analysis methods, which demonstrates the validity of the analysis proposed in this paper.

Figure 4 shows the diversity multiplexing tradeoff curve of a 16×8 LSSTC system. As we can see from the figure, two points of interest can be identified $d_{max} = d(r_{min}, L_{min}) =$



Fig. 3. Layers' SER of 16×2 LSSTC employing SGIC without ordering and BPSK modulation with K = 2 & L = 4 (comparing analysis to simulation results).

 $d(0,1) = \left(\frac{N_T N_R}{L}\right)$ and $r_{max} = min\left(\frac{N_T N_R}{L}, N_R\right)$. It can be noted that increasing the diversity advantage at a specific beamforming gain comes at a price of decreasing the spatial multiplexing gain, and vice versa.

Figure 5 shows the diversity-beamforming tradeoff of a $16 \times$ 8 LSSTC system. In Figure 3 it should be noted that the points to the right of each curve represent an achievable diversity gain for that specific configuration, whereas the points to the left of each curve are not achievable. the same applies to Figure 5.

Figure 6 shows the diversity-multiplexing-beamforming tradeoff of a 16×8 LSSTC system plotted in 3 - D format. It can be noted that increasing the diversity advantage and/or the beamforming gain comes at a price of decreasing the spatial multiplexing gain, and similar relationship is observed with replacing the last three parameters, and the opposite applies.

VI. CONCLUSIONS

In this paper, we analyzed the performance of LSSTC. A recursive formula for the probability of error of LSSTC was derived. The main results of this study showed that combining beamforming, STBC, and VBLAST has better performance than VBLAST at high SNR range. In addition, we analyzed the diversity, multiplexing, and beamforming tradeoff curve for LSSTC. This curve links between these three extremes, where increasing one parameter causes the other parameters to decrease and vice versa.

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Fig. 4. Diversity-Multiplexing tradeoff ($N_T = 16 \& m_k = 2 \& N_R = 8$).



Fig. 5. Diversity–Beamforming tradeoff ($N_T = 16 \& m_k = 2 \& N_R = 8$).

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Fig. 6. Diversity-Multiplexing–Beamforming tradeoff ($N_T=16$ & $m_k=2$ & $N_R=8).$

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