

# A NUMERICAL COMPARISON BETWEEN LSSTC AND VBLAST IN WIRELESS SYSTEMS

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**Abstract:** In this work we evaluate a recently proposed multiple-input multiple-output (MIMO) system called the Layered Steered Space-Time Codes (LSSTC) that combines the benefits of vertical Bell Labs space-time (VBLAST) scheme, space-time block codes (STBC) and beamforming. This evaluation is done by comparing the capacity and the error rate of LSSTC to the well-known MIMO system, known as VBLAST. For that, we derive a formula for the instantaneous capacity of single-user LSSTC. In addition, an adaptive scheme that is based on LSSTC and VBLAST systems is proposed. This scheme selects the configuration and the modulation scheme in order to improve the performance.

## 1 INTRODUCTION

Various techniques have been proposed to counter the problem of propagation conditions, and to achieve data rates that are very close to the Shannon limit. One of these techniques is using MIMO systems which uses antenna arrays at both the transmitter and the receiver. Wolniansky *et al.* has proposed in (?) the well-known MIMO scheme, known as VBLAST. In VBLAST architecture, parallel data streams are sent via the transmit antennas at the same carrier frequency.

While MIMO systems as VBLAST can improve the system capacity greatly, it is difficult to implement antenna arrays on hand-held terminals due to size, cost and hardware limitation(?), also it has poor energy performance and doesn't fully exploit the available diversity. In order to overcome these problems, Alamouti has presented in (?) a new scheme called STBC with two transmit and one receive antennas that provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit and two receive antennas. With the tempting advantages of VBLAST and STBC, many researchers have attempted to combine these two schemes to result in a multi-layered architecture called MLSTBC (?) with each layer being composed of antennas that corresponds to a specific STBC. This combined scheme arises as a solution to jointly achieve spatial multiplexing and diversity gains simultaneously. With ML-

STBC scheme, it is possible to increase the data rate while keeping a satisfactory link quality in terms of symbol error rate (SER).

In (?) beamforming was combined with MLSTBC to produce a hybrid system called the layered steered space time codes (LSSTC). The addition of beamforming to MLSTBC further improves the performance of the system by increasing the antenna gain in the direction of the desired user, while reducing it towards the interfering users. In this paper, we show the superiority of LSSTC over VBLAST by comparing their capacity and SER. Also we derive a formula for the instantaneous capacity of a single-user LSSTC system. In addition, an adaptive scheme based on LSSTC and VBLAST systems is proposed. This scheme selects the configuration and the modulation scheme to improve the performance.

## 2 SYSTEM MODEL OF LSSTC

Figure ?? shows the block diagram of a single-user LSSTC system proposed in (?). The system has  $N_T$  total transmitting antennas and  $N_R$  receiving antennas and is denoted by an  $N_T \times N_R$  LSSTC. The antenna architecture employed in Figure ?? has  $M$  transmit adaptive antenna arrays (AAs) spaced sufficiently far apart in order to experience independent fading and hence achieve transmit diversity. Each of the AAs consists of  $L$  elements that are spaced at a distance of  $d = \lambda/2$  to ensure achieving beamforming. A block

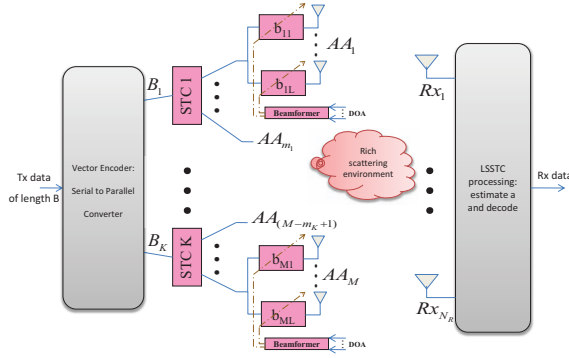


Figure 1: Block diagram of a single user LSSTC system.

of  $B$  input information bits is sent to the vector encoder of LSSTC and serial-to-parallel converted to produce  $K$  streams (layers) of length  $B_1, B_2, \dots, B_K$ , where  $B_1 + B_2 + \dots + B_K = B$ . Each group of  $B_k$  bits,  $k \in [1, K]$ , is then encoded by a component space-time code  $STC_k$  associated with  $m_k$  transmit AAs, where  $m_1 + m_2 + \dots + m_K = M$ . The output of the  $k^{\text{th}}$   $STC$  encoder is a  $m_k \times l$  codeword,  $\mathbf{c}_k$ , that is sent over  $l$  time intervals. The space-time coded symbols from all layers can be written as  $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_K]^T$ , where  $\mathbf{C}$  is an  $M \times l$  matrix.

The coded symbols from  $\mathbf{C}$  are then processed by the corresponding beamformers, and then transmitted simultaneously over the wireless channels. The transmit antennas of all the groups are synchronized and allocated equal power. Moreover, the total transmission power is fixed, where the transmitted symbols have an average power of  $P_T = 1$ , where the average is taken across all codewords over both spatial and temporal components. For the LSSTC system to operate properly, the number of receive antennas  $N_R$  should be at least equal to the number of layers  $K$ .

We formulate the system model as follows. The channel model is a MIMO quasi-static Rayleigh flat-fading channel with  $N_T$  transmit antennas and  $N_R$  receive antennas. The quasi-static assumption indicates that the channel gain coefficients remain constant for the duration of the STBC block and change independently for each STBC block. The flat-fading assumption allows each transmitted symbol to be represented by a single-tap in the discrete-time model with no inter-symbol interference (ISI). We assume independent Rayleigh coefficients, i.e., fading coefficients are independent and identically distributed (i.i.d.) circular-complex normal random variables with zero-mean and 0.5 variance per dimension, abbreviated as  $\mathcal{CN}(0, 1)$ . The correlation caused by the small distance separation is approximately removed using the beamforming processing as we will show in

this Section.

The system model can be described in matrix notation, where the received baseband data matrix  $\mathbf{Y}$  can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{W}\mathbf{C} + \mathbf{N}, \quad (1)$$

where  $\mathbf{Y}$  is the received signal over  $l$  time intervals and has a dimension of  $N_R \times l$ ,  $\mathbf{H}$  is an  $N_R \times M$  matrix whose entries are  $\mathbf{h}_{n,m}$ , and  $\mathbf{N}$  is an  $N_R \times l$  matrix that characterizes the additive white Gaussian noise (AWGN). The  $n^{\text{th}}$  row of  $\mathbf{N}$  denoted as  $\mathbf{z}_n$ , where  $n \in [1, \dots, N_R]$ , is a row vector of  $l$  columns, the  $i^{\text{th}}$  entry of  $\mathbf{z}_n$  is a spatially uncorrelated circular-complex normal random variable, and can be written as  $z_n^i = z_{I,n}^i + jz_{Q,n}^i$ , where  $z_{I,n}^i$  and  $z_{Q,n}^i$  are two independent zero-mean Gaussian random variables having a variance of  $N_0/2$ , we will represent  $z_n^i$  as  $\mathcal{CN}(0, N_0)$ . Furthermore,  $\mathbf{W}$  is an  $M \times M$  diagonal weight matrix, whose diagonal entry  $\mathbf{w}_{m,m}$  is the  $L$ -dimensional beamforming weight vector for the  $m^{\text{th}}$  beamformer AA and the  $n^{\text{th}}$  receive antenna, and can be written as  $\mathbf{w}_{m,m} = [b_{m1}, \dots, b_{mL}]$ , where  $b_{mi}$ ,  $i \in [1, \dots, L]$ , is the  $i^{\text{th}}$  weighting gain of the  $m^{\text{th}}$  AA. The received signal  $\mathbf{Y}$  can be written in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_R} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{1,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{1,M}\mathbf{w}_{M,M} \\ \mathbf{h}_{2,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{2,M}\mathbf{w}_{M,M} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{N_R,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{N_R,M}\mathbf{w}_{M,M} \end{bmatrix} \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_K \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{N_R} \end{bmatrix}. \quad (2)$$

The beamforming vector  $\mathbf{w}_{m,m}$  is given by (2) as  $\mathbf{w}_{m,m} = \mathbf{d}_{n,m}^*$ , where the superscript  $*$  represents the conjugate of the matrix. Referring to (2), a modified channel matrix is defined as

$$\hat{\mathbf{H}} = \begin{bmatrix} \mathbf{h}_{1,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{1,M}\mathbf{w}_{M,M} \\ \mathbf{h}_{2,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{2,M}\mathbf{w}_{M,M} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{N_R,1}\mathbf{w}_{1,1} & \cdots & \mathbf{h}_{N_R,M}\mathbf{w}_{M,M} \end{bmatrix}, \quad (3)$$

where  $\hat{\mathbf{H}}$  is the reconstructed channel matrix comprising the MIMO fading channel and the DOA information. Note that we assumed that the nulling vector for all the paths corresponding to one AA ( $\mathbf{w}_{m,m}$ ) is the same. This follows from the assumption that the separation between the receive antennas is much less than the distance between the AA and the receiver, then roughly speaking, they will have the same direction of arrival, which will result in having the same nulling vector.

According to Equation (2)  $\mathbf{Y}$  can be rewritten as:

$$\mathbf{Y} = \hat{\mathbf{H}}\mathbf{C} + \mathbf{N}. \quad (4)$$

The channel coefficient  $\hat{\mathbf{H}}_{n,m}$  can roughly ex-

pressed as

$$\begin{aligned}\hat{\mathbf{H}}_{n,m} &= \mathbf{h}_{n,m} \mathbf{w}_{m,m} \\ &= \alpha_{n,m} \cdot [\mathbf{d}_{n,m}]^T [\mathbf{d}_{n,m}]^* \\ &= L \cdot \alpha_{n,m}.\end{aligned}\quad (5)$$

Therefore the received signal can be expressed as in (?):

$$\mathbf{Y} = L\tilde{\mathbf{H}}\mathbf{C} + \mathbf{N}, \quad (6)$$

where  $\tilde{\mathbf{H}}$  is an  $(N_R \times M)$  matrix whose entries are  $\alpha_{n,m}$ . Looking at (?), the effect of beamforming can be clearly seen, which is a direct SNR gain.

### 3 CAPACITY OF LSSTC

The system capacity of VBLAST is given by (?) as

$$C_{VBLAST} = N_T \min_{i=1,2,\dots,N_T} \left\{ \log_2 \left( 1 + \frac{SNR}{N_T \|W_{ZF,i}\|^2} \right) \right\}, \quad (7)$$

where SNR is the average signal-to-noise ratio, and  $\|W_{ZF,i}\|^2$  is the squared Frobenius norm of the zero-forcing projection row for the  $i^{th}$  layer.

To derive a formula for the capacity of LSSTC per user, we will follow the derivation of (?). First, the instantaneous capacity was found in (?) for an orthogonal STBC with  $M_s$  transmit antennas and  $R_s$  code rate,

$$C_{STBC} = R_s \cdot \log_2 \left( 1 + \frac{P_T}{M_s} \|\mathbf{H}\|^2 \right) \quad (8)$$

In MLSTBC which is a combination of VBLAST and STBC, an outage occurs if an outage happens in any layer because all the STBC encoders (layers) are transmitting at the same rate. The layer that is the most probable to fall in an outage is the weakest layer, i.e. the one that has the least value of  $\|\mathbf{H}_i\|^2$ ,  $i = 1, 2, \dots, K$ , where  $\mathbf{H}_i$  is the  $i^{th}$  matrix of  $\mathbf{H}$ . Therefore, the instantaneous capacity of a  $K$  layered STBC system with a sub-stream SNR of  $\rho$  can be written as:

$$\begin{aligned}C &= K \cdot R_s \cdot \log_2 \left( 1 + \rho \cdot \min_{k=1,2,\dots,K} \{ \|\mathbf{H}_i\|^2 \} \right) \\ &= K \cdot R_s \cdot \min_{k=1,2,\dots,K} \{ \log_2 (1 + \rho \cdot \|\mathbf{H}_i\|^2) \}.\end{aligned}\quad (9)$$

Extending the last results, the instantaneous capacity of LSSTC can be expressed as:

$$C_{LSSTC} = KR_s \min_{k=1,2,\dots,K} \left\{ \log_2 \left( 1 + \frac{L^2 \cdot P_T}{M \cdot N_0} \cdot \|\mathbf{H}_{PP,k}\|^2 \right) \right\} \quad (10)$$

where  $\mathbf{H}_{PP,k}$  is the Post-Processing (PP) matrix corresponding to the  $k^{th}$  layer after nulling out the interference from the yet-to-be-detected layers. It is clear that the LSSTC capacity is dominated by the worst group which has the minimum value of  $\mathbf{H}_{PP,k}$ ,  $k = 1, 2, \dots, K$ .

## 4 NUMERICAL RESULTS

In all the Monte-Carlo simulations conducted in this work, we used Alamouti's STBC matrix of unity rate for the STBC encoders in each layer. In addition, unless otherwise mentioned, non-ordered serial group interference cancellation (SGIC) detector is used.

Figure ?? shows a fair comparison between LSSTC and VBLAST in terms of the symbol error rate. The two systems use a total number of transmit antennas,  $N_T = 8$ , and the receiver is equipped with 4 antennas. In this comparison we have also compared many transmitter configurations, in each a different modulation scheme is used such that the spectral efficiency would be the same for all of them, which is set to 4 bps/Hz. From Figure ?? it can be clearly seen that VBLAST outperforms LSSTC in the low range of SNR, whereas for values of SNR that exceed 9 dB, the LSSTC outperforms VBLAST because it has a higher diversity order resulting from using STBC, which drives the SER to decay sharply. Next, we pro-

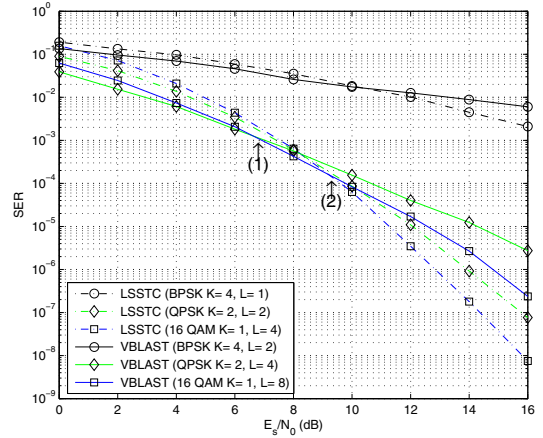


Figure 2: SER of LSSTC employing non-ordered SGIC at 4 bps/Hz and different modulation schemes with  $N_T = 8$  &  $N_R = 4$  (comparing VBLAST to LSSTC fairly).

pose an adaptive transmission scheme that selects the configuration and the modulation scheme in order to improve the performance. Table ?? lists the proposed transmitter configuration and modulation scheme depending on the SNR level. The adaptive scheme can

Table 1: Proposed Tx. and modulation configuration.

| SNR level (dB) | configuration | Modulation |
|----------------|---------------|------------|
| < 6.6          | VBLAST        | QPSK       |
| 6.6 – 9.2      | VBLAST        | 16-QAM     |
| > 9.2          | LSSTC         | 16-QAM     |

be designed using an antenna array with the capability

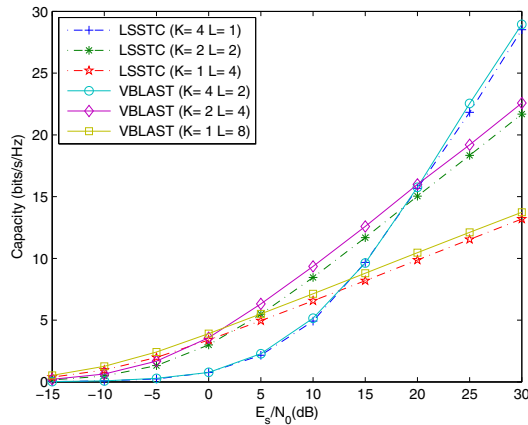


Figure 3: Outage Capacity vs.  $E_s/N_0$  for an  $8 \times 4$  MIMO at 10% Outage probability, and 15 dB average SNR (comparing VBLAST to LSSTC fairly).

of electronically activating specific antenna elements and deactivating the remaining ones. This is done to meet the antenna separation conditions of each mode in the multi-configuration system. In LSSTC, there are two conditions for the antenna element separation. (1) The AAs should be sufficiently far apart in order to experience independent fading. (2) Beamforming elements within each AA should be spaced at small distance (less than  $\lambda/2$ ) to achieve beamforming. On the other hand, VBLAST requires all the antennas to be spaced sufficiently far from each other.

Figure ?? fairly compares LSSTC to VBLAST in terms of the outage capacity of an  $8 \times 4$  MIMO using non-ordered SGIC at 15 dB average SNR. Several configurations are considered, and the capacity is plotted versus  $E_s/N_0$ . As it can be seen from the figure, the capacity is approximately linearly increasing with increasing  $E_s/N_0$ . It is clear to see that VBLAST outperforms LSSTC, which is actually expected, since VBLAST is a pure spatial multiplexing unlike LSSTC, where some antennas are assigned for diversity. An adaptive system can be designed to maximize the capacity for all values of SNR. For the fore-mentioned configuration we choose the single-layer VBLAST system for the first range (-15 dB up to 1 dB), and for the second range (1 dB up to 20 dB) the dual-layer VBLAST system gives the highest capacity. If the SNR lies in the last range (>20 dB), then using either LSSTC or VBLAST with 4 layers will have approximately the same capacity. However, Figure ?? shows that LSSTC has a lower SER in the last range, and therefore, choosing LSSTC is better.

## 5 CONCLUSIONS

In this paper, we evaluated the performance of LSSTC by comparing it to VBLAST. Also an adaptive system that selects between LSSTC and VBLAST was proposed. This study showed that combining beamforming, STBC, and VBLAST in LSSTC has better performance than VBLAST at high SNR range.

## ACKNOWLEDGEMENTS

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## REFERENCES

- Al-Ghadhban, S., Buehrer, R., and Woerner, B. (2005). Outage Capacity Comparison of Multi-Layered STBC and V-BLAST Systems. *Vehicular Technology Conference, IEEE 62nd*, 1:24–27.
- Alamouti, S. (1998). A Simple Transmit Diversity Technique for Wireless Communications. *Selected Areas in Communications, IEEE Journal on*, 16(8):1451–1458.
- El-Hajjar, M. and Hanzo, L. (2007). Layered steered space-time codes and their capacity. *IEEE Electronics Letters*, pages 680–682.
- Mohammad, M., Al-Ghadhban, S., Woerner, B., and Tranter, W. (2004). Comparing Decoding Algorithms for Multi-Layered Space-Time Block Codes. *IEEE SoutheastCon Proceedings*, pages 147–152.
- Sandhu, S. and Paulraj, A. (2000). Space-Time Block Codes: a Capacity Perspective. *Communications Letters, IEEE*, 4(12):384–386.
- Shu, F., Lihua, L., Xiaofeng, T., and Ping, Z. (2007). A Spatial Multiplexing MIMO Scheme with Beamforming for Downlink Transmission. *IEEE Vehicular Technology Conference*, pages 700–704.
- Wolniansky, P. W., Foschini, G. J., Golden, G. D., and Valenzuela, R. A. (1998). V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel. *URSI International Symposium on Signals, Systems and Electronics*, pages 295–300.