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Performance of CDMA-Based Multi-hop Wireless Networks in Fading

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Abstract*— n this paper, we derive the end-to-end error probability of multi-hop networks employing CDMA transmission in a multiuser environment. The standard Gaussian approximation (SGA) is used to model the multiple access interference (MAI) in the system. We consider multihop networks employing serial, parallel and selection relaying using decode-and-forward (DF) technique. Additionally, we investigate the effect of different system parameters on the error performance of the system, such as the number of users and the path loss exponent. Results show that as the number of users increases, the improvement coming from the relaying decreases. However, relaying still gives significant improvement, especially in parallel relaying.*n this paper, we derive the end-to-end error probability of multi-hop networks employing CDMA transmission in a multiuser environment. The standard Gaussian approximation (SGA) is used to model the multiple access interference (MAI) in the system. We consider multihop networks employing serial, parallel and selection relaying using decode-and-forward (DF) technique. Additionally, we investigate the effect of different system parameters on the error performance of the system, such as the number of users and the path loss exponent. Results show that as the number of users increases, the improvement coming from the relaying decreases. However, relaying still gives significant improvement, especially in parallel relaying.I**

I. INTRODUCTION

Multipath fading is a serious problem to wireless systems, for which diversity is a fundamental solution. Multi-hop networking can enhance the system capacity and provide diversity in a distributed manner. In multi-hop networks, mobile terminals are used to relay the data for other users in order to provide larger coverage area, low power consumption and better quality of service in fading conditions [1]. Multihop topology can be used in existing neworks employing code division multiple access (CDMA) for multiple access. Users' signals in a CDMA network are distinguished by spreading the information signal of each user using a different pseudo-random noise (PN) code [2]. Multiple access interference (MAI) arises in CDMA networks due to the imperfect PN codes or time jitter.

In [3], Hasna *et al.* derived the end-to-end error performance of a two-hop network over Rayleigh fading channels, which was extended in [4] to N-hop serial non-regenerative relaying model. In [5], Hasna *et al.* studied the performance of a two-hop wireless networks with amplify-and-forward (AF) relays having fixed gains in terms of outage probability and average BER. In [6], Yang *et al.* presented a closed-form expression for the average outage duration (AOD) of multi-hop networks employing decode-and-forward (DF) relays over different fading channel models. In [7], Sunay *et al.* derived the error probabilities for various CDMA systems using the standard Gaussian approximation (SGA) and other approximations.

In this paper, we derive and investigate the error performance of multihop CDMA-based networks employing serial, parallel and selection relaying over different values of spreading gain, number of users and path-loss exponent. The main contribution of this paper lies in the analysis of multihop network topologies in CDMA environments, by considering combining signals received from the source as well as the relays. This assumption is very practical and has never been analyzed up to the author knowledge. The rest of this paper is organized as follows. Section II describes the system model. The performance analysis of CDMA-based multihop networks is presented in Section III. In Section IV, numerical results are discussed. Finally, Section V presents the main conclusions of this work.

II. SYSTEM MODEL

In this paper, we consider multihop wireless networks employing CDMA transmission in the physical layer. The CDMA tranmitter is composed of a PN code generator and a BPSK modulator, whereas the receiver is composed of a BPSK demodulator and a correlator. We assume that the information rate at the input to the modulator is R bits/sec, the available channel bandwidth is W Hz, and the spreading gain is $V = W/R$. Assuming that there are K active users in the network, the

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received signal at any of the users is

$$r(t)=\sum_{k=1}^{K}\sqrt{2P_k d_k^{-n}}\alpha_k b_k(t-\tau_k)c_k(t-\tau_k)\cos(\omega_c t+\phi_k)+\eta(t), \tag{1}$$

where P_k is the power of the transmitted signal of user k, $b_k(t)$ and $c_k(t)$ are the data and spreading signals of user k respectively, ω_c is the signaling frequency and θ_k is the phase shift of user k, α_k is the fading component of user k, d_k is the distance from user k to the destination, n is the path loss exponent, τ_k and ϕ_k are the phase and time delay introduced by the channel of user k, and $\eta(t)$ is the AWGN introduced at the receiver. At each user, a correlation receiver is typically used to filter the desired user's signal from all the other users' signals which share the same channel. Assuming the receiver has perfect estimate about the delay of the signal of user 1, the decision statistics for user 1 are given by [7]

$$Z_1=\sqrt{\frac{1}{2}P_1T_b^2 d_1^{-n}}\alpha_1 b_1+\mathrm{MAI}+\hat{\eta}, \tag{2}$$

where $\hat{\eta}$ is an AWGN random variable with variance equal to $N_0T_b/4$ and MAI denotes the multiple access interference term derived in [7].

Figures 1 and 2 show the network topology of the serial and parallel schemes, in which the source node (S) transmits information to the destination node (D) with the help of other users referred to as relays (R). In this paper, all relays employ DF relaying [6]. For fair comparison between the direct and multihop tranmission, we assume that transmission takes place according to the time slotting shown in Figures 1 and 2. In order to get the maximum of the system, the destination uses maximal-ratio combining (MRC) to combine the received signals from both the relay and the source. In order to be fair in comparing the relayed system with the direct system, we reduce the spreading gain in the relayed systems. By doing so, we maintain the system throughput to be equal in all cases.

III. PERFORMANCE ANALYSIS

A. Direct Transmission

A method called the standard Gaussian approximation (SGA) [8] is used to derive the BEP of CDMA systems based on the argument that the decision statistic, Z_1 given in (2), may be modeled as a a zero-mean Gaussian random variable with variance [7] given by

$$\sigma_m^2=\frac{T_b^2}{6V}\sum_{k=2}^{K}P_k d^{-n}\alpha_k^2. \tag{3}$$

In this paper, we employ random PN sequences and assume that $d_k=d, k=1,\ldots K$, and that $P_1=P_k$ for all k. Define $E_b=P_1T_bd^{-n}$ as the average received energy per bit of User 1. For a receiver that combines the signals of the intended user from L diversity branches using MRC, the conditional BEP [9] is given by

$$P(E|\{\alpha_1\})=Q\left(\sqrt{\frac{\sum_{i=1}^{L}2\alpha_i^2E_b/N_0}{\frac{2E_b(K-1)}{3VN_0}+1}}\right). \tag{4}$$

Define the signal-to-interference-and-noise-ratio (SINR) as

$$\mathrm{SINR}(V,K)=\frac{2E_b/N_0}{\frac{2(K-1)E_b/N_0}{3V}+1}. \tag{5}$$

For generality, we assume that all α_i's are independent and identically distributed (i.i.d.) Rayleigh random variables, then the unconditional average BEP of a CDMA system with L-branch diversity, spreading gain V, and K users

$$P_b(L,V,K)=\frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L}\left(\frac{1}{1+\frac{\mathrm{SINR(V,K)}}{2\sin^2(\theta)}}\right)d\theta. \tag{6}$$

B. Serial Relaying

Figure 1 shows examples of serial relaying network. The source sends the data to all the relays in the first time slot, and then the relays send it in the subsequent time slots to the destination node in order. In order to maintain the same throughput and energy efficiency of the direct system, we reduce the spreading gain and the transmission energy in the relayed systems. In particular, the spreading gain is set to $V/(J+1)$, where J is the number of relays. The receiver combines signals from the source and the relays which have successfully detected the source data. This combining will result in different scenarios, which need to be accounted for in the analysis. In order to simplify the analysis, we define the following events, and we will use them in the rest of the analysis.

- E = Detection error at the destination.
- $P_b(L,V,K)$ = BEP in (6) given diversity order L, spreading gain V, and number of users K.

A general formula for the end-to-end error probability for serial relaying employing J relays [10] can be expressed as

$$\begin{aligned}P(E)&=P_b\Big(J+1,\frac{V}{J+1},K\Big)\Big[1-P_b\Big(1,\frac{V}{J+1},K\Big)\Big]^J\\&+\sum_{i=0}^{J-1}P_b\Big(i+1,\frac{V}{J+1},K\Big)\Big[1-P_b\Big(1,\frac{V}{J+1},K\Big)\Big]^i\\&\times P_b\Big(1,\frac{V}{J+1},K\Big),\end{aligned} \tag{7}$$

which can be easily evaluated using equation (6).

C. Parallel Relaying

Figure 2 shows an example of parallel relaying. The source sends the data to all the relays in the first time slot, and then all the relays send it in the next time slot to the destination node using CDMA fashion. The destination uses MRC to combine the received signals from all the relays and the source node. In order to be fair in the comparison, the spreading gain is always $V/2$ and the energy in the second hop is divided by the number of the relays. In addition, the number of interfering users will depend on the number of relays which detected the source data correctly, and hence participated in relaying the data to the destination. A general formula for the end-to-end error probability for parallel relaying with J relays [10] can be easily expressed as

$$P(E) = \sum_{i=0}^{J} \binom{J}{i} P_b(i+1, V/2, K+i)[1-P_b(1, V/2, K)]^i \times [P_b(1, V/2, K)]^{J-i}, \tag{8}$$

which can be easily evaluated using equation (6).

D. Selection Relaying

Selection relaying uses the same as the parallel topology in Figure 2 with specific relays cooperation according to certain criteria. The fading channels between the source and the relays are measured before each transmission in order to specify the relays which are going to cooperate in the transmission. We consider the following two selection schemes:

- Threshold-based relay selection (TRS): In this scheme, only the relays with fading above a certain threshold α_{th} join in the relaying. The spreading gain V is selected according to the number of the relays joining in the transmission. For example, when only one relay is joining in the transmission, the spreading gain becomes $V/2$, in order to be fair when comparing with the direct transmission case.
- Maximum relay selection (MRS): In this scheme, only one relay which has the maximum fading will relay the source information. Here, the spreading gain is always $V/2$.

For the sake of analysis, we define the outage probability as the probability that the fading value α is less than the threshold α_{th} required for relaying, which is given by $P_{\text{out}} = \int_0^{\alpha_{th}} f(\alpha) d\alpha$. A general formula for the end-to-end error probability for TRS relaying employing J relays [10] can be expressed as

$$P(E) = \sum_{j=0}^{J} \binom{J}{j} [1-P_{\text{out}}]^j P_{\text{out}}^{J-j} \sum_{i=0}^{j} \binom{j}{i} P_b(i+1, V/2, K+i) \times [1 - P_b(1, V/2, K)]^i [P_b(1, V/2, K)]^{j-i}, \tag{9}$$

which can be easily evaluated using equation (6).

For the MRS scheme, the end-to-end error probability is the same as for the single-relay serial scheme in (7), but with a fading distribution, which will lead to a different expression for $P_b(L, V, K)$. For the two-relay MRS scheme, the distribution [11] of the random variable defined as $\gamma = \max(\alpha_1^2, \alpha_2^2)$ is given by

$$f_\gamma(x) = \frac{1}{\sigma^2} \exp\left(\frac{-x}{2\sigma^2}\right) \left[1 - \exp\left(\frac{-x}{2\sigma^2}\right)\right]. \tag{10}$$

By averaging the conditional BEP in (4) over (10), we get the unconditional BEP to be

$$P_b(L, V, K) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} \left[\frac{2}{1 + \frac{\text{SINR(V,K)}}{\sin^2(\theta)}} - \frac{1}{1 + \frac{\text{SINR(V,K)}}{2\sin^2(\theta)}} \right] d\theta. \tag{11}$$

For the three-relay MRS, using the same procedure, the unconditional BEP becomes

$$P_b(L, V, K) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{L} \left[\frac{3}{1 + \frac{\text{SINR(V,K)}}{\sin^2(\theta)}} - \frac{3}{1 + \frac{\text{SINR(V,K)}}{2\sin^2(\theta)}} + \frac{3}{3 + \frac{\text{SINR(V,K)}}{\sin^2(\theta)}} \right] d\theta. \tag{12}$$

In order to find the end-to-end error probability for the MRS scheme, (7) is used with (11) and (11) for the cases of two-relay and three-relay schemes, respectively.

IV. NUMERICAL RESULTS

In this section, we show analytical and simulation results for the discussed multihop network topolgies. For the sake of simplicity, we assume that the relays are at equal distances from the source and the destination. The path loss exponent n is set to 3, except when we test the effect of the path loss on the system. Figures 3 and 4 show the analysis and simulation results for the serial and parallel topologies, respectively. We notice that the overall performance is bad due to MAI, but relaying improves the performance significantly. In addition, the performance of the parallel relaying is much better than the serial relaying due to to the added diversity in the parallel case. Also the three-relay scheme is much better than the two-relay scheme, even though it is using a smaller spreading gain, which is due to more dominant effect of diversity gain as compared to the loss due to MAI.

Figures 5 and 6 show the results of the TRS and MRS schemes, respectively. The selection threshold was set to $\alpha_{th} = 0.5$. This value of $\alpha_{th} = 0.5$ is chosen such that the outage probability is 0.1175. We note that the performance of TRS is better than the serial scheme but worse than the parallel scheme. Further, the TRS performance depends on the choice of the threshold value α_{th}. As the threshold increases, fewer relays join, and so the performance approaches the serial case. On the other hand, as the threshold decreases, more relays join, and the performance approaches the parallel case. It can be noticed that the performance of MRS is worse than the TRS scheme because it uses only one relay all the time, and hence has less diversity gain. However, it performs better than the single-relay serial scheme because the reduced probability of falling into deep fading.

Figure 7 shows the performance of the direct, serial, parallel, MRS and TRS relaying systems against the number of users when the SNR = 20 dB and spreading gain $V = 128$. We notice that both the error performance and the relaying gain decrease as the number of users increases. Increasing the spreading gain reduces the effect of MAI, and hence the diversity effect becomes more visible. In Figure 8, we show the effect of the path loss exponent on the performance of multihop networks. Results show that the higher the path loss exponent the better the performance of the relayed system. In environments with large path loss exponent, sending the data over short distances saves more power than in environments with small path loss. So the gain given by relaying to the system gets better as the path loss exponent increases.

V. CONCLUSIONS

In this paper we derived the end-to-end error probability for multihop networks employing CDMA and serial, parallel and selection topologies. Results showed that the analysis and simulation results are close to each other. Additionally, the error performance of the system with serial, parallel, and selection relaying was investigated with different values of spreading gain and number of users.

VI. ACKNOWLEDGEMENTS

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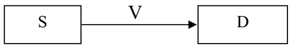
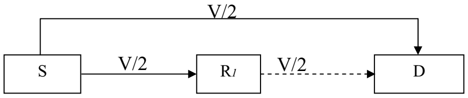
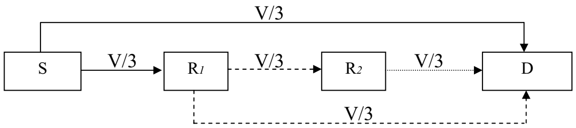


(a) Direct Transmission



(b) Single-Relay



(c) Two-Relays

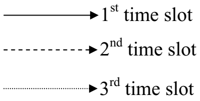
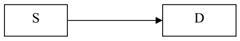
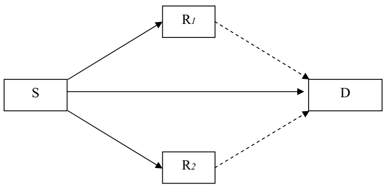


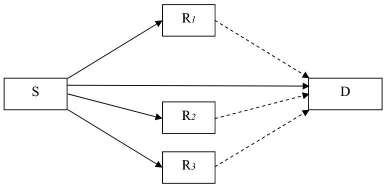
Fig. 1. Serial multihop network. (a) Direct transmission with spreading gain V, (b) single-relay network with spreading gain V/2, and (c) Two-relay network with spreading gain V/3.

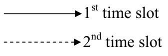


(a) Direct Transmission



(b) Two-Relays





(c) Three-Relays

Fig. 2. Parallel multihop network. (a) Direct transmission, (b) Two-relay, and (c) Three-relay.

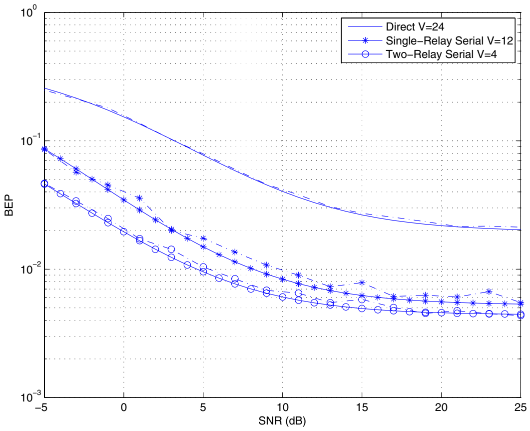


Fig. 3. Performance of the direct, single-relay and two-relay serial networks over Rayleigh fading channels with $K = 4$: (solid) analysis, (dashed) simulation.

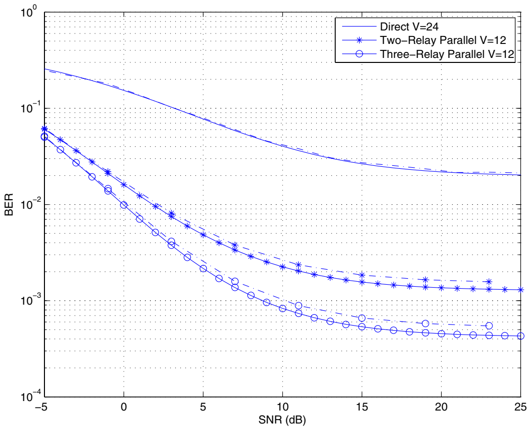


Fig. 4. Performance of the direct, two-relay and three-relay parallel networks over Rayleigh fading channels with $K = 4$: (solid) analysis, (dashed) simulation.

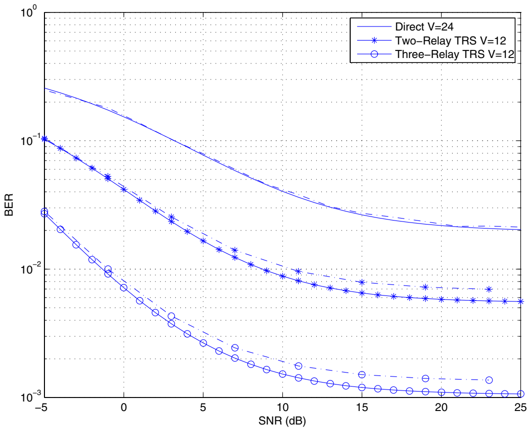


Fig. 5. Performance of the direct, two-relay and three-relay TRS networks over Rayleigh fading channels with K=4 and $\alpha_{th} = 0.5$: (solid) analysis, (dashed) simulation.

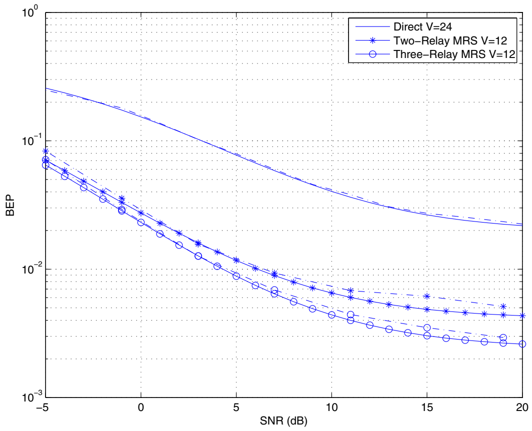


Fig. 6. Performance of the direct, two-relay and three-relay MRS networks over Rayleigh fading channels with $K = 4$: (solid) analysis, (dashed) simulation.

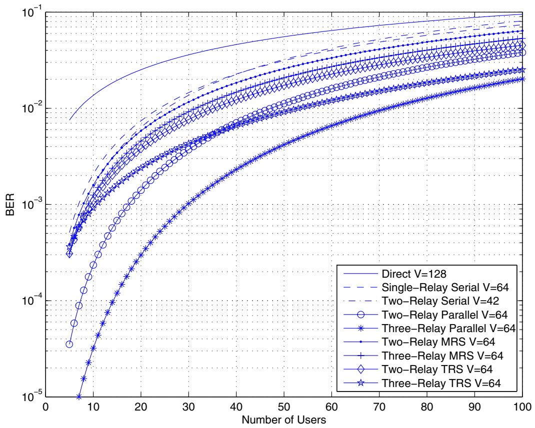


Fig. 7. Performance analysis for the direct, serial, parallel, MRS and TRS networks in Rayleigh fading channels against K when SNR = 20 dB and V = 128.

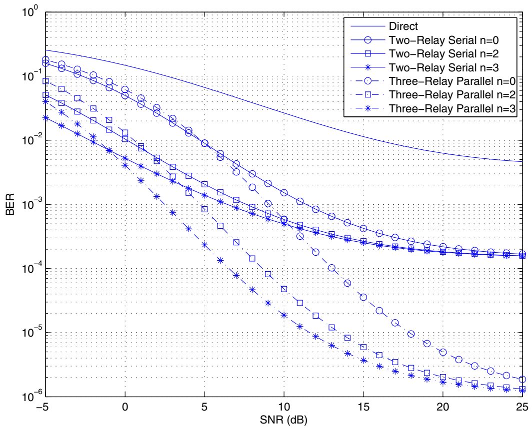


Fig. 8. Effect of path loss exponent of the performance of the serial two-relay and the parallel three-relay schemes over Rayleigh fading channel with $K = 4$, $V = 128$, and $n = 0, 2,$ and 3.