Spectral Efficiency Evaluation of Generalized Selection Combiners (GSC) over Slow Fading with Estimation Errors

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Abstract—**In this paper we derive closed-form expressions for the single-user capacity of generalized selection combiners (GSC) system, taking into account the effect of imperfect channel estimation at the receiver. The channel considered is a slowly varying spatially independent flat Rayleigh fading channel. The complex channel estimate and the actual channel are modelled as jointly Gaussian random variables with a correlation that depends on the estimation quality. Two adaptive transmission schemes are analyzed: 1) optimal power and rate adaptation; and 2) constant power with optimal rate adaptation, including derivation of an upper bound and asymptotically tight approximations at high and low SNR regions. Our numerical results show the effect of Gaussian channel estimation error on the achievable spectral efficiency.**
-Keywords: Generalized Selection Combiner (GSC), Channel Estimation Error, Fading Channels, Spectral Efficiency.

I. INTRODUCTION

Spatial diversity is a well-known and efficient way to combat multi-path fading and improve the system performance over fading channel. The most commonly used received diversity techniques include equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), and a hybrid combination of SC and MRC, called generalized selection combining (GSC). In the GSC, a fixed subset of size M of a large number of available diversity channels of size L is chosen and then combined using MRC. The MRC combines the branch signals such that the instantaneous output signal-to-noise ratio (SNR) is maximized [1], [2], [3].

System performance is often analyzed with the assumption of perfect channel estimate. However, in practice the branch signal-to-noise ratio (SNR) estimates are corrupted with channel impairments making it difficult to achieve perfect estimation. Normally, a diversity branch SNR estimate can be obtained either from a pilot signal or data signals (by applying a clairvoyant estimator) [4]. For example, if a pilot signal is inserted to estimate the channel, a Gaussian error may arise in due the large frequency separation or time dispersion. Therefore, it is important to include estimation errors in system performance analysis. Previous works on the analysis of imperfect channel estimation with and without diversity can be found in [4]-[8]. The paper in [8] considers the channel estimation error of the GSC per branch SNR as being complex Gaussian and derives the probability density function (PDF) of the output combiner. The pioneering work of Shannon [9] has established the significance of channel capacity as the maximum possible rate at which information can be transmitted over a channel. Spectral efficiency of adaptive transmission techniques has received extensive interest in the last decade. In [12], the capacity of a single user flat fading channel with perfect channel information at the transmitter and the receiver is derived for various adaptation policies, namely, 1) optimal rate and power adaptation (*opra*), 2)optimal rate adaptation and constant power (*ora*), and 3) channel inversion with fixed rate (*cifr*). The first scheme requires channel information at the transmitter and receiver, whereas the second scheme is more practical since the transmission power remains constant. The last scheme is a suboptimal transmission adaptation scheme, in which the channel side information is used to maintain a constant received power by inverting the channel fading [12]. In [13], the general theory developed in [12] is applied to achieve closed form expressions for the capacity of Rayleigh fading channel under different adaptive transmission and diversity combining techniques, also this work has been extended to many fading scenarios environments (here within [14],[15]). In this paper, we extend the results in [13] to obtain closed-form expressions for the single-user capacity of SCD system, in the presence of Gaussian channel estimation errors.

The contribution of this paper is to derive closed-expressions for two adaptive transmission schemes including their asymptotic approximations and upper bounds and these schemes are: (1) optimal simultaneous power and rate adaptation (*opra*). (2) optimal rate adaptation with constant transmit power (*ora*) including all its approximations that provide good measures in high and low SNR.

The paper is organized as follows. In Section II, the system model used in this paper is discussed. In Section III, we derive closed-form expressions for the channel capacity under two adaptation schemes; *opra* and *ora* including their asymptotic approximations and upper bounds in sub-sections III-A and in III-B, respectively. Results are presented and discussed in Section IV. The main outcomes of the paper are summarized in Section V.

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II. System Model

We consider an L-branch diversity receiver in slow fading channels. Assuming perfect timing and inter-symbol interference (ISI) free transmission, the received signal on the lth branch due to the transmission of a symbol s can be expressed as

$$r_l = g_l s + n_l, \quad l = 1 \dots L, \tag{1}$$

where g_l is a zero-mean complex Gaussian distributed channel gain, n_l is the complex additive white Gaussian noise (AWGN) sample with a variance of $N_0/2$, and s is the data symbol taken from a normalized unit-energy signal set with an average power P_s. Under channel estimation error, the PDF of the received instantaneous SNR γ is given by [8]

$$\begin{aligned} p_\gamma(\gamma) \;=\; & \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} [\overline{\gamma}(1-\rho^2)]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \\ & \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \exp\left(\frac{-\gamma}{\Lambda_l}\right) \end{aligned} \tag{2}$$

Where μ denotes the coefficients of partial fractions, $\mu_l^0 = \binom{L}{M}$for $l = 0,$ and $(-1)^{M+l-1}\left(\frac{M}{M+l}\right)\left(\frac{M}{l}\right)^{M-1} \binom{L}{M}\binom{L-M}{l}$ for $l \neq 0$. For $\mu_0^{M-k} = \sum_{i=l}^{L-M}(-1)^{i+M-k-1}\binom{L}{M}\binom{L-M}{i}\left(\frac{M}{i}\right)^{M-k}$ if $k < M < L$,and $(-1)^{M+l-1}\left(\frac{M}{M+l}\right)\left(\frac{M}{l}\right)^{M-1}\binom{L}{M}\binom{L-M}{l}$ for$l \neq 0$. $\Lambda_l = \overline{\gamma}\left[(M + l(1-\rho^2))/(l+M)\right]$ and ρ denotes the correlation between the actual channel gains and their estimates and it can be expressed as:

$$\rho = \frac{\operatorname{cov}(g_l, \widehat{g}_l)}{\sqrt{\operatorname{var}(g_l)\operatorname{var}(\widehat{g}_l)}} = \sqrt{1-\epsilon^2} \tag{3}$$

The actual channel gain g is related to the channel estimate $\widehat{g}$ [4] as follows

$$\widehat{g}_l = \sqrt{1-\epsilon^2} g_l + \epsilon z_l, \tag{4}$$

Where z_l is a complex Gaussian random variable independent of $\widehat{g}$ with zero-mean and a unit variance and $\epsilon \in [0, 1]$ is a measure of the accuracy of the channel estimation. The true channel is scaled to keep the covariance of the estimated channel and the true channel to be the same. For $\epsilon = 0$, the estimated channel is fully correlated with the true channel (perfect channel estimation $\rho = 1$). Note that for perfect channel estimation, the pdf of (2) reduces to

$$p_\gamma(\gamma) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}\gamma^{k-1}(l+M)}{M(k-1)!\overline{\gamma}} \exp\left(\frac{-(l+M)\gamma}{M\overline{\gamma}}\right) \tag{5}$$

III. ADAPTIVE CAPACITY POLICIES

In this section, we derive close-form expressions for different adaptive schemes with GSC over Rayleigh fading channels. In the derivation, we will rely on the main results from [13].

A. Power and Rate Adaptation

Given an average transmit power constraint, the channel capacity C_{opra} in (bits/seconds) of a fading channel [12], [13] is given by

$$C_{opra} = \frac{B}{\ln 2} \int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) p_\gamma(\gamma) d\gamma, \tag{6}$$

where B (in hertz) is the channel bandwidth and γ_0 is the optimum cutoff SNR satisfying the following condition

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p_\gamma(\gamma) d\gamma = 1. \tag{7}$$

To achieve the capacity in (6), the channel fading level must be tracked at both transmitter and receiver. The transmitter has to adapt its power and rate accordingly by allocating high power levels and transmission rates for good channel conditions (large γ). Since the transmission is suspended when $\gamma < \gamma_0$, this policy suffers from outage, whose whose probability P_{out} is defined as the probability of no transmission and is given by $P_{out} = 1 - \int_{\gamma_0}^{\infty} p_\gamma(\gamma) d\gamma$. Substituting (2) into (7) yields the equality

$$\begin{aligned} & \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} [\overline{\gamma}(1-\rho^2)]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \binom{k-1}{n} \\ & \left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \left[\frac{\Lambda_l}{\gamma_0}\Gamma\left(n+1, \frac{\gamma_0}{\Lambda_l}\right) - \Gamma\left(n, \frac{\gamma_0}{\Lambda_l}\right)\right] = 1 \end{aligned} \tag{8}$$

To obtain the optimal cutoff SNR γ_0, in (8), we follow the following procedure. Let $x = \frac{\Lambda_l}{\gamma_0}$ and define $f_{GSC}(x)$. Now, differentiating the function $f_{GSC}(x)$ with respect to x over the interval $]0, +\infty[$ resulting in $f'_{GSC}(x) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \sum_{n=0}^{k-1} \frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n \left[\frac{\Gamma\left(n+1,x\right)}{x^2}\right]$.

Hence, $f'_{GSC}(x) < 0, \forall x > 0$, meaning that f'_{MRC} is a strictly decreasing function of x. From (8) in terms of x it can be observed that $\lim_{x\to 0} f_{GSC}(x) = +\infty$ and $\lim_{x\to+\infty} f_{GSC}(x) = \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} \sum_{n=0}^{k-1} \frac{1}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n$. Note however that $f_{GSC}(x)$ is a continuous function of x, which leads to a unique positive γ_0 such that $f_{GSC}(x) = 0$. We thereby conclude that for each $\overline{\gamma} > 0$ there is a unique γ_0 satisfying $f_{GSC}(x)$. Numerical results using MATLAB shows that $\gamma_o \in [0, 1]$ as $\overline{\gamma}$ increases, and $\gamma_0 \to 1$ as $\overline{\gamma} \to \infty$. Now, substituting (2) into (6) yields the channel capacity with *opra* scheme as follows

$$\begin{aligned} C_{opra} \;=\; & \sum_{l=0}^{L-M} \sum_{k=1}^{D} \frac{\mu_l^{D_l-k}}{\Lambda_l^k} [\overline{\gamma}(1-\rho^2)]^{k-1} \sum_{n=0}^{k-1} \frac{1}{n!} \\ & \times \binom{k-1}{n} \left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n \\ & \underbrace{\int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) \gamma^n \exp\left(\frac{-\gamma}{\Lambda_l}\right) d\gamma}_{I_1} \end{aligned} \tag{9}$$

We evaluate I_1 by taking the help of the following identity [13] given by $J_s(\mu) = \int_1^{\infty} t^{s-1}\ln(t)e^{-\mu t}dt = \frac{\Gamma(s)}{\mu^s}\{E_1(\mu) + \sum_{k=1}^{s-1}\frac{1}{k}P_k(\mu)\}$, where$E_1$ denotes the Exponential integral of the first order [17] defined as $E_1(x) = \int_1^{\infty}\frac{e^{x\gamma}}{\gamma}d\gamma, \quad x \geq 0$ and $P_k(\mu)$ denotes the poisson distribution [17] given $P_k(x) = \frac{\Gamma(k,x)}{\Gamma(k)} = e^{-x}\sum_{i=0}^{k-1}\frac{x^n}{n!}$. Up on substituting $J_s(\mu)$ into (9), the following closed-form expression for capacity C_{opra} per unit bandwidth (in bits/seconds/Hz) can be obtained as follows:

$$C_{opra} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}[\overline{\gamma}(1-\rho^2)]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!} \times\binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \times\left\{E_1\left(\frac{\gamma_0}{\overline{\gamma}}\right)+\sum_{k=1}^{n+1}\frac{1}{k}P_k\left(\frac{\gamma_0}{\overline{\gamma}}\right)\right\} \quad (10)$$

The above capacity expression allows us to examine the limiting cases for ($M = L$, and $M = 1$) more conveniently for MRC and SC, respectively.

1) Asymptotic Approximation: We can obtain asymptotic approximation C_{opra} using the series representation of Exponential integral of first order function [17] expressed as $E_1(x) = -E - \ln(x) - \sum_{i=1}^{+\infty}\frac{(-x)^i}{i.i!}$ where $E = 0.5772156659$ is the Euler-Mascheroni constant. Then, the asymptotic approximation C_{opra}^{∞} per unit bandwidth (in bits/seconds/hertz) can be shown to be

$$C_{opra} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}[\overline{\gamma}(1-\rho^2)]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!} \binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n\left\{\left(-E-\ln\left(\frac{\gamma_0}{\overline{\gamma}}\right) +\left(\frac{\gamma_0}{\overline{\gamma}}\right)\right)+\sum_{k=1}^{n+1}\frac{1}{k}P_k\left(\frac{\gamma_0}{\overline{\gamma}}\right)\right\} \quad (11)$$

2) Upper Bound: The capacity expression of C_{opra} can be upper bounded by applying Jensen's inequality to (6) as follows $C_{OPRA}^{UP} = \ln\Big(\mathbb{E}[\gamma]\Big)$, we can evaluate C_{OPRA}^{UP} using the pdf γ given in (2) and the identity [17],$\int_0^{\infty}x^n e^{-\mu x}dx = n!\mu^{-n-1}$ for Re$[\mu] > 0$ and simplify the resulting expression to obtain the capacity (6) upper bound

$$\frac{C_{opra}^{UB}}{B} = \ln\Bigg(\sum_{l=0}^{L-M}\sum_{k=1}^{D}\mu_l^{D_l-k}\Lambda_l^{2-k}[\overline{\gamma}(1-\rho^2)]^{k-1} \sum_{n=0}^{k-1}\frac{(n+1)!}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\Bigg) \quad (12)$$

B. Constant Transmit Power

By adapting the transmission rate to the channel fading condition with a constant power, the channel capacity C_{ora} [9], [10] is given by

$$C_{ora} = \frac{B}{\ln 2}\int_0^{\infty}\ln\big(1+\gamma)p_{\gamma}(\gamma)d\gamma. \quad (13)$$

Substituting (2) into (13) results in

$$\frac{C_{ora}}{B} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^k}[\overline{\gamma}(1-\rho^2)]^{k-1}\sum_{n=0}^{k-1}\frac{1}{n!} \binom{k-1}{n}\left[\frac{M\rho^2}{\Lambda_l(1-\rho^2)(l+M)}\right]^n \underbrace{\int_0^{\infty}\ln\Big(1+\gamma\Big)\gamma^n\exp\Big(-\frac{\gamma}{\Lambda_l}\Big)d\gamma}_{I_2}. \quad (14)$$

The integral I_2 in (14) can be evaluated by taking use of equality [17]$\int_0^{\infty}\ln(1+y)y^{t-1}e^{xy}dy = (t-1)!e^x\sum_{i=1}^{t}\frac{\Gamma(-t+i,x)}{x^i}$ where $\Gamma(.,.)$ is the complementary incomplete Gamma function which can be related to the exponential integral function $E_t(x)$ through [17]$E_t(x) = x^{t-1}\Gamma(1-l,x)$. The integral I_2 can be obtained as $\int_0^{\infty}\ln\big(1+\gamma)\gamma^n\exp\Big(\frac{-\gamma}{\Lambda_l}\Big)d\gamma = \exp\Big(\frac{1}{\Lambda_l}\Big)\sum_{j=1}^{n+1}\Lambda_l^{n+1}E_{n+2-j}\Big(\frac{1}{\Lambda_l}\Big)$ which leads to a closed-form expression for the capacity C_{ora} per unit bandwidth (in bits/seconds/hertz). The capacity C_{ora} can be expressed in another form. It has been shown that the integral in I_2 has the following form which is derived in [13] $\int_0^{\infty}\ln(1+y)y^{t-1}e^{xy}dy = \frac{\Gamma(t)}{x^t}\big[P_t\big(-x\big)E_1(x) + \sum_{i=1}^{t-1}\frac{P_i\big(x\big)P_{t-i}\big(-x\big)}{i}\big]$. Upon substituting this result into (14) yielding another closed-form expression for the capacity C_{ora} per unit bandwidth (in bits/seconds/hertz)

$$C_{ora} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\mu_l^{D_l-k}\Lambda_l^{1-k}[\overline{\gamma}(1-\rho^2)]^{k-1}\sum_{n=0}^{k-1} \binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\Bigg[P_{n+1}\left(\frac{-1}{\Lambda_l}\right) \times E_1\left(\frac{1}{\Lambda_l}\right)+\sum_{i=1}^{n}\frac{P_i\big(\frac{1}{\Lambda_l}\big)P_{n+1-i}\big(\frac{-1}{\Lambda_l}\big)}{i}\Bigg] \quad (15)$$

1) Asymptotic Approximation: Following the same procedure in Section III-A, the asymptotic approximation C_{ora}^{∞} per unit bandwidth (in bits/seconds/hertz) can be computed as

$$C_{ora} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\mu_l^{D_l-k}\Lambda_l^{1-k}[\overline{\gamma}(1-\rho^2)]^{k-1}\sum_{n=0}^{k-1} \binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\Bigg[P_{n+1}\left(\frac{-1}{\Lambda_l}\right) \quad (16) \times\left(-E-\ln\left(\frac{1}{\Lambda_l}\right)+\frac{1}{\Lambda_l}\right)+\sum_{i=1}^{n}\frac{P_i\big(\frac{1}{\Lambda_l}\big)P_{n+1-i}\big(\frac{-1}{\Lambda_l}\big)}{i}\Bigg]$$

2) Upper Bound: The capacity C_{ora} can be upper bounded by applying Jensen's inequality to (6) as follows

$$\frac{C_{ora}^{UB}}{B} = \ln\Bigg(1+\sum_{l=0}^{L-M}\sum_{k=1}^{D}\mu_l^{D_l-k}\Lambda_l^{2-k}[\overline{\gamma}(1-\rho^2)]^{k-1} \sum_{n=0}^{k-1}\frac{(n+1)!}{n!}\binom{k-1}{n}\left[\frac{M\rho^2}{(1-\rho^2)(l+M)}\right]^n\Bigg) \quad (17)$$

3) Higher SNR Region: The Shannon capacity can be approximated at high SNR region using the fact $\log_2(1+\gamma) = \log_2(\gamma)$ as $\gamma \to \infty$ for x > 0 yields an asymptotically tight bounds for (15) in high SNR per unit bandwidth (in bits/seconds/hertz) as

$$C^{high} = \sum_{l=0}^{L-M}\sum_{k=1}^{D}\mu_l^{D_l-k}\Lambda_l^{1-k}[\overline{\gamma}(1-\rho^2)]\sum_{n=0}^{k-1}\binom{k-1}{n} \times\left[\frac{M\rho^2\gamma}{\Lambda_l(1-\rho^2)(l+M)}\right]^n\left[\psi(n+1)-\ln\left(\frac{1}{\Lambda_l}\right)\right] \quad (18)$$

where $\psi(x)$ denotes Psi function defined as $\psi(x) = \frac{d}{dx}\ln\left(\Gamma(x)\right)$. For integer values of x, Psi can be represented as $\psi(x) = -E + \sum_i^{x-1}\frac{1}{i}$.

4) Lower SNR Region: We can approximate Shannon capacity in low SNR region by the square capacity of the argument (*gamma*) in low SNR region as $\log_2(1+\gamma) \approx \sqrt{\gamma}$ [16]. Upon using this approximation along with definition of incomplete gamma function yields the approximated Shannon capacity at low SNR per unit bandwidth (in bits/seconds/hertz) as

$$C^{Low^1} \approx \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}}{\Lambda_l^{k-3/2}}[\overline{\gamma}(1-\rho^2)]\sum_{n=0}^{k-1} \times\frac{\Gamma(n+3/2)}{n!}\binom{k-1}{n}\left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n \quad (19)$$

5) Lower SNR Region II: The Shannon capacity can be approximated as well in low SNR region by exploiting the fact $\log_2(1+\gamma) \approx \frac{1}{\ln(2)}(\gamma - \frac{1}{2}\gamma^2)$ gives the approximated Shannon capacity in low SNR region per unit bandwidth (in bits/seconds/hertz)

$$C^{Low^2} \approx \sum_{l=0}^{L-M}\sum_{k=1}^{D}\frac{\mu_l^{D_l-k}\Lambda_l^{2-k}}{\ln(2)n!}[\overline{\gamma}(1-\rho^2)]\sum_{n=0}^{k-1}\binom{k-1}{n} \times\left[\frac{M\rho^2\gamma}{(1-\rho^2)(l+M)}\right]^n\left[\Gamma(n+2)-\frac{1}{2}\Gamma(n+3)\Lambda_l\right] \quad (20)$$

IV. Numerical Result

In this section we provide some numerical results that illustrate the mathematical derivation of the channel capacity per unit bandwidth as a function of average receiver SNR $\overline{\gamma}$ in dB for two different adaptation policies with (GSC) over slow Rayleigh fading with weight estimation errors. All curves provided are obtained using the closed-form expressions.

Figure 1 shows the comparison of the capacity per unit bandwidth for *opra*, and *ora* policies for GSC ($L = 4$,$M = 3$). The result indicates how the *opra* policy achieves the highest capacity for any average receive SNR, $\overline{\gamma}$. From the same figure, it can be noticed that *ora* achieves less capacity than *opra*. However, both *opra* and *ora* achieve the same result when there is no power adaptation implemented at the transmitter as in *opra*. The results in Figure 1 is plotted for the case of fully estimated channel ($\rho^2 = 1$). Figure 2 compares C_{opra} for different values of correlation between the channel and

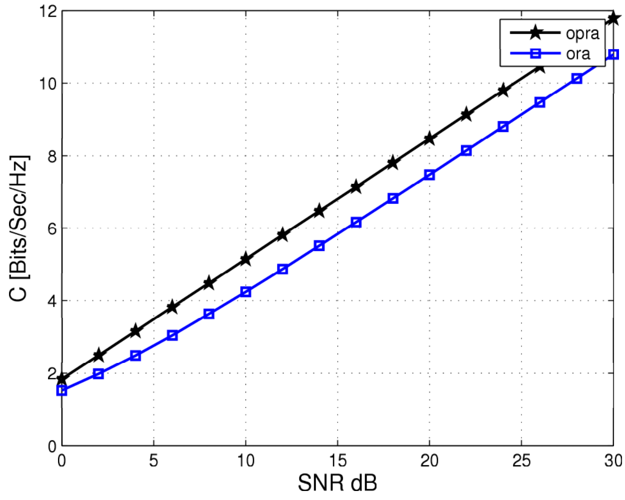

Fig. 1. Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$, $M = 3$) for two adaptation schemes with perfect estimation $\rho^2 = 1$

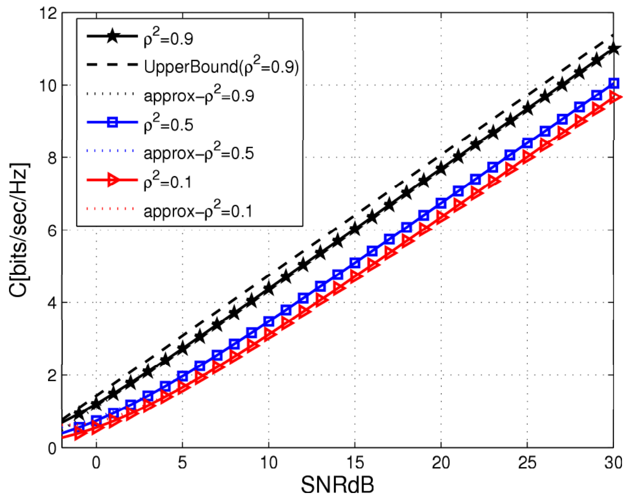

Fig. 2. Capacity per unit bandwidth for a Rayleigh fading with GSC diversity ($L = 4$,$M = 3$) and various values of different ρ^2 under power and rate adaptation.

its estimate; namely, $\rho^2 = 0.1$, $\rho^2 = 0.5$,and $\rho^2 = 0.9$. It can be noticed that the highest C_{opra} that can be achieved is when $\rho^2 = 1$ as shown in figure 1 with perfect estimation . Furthermore, C_{opra} decreases when the value of ρ^2 decreases where in this case the weight error increases. It can be observed from Figure 2 that there is almost a 3 dB difference in C_{opra} between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure (3) shows the plot of C_{ora} as well as its asymptotic approximation and upper bound as a function of the average received SNR $\overline{\gamma}$ for ($L = 4$,$M = 3$). As can be seen from the same figure that the *ora* policy is more sensitive to the estimation error than the *opra* policy by 1 dB difference between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Figure 4 depicts the exact closed-form expression

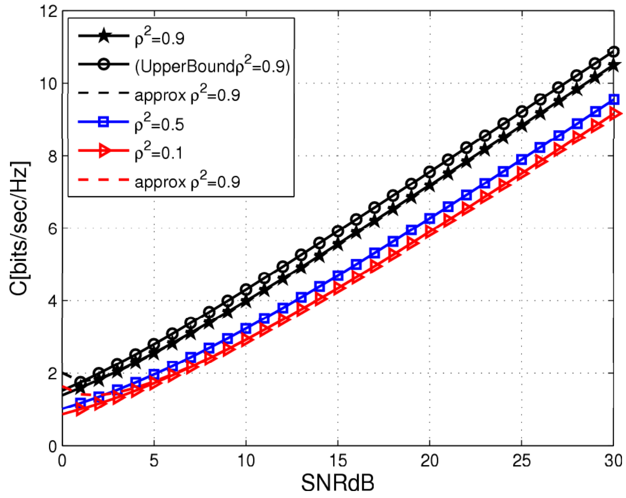

Fig. 3. Capacity per unit bandwidth for a Rayleigh fading channel with SCD ($L = 4$,$M = 3$) and various values of different ρ^2 under rate adaptation and constant power.

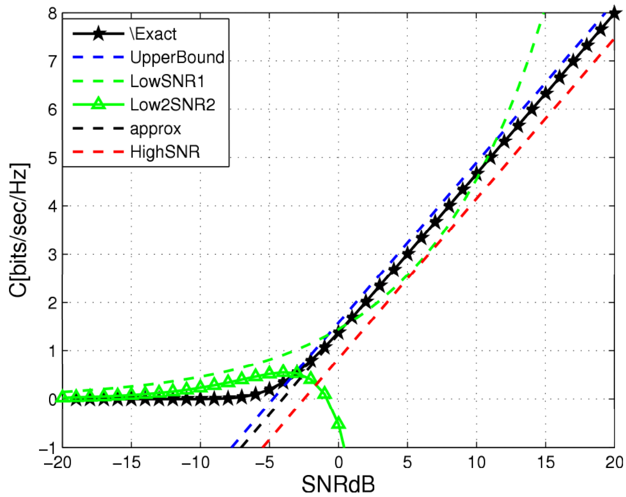

Fig. 4. Exact and approximated capacity per unit bandwidth for a Rayleigh fading with GSC diversity for ($L = 4$,$M = 3$) under imperfect channel estimation $\rho^2 = 0.9$

capacity of (15) for ($L = 4$,$M = 3$) and $\rho^2 = 0.9$ as well as the corresponding approximations given in (16)-(20). It can be observed that (17) and (18) correspond to each other for SNR ≥ 5 dB which show a tight approximation of the exact average capacity. Furthermore, the approximations for low SNR region are two fold: 1) the expression in (19) becomes tight in SNRs $<$ -15 dB, whereas; 2) the expression in (20) becomes tight to exact capacity between 0 and 10 dB as shown in Figure 4.

V. Conclusions

The closed-form expressions for the channel capacity per unit bandwidth for two different adaptation policies including their approximations and upper bounds over a slow Rayleigh fading channel for GSC (L,M) with estimation error is derived. Furthermore, we presented an upper bound as well as asymptotically tight approximations for *ora* policy for the high and low SNR regions. The results showed that *opra* outperforms *ora* by 1 dB difference between $\rho^2 = 0.9$ and $\rho^2 = 0.1$. Finally, it is worth to mention that the derived capacity expressions represent general formulas for GSC (L,M) with estimation error over slow Rayleigh fading channel from which those spacial limited cases of GSC (i.e, $L = M$, MRC; $M = 1$, SC; $\rho^2 = 1$,perfect estimation; $\rho^2 = 0$ no diversity) can be derived.

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