Performance of Coded Multi-hop Ad-hoc Networks in OFDM Wireless Environments

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***Abstract*— Multi-hop networks help in combating fading by employing a distributed space diversity system. The performance of the system can be further enhanced by employing a good error control code such as convolutional codes, which provide low implementation complexity and short delays. In this paper we investigate the error performance of a convolutionally coded multi-hop ad-hoc network in an OFDM wireless environment with serial and selection relaying. In particular, we investigate the effect of relaying scheme, channel, bandwidth, and path loss exponent on the performance. Results show that the relaying improves the system greatly especially in the case of selection relaying and in high pat loss environments.**

I. Introduction

Next generation wireless networks are expected to provide services that require high data rates with very high bandwidth efficiency compared to previous generations. As the number of wireless terminals increases, higher system capacity is needed to provide the high data rate required by all these terminals. It is not practical to install more base stations to serve the large number of terminals, especially when these terminal are located in a pico-cell environment. In addition, there is the multipath fading problem which arises from the reflection of the transmitted signal over many objects in the environment surrounding a transmitter and receiver. The effects of multipath fading can be combated by using diversity.

Diversity is based on the fact that several independent fading channels are unlikely to fade simultaneously, and so by transmitting several replica of the message over different paths (time slots, frequencies, or antennas) we can reduce the multipath fading effect. Another way to reduce fading errors is by implementing an error control code with low implementation complexity and low power consumption in order to reduce the bit error rate (BER).

To further inhance the system's ability to combat frequency selectivity and multipath fading without the need for complex equalization filters is to use OFDM. In OFDM, the data is divided into several parallel data streams or channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme (such as Binary phase shift keying) at a low symbol rate, reducing the effect of intersymbol interference resulting from the selectivity of the channel.

Much work has been done in literature of the subject of multi-hop networks. In [1], Hasna *et.al.* focused on two-hop wireless networks and derived the end-to-end error performance over independent Rayleigh fading channels. In particular, they presented closed-form expressions for the statistics of the harmonic mean of two independent exponential variates, which is used to characterize the error performance of the system. The main conclusion was that networks employing regenerative relays outperform those employing non-regenerative relays at low average SNR. However, at high average SNR, it was shown that the two systems are essentially equivalent in terms of outage probability average BER and outage capacity. In [2], Zummo investigated the performance of coded cooperation diversity with multiple cooperating users. Where the users form a cluster and cooperate their coded data to a common base station. The researcher derived the end-to-end bit error probability for users in a cluster, and showed results for different cluster sizes and SNRs. He also derived a union bound on the end-to-end bit error probability averaged over different cooperation scenarios, such as, no cooperation and all cooperation scenarios considering both correlated and un-correlated Rayleigh fading channels. Numerical results showed that for a fixed inter-user channel quality, the small-sized cluster perform better than the large-sized clusters in terms of probability of error.

In this paper we investigate the error performance of convolutionaly coded multi-hop network with serial and selection relaying topologies over an OFDM wireless environment. We used convolutional codes with rates $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$, polynomials $(5,7), (5,7,7), (5,5,5,7,7,7)$ respectively, and memory order=2. For the OFDM, we used the number of sub-carriers $N_c = 64$ and the number of FFT points $N_{FFT} = 64$.

The rest of the paper is organized as follows. In section II, we describe the system model we are using in this work. Section III, show the forwarding techniques and the multi-hop topologies we are using. The effects of system parameters are showed in section IV. Finally Section V is the conclusion.

II. System Model

Figure 1 shows the system model of coded OFDM systems.

Every k source bits in a length-k vector $\mathbf{u}$, are encoded into an n-bit codeword $\mathbf{v}$. The coded vector $\mathbf{v}$ is interleaved

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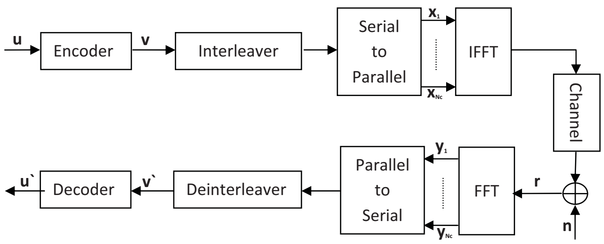

Fig. 1. Model of coded OFDM system.

and mapped to the N_c symbol vectors $\{\mathbf{x}_l\}_{l=1}^{N_c}$, where $\mathbf{x}_l = (x_{l1}, x_{l2}, ..., x_{lM})$, and $x_{l,i}$ is a symbol drawn from a signal constellation. At every time instance t, the t^{th} elements of the N_c signal vectors $\{\mathbf{x}_l\}_{l=1}^{N_c}$ are input to an IFFT block to output the t^{th} OFDM symbol s_t, whose k^{th} component is expressed as

$$s_{t,k} = G_T \sum_{l=1}^{N_c} x_{l,t} \exp\left(j2\pi \frac{kp_l}{N_{FFT}}\right), \quad (1)$$
$$k = 0, ..., N_{FFT}-1, t = 1, ..., M,$$

where G_T is the IFFT gain factor, p_l determines the corresponding subcarrier of the l-th vector $\mathbf{x}_l$, and N_{FFT} is the number of the FFT points used in the OFDM modulation. The resulting signal is transmitted over a p-path fading channel with a profile $\{g_p, \tau_p\}_{p=l}^{P}$, where g_p, and τ_p, are the gain and delay of the channel's p-th path with $\sum_{p=1}^{P} g_p^2 = 1$ satisfied.
At the receiver side, the received sample for the k^{th} subcarrier at the t^{th} time instance $r_{t,k}$ is written as

$$r_{t,k} = G_T \sum_{l=1}^{N_c} \sum_{p=1}^{P} g_p \beta_p x_{l,t} H\left(j2\pi \frac{p_l}{N_{FFT}}\right) * \exp\left(j2\pi \frac{(k-\tau_p)p_l}{N_{FFT}}\right) + \tilde{n}_{t,k} \quad (2)$$

$$= G_T \sum_{l=1}^{N_c} x_{l,t} \alpha_l \exp\left(j2\pi \frac{kp_l}{N_{FFT}}\right) + \tilde{n}_{t,k}, \quad (3)$$

where β_p's are complex zero-mean Gaussian independent random variables with unit variance representing the fading coefficients of each path of the channel. Due to the small packet size compared to the coherence time of fading channel, channel condition does not vary significantly during the transmission of a packet, hence β_p's are assumed to be constant within a packet. In equation 3, $H(.)$ is the frequency response of the cascaded transmit and receive filters, and $\tilde{n}_{t,k}$ denote the noise samples filtered by the receive filter, $n_{t,k}$'s are independent zero-mean Gaussian noise components with variance σ_n^2.
Finally α_l's are the frequency domain fading coefficients defined as follows

$$\alpha_l = H\left(j2\pi \frac{p_l}{N_{FFT}}\right) \left(\sum_{p=1}^{P} g_p \beta_p \exp\left(-j2\pi \frac{\tau_p p_l}{N_{FFT}}\right)\right), \quad (4)$$

After applying the FFT to the received samples $r_{t,k}$, we get

$$y_{l,t} = G_R \sum_{k=0}^{N_{FFT}-1} \left[\sum_{i=1}^{N_c} \alpha_i x_{i,t} \exp\left(j2\pi \frac{kp_i}{N_{FFT}}\right) + \tilde{n}_{t,k}\right] * \exp\left(-j2\pi \frac{kp_l}{N_{FFT}}\right) \quad (5)$$

$$= \alpha_l x_{l,t} + \nu_{l,t}, \quad (6)$$

where G_R is the FFT gain factor and $\nu_{l,t}$ is the frequency domain noise samples.
In packet switched communications such as the OFDM, the channel can be modeled as a block fading (BF) channel. In BF model, transmitted sequence is divided into blocks and all the symbols belonging to the same block suffer the same fade. In some cases, the fade blocks are assumed to be independent from each other, but in some applications such as the OFDM system there is a considerable correlation among the fade blocks. In this case, the fading is referred to as correlated block fading (CBF). It was proved in [3] that OFDM systems can be simulated using CBF.
In CBF channels, each symbol vector $\mathbf{x}_l = (x_{l1}, x_{l2}, ..., x_{lM})$ is transmitted through a block fading subchannel with a multiplicative distortion $\boldsymbol{\alpha}_l = (\alpha_1, ..., \alpha_{N_c})$, which is a complex Gaussian random variable that stays constant during the transmission of $\mathbf{x}_l$. The fading coefficients $\{\boldsymbol{\alpha}_l\}_{l=1}^{N_c}$ are correlated with correlation matrix $C_\alpha = E[\boldsymbol{\alpha}\boldsymbol{\alpha}^H]$, where $\boldsymbol{\alpha}_l = (\alpha_1, ..., \alpha_{N_c})^T$
The received vectors $\mathbf{y}_l = (y_{l1}, ..., y_{lM})$ corresponding to the transmission of $\mathbf{x}_l$ over the CBF channel is given by

$$\mathbf{y}_l = \boldsymbol{\alpha}_l \mathbf{x}_l + \mathbf{n}_l, \quad (7)$$

where the noise vector $\mathbf{n}_l$ consists of a zero-mean independent Gaussian components with variance σ_n^2.
Figure 2 shows the normalized gain profiles of the two channels we are going to use. The delay profile is uniformly spaced [3].

III. Forwarding Techniques and Multi-hop Topologies

A. Forwarding Techniques

There are two basic forwarding techniques used in literature, amplify-and-forward (AF), and decode-and-forward (DF).
In AF, the relays amplify the data by a certain gain G and then forward it to the next relay or the destination. The most widely used formula for the forwarding gain is [4]:

$$G^2 = \frac{E_2}{E_1\alpha_1^2 + N_{0_1}}, \quad (8)$$

where E_2 is the power of the transmitted signal at the output of the relay, α_1 is the fading amplitude of the first hop, and

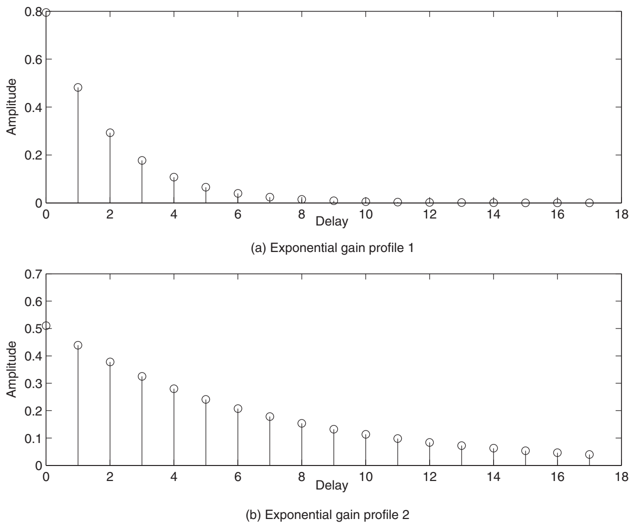

Fig. 2. Channel Spread Profile, (a) Exponential Channel 1. (b) Exponential Channel 2.

N_{0_1} is the noise average power of the first hop. The choice of this gain aims to invert the fading effect of the first hop while limiting the output power of the relay if the fading amplitude of the first hop is low.
In DF, the relays receive the data, completely decode it and then re-encode it and forward it to the next relay. If the data is not coded, then this technique is referred to as detect-and-forward and it performs very close to AF [4]. In this work, we are going to consider DF only.

B. Multi-hop Topologies

In this work, we are considering two multi-hop topologies, serial relaying and selection relaying.
In serial relaying, shown in Figure 3, in the first time slot, the source sends the data to the relay which in turn forwards it to the destination in the next time slot and so on. Figure 4 shows the time allocation of the transmission.

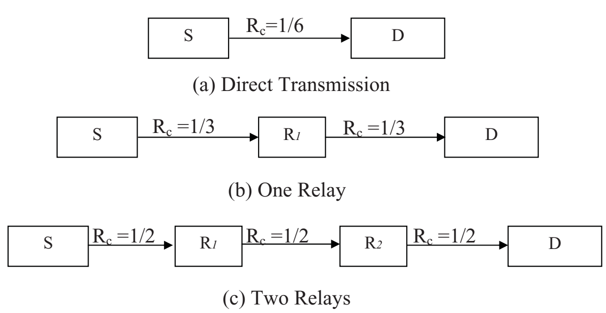

Fig. 3. Serial Topology. (a) Direct Transmission with code rate=1/6, (b) One relay with coderate=1/3, and (c) Two relays with code rate=1/2.

The discrete-time signal model of the received signal at the relay is given by

$$y_r = D_{sr}^{-n} \alpha_{sr} x_s + n_r, \tag{9}$$

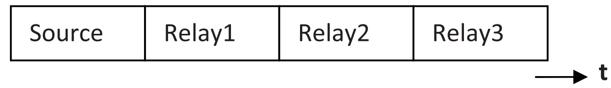

Fig. 4. Time Allocation

where y_r is the received signal at the relay, α_{sr} is the channel fading amplitude between the source and the relay, x_s is the transmitted signal from the source, D_{sr} is the distance between the source and the relay usually measured in meters, n is the path loss exponent which depends on the environment, and finally n_r is the AWGN component at the destination. The relay then forwards the signal to the destination. the signal model is given by The received signal at the destination node using DF scheme is given by,

$$y_d = D_{rd}^{-n} \alpha_{rd} \hat{y}_r + n_d, \tag{10}$$

where, $\hat{y}_r$ is the signal after being encoded by the relay node. The destination then uses maximum ratio combining (MRC) to combine the data received from the source and the relays. To implement MRC in our system, we change the metric of the Viterbi decoder to $(y_{s,d} - D_{sd}^{-n} \alpha_{sd} x_d)^2 + (y_{r,d} - D_{rd}^{-n} \alpha_{rd} x_d)^2$, where the receiver uses the signals coming from both the source and the relay together.
For the sake of simplicity, we assume that the relays are equally-spaced between the source and the destination. In order to be fair when comparing the different scenarios, we change the coding rate at the transmitter and relays in order to fix the total transmitted number of bits and the total energy used. In the case of direct transmission, we use $\frac{1}{6}$ code rate while in the one relay case, we use $\frac{1}{3}$ code rate, and finally in the two relays case, we use $\frac{1}{2}$ code rate.
In selection relaying, we use the parallel topology as in Figure 5, but we select only one relay which then forwards the data to the destination using all the subcarriers.

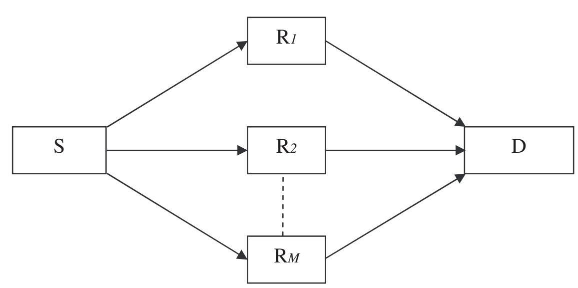

Fig. 5. Parallel Topology.

The relay selection is based on the fading vector of both the source-relay channel and the relay-destination channel $\boldsymbol{\alpha}_{eq} = \boldsymbol{\alpha}_{sr}.*\boldsymbol{\alpha}_{rd}$. Three criteria were considered for the selection, the mean of the fading vector $\mu_{\boldsymbol{\alpha}_{eq}}$, the variance of the fading vector $\sigma^2_{\boldsymbol{\alpha}_{eq}}$, and the ratio between the maximum

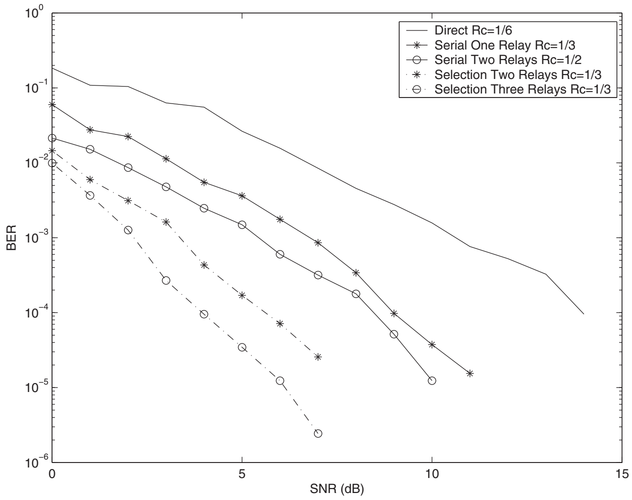

Fig. 6. Performance of convolutionally coded multi-hop network using serial and selection relaying over OFDM with exponential channel 1 and $N_c = 64$.

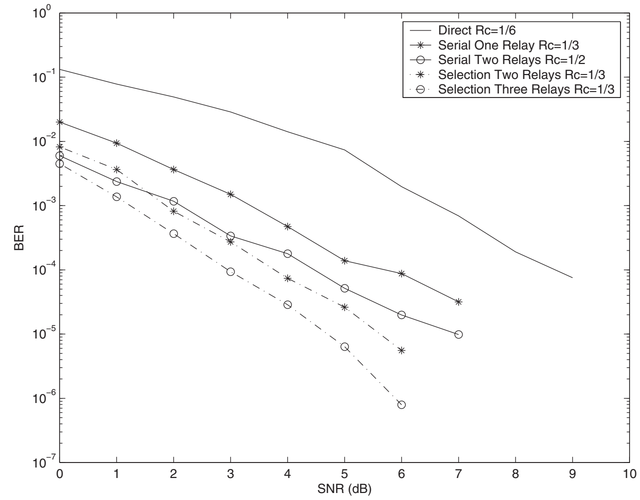

Fig. 7. Performance of convolutionally coded multi-hop network using serial and selection relaying over OFDM with exponential channel 2 and $N_c = 64$.

and minimum eigen values of the fading vector $\frac{max(\lambda_{\alpha_{eq}})}{min(\lambda_{\alpha_{eq}})}$. It was found from the simulation that the criteria that gives the best performance is the mean criteria, which is correspondent with [5]. In this work, we selected the relay with the highest mean of its fading vector and only use this relay to forward the data to the destination.

IV. Effect of System Parameters

In this section we discuss the various simulation results we found for different system parameters.

A. Effect of relaying scheme and channel

Figure 6 shows the error performance of coded multi-hop network using serial and selection relaying over OFDM with exponential channel 1 and $N_c = 64$. It is clear from the figure the improvement relaying gives to the system, not only in terms of SNR gain but also in terms of diversity gain. The diversity gain is coming from both the coding and from using DF and it is seen in the change of the slope of the curve. Also selection relaying gives more improvements than serial relaying, this is because the selection algorithm makes the system acts like a one relay serial relaying system but with always good channel.

Figure 7 shows the error performance of coded multi-hop network using serial and selection relaying over OFDM with exponential channel 2 and $N_c = 64$. By using a more frequency selective channel, we introduce more frequency diversity gain to the system. Which can be seen from the SNR gain and the change of the slope of the curves.

B. Effect of OFDM Bandwidth

Further we tested the effect of the bandwidth used in OFDM by changing the number of carriers N_c. Figure 8 shows the error performance of coded multi-hop network using serial and selection relaying over OFDM with $N_c = 4, 8, 16, 32, and, 64$, and SNR=7 dB.

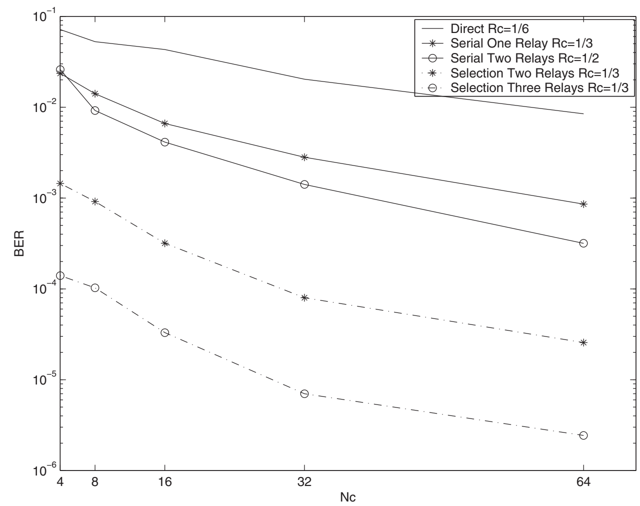

Fig. 8. Performance of convolutionally coded multi-hop network using serial and selection relaying over OFDM with $Nc = 4, 8, 16, 32, and, 64$, and SNR=7 dB.

It is clear from the figure the improvement the increase in number of carriers gives to the error performance. By increasing the number of carriers, we add more frequency diversity to the system, which in turn improves the error performance of the system.

C. Effect of the path loss exponent

Here we investigated the effect changing the path loss exponent has on the system. Figure 9 show the error performance of convolutionally coded multi-hop network using one relay serial relaying over OFDM with $N_c = 64$, and path loss exponent $n = 0, 2, 3$.

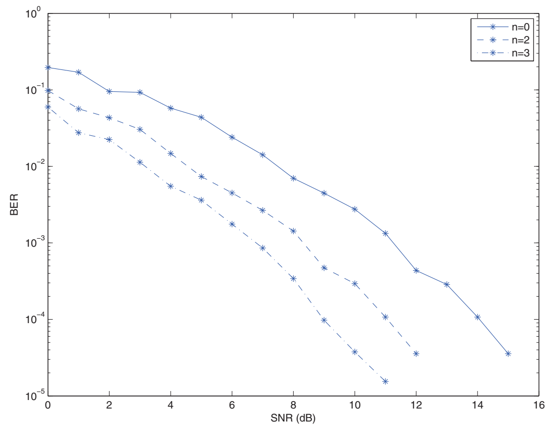

Fig. 9. Performance of convolutionally coded multi-hop network using one relay serial relaying over OFDM with $N_c = 64$, and path loss exponent $n = 0, 2, 3$.

We can see from the figure that the performance improves as the path loss exponent increases. In high path loss exponent environments, sending the data over short distances saves more power than doing the same thing in low path loss environments. So the gain relaying gives to the system gets better as the path loss exponent increases.

V. Conclusion

In this paper we investigated the performance of convolutionally coded multi-hop networks in OFDM wireless environment. We discussed the different forwarding techniques and the multi-hop topologies used in literature. We showed the performance of the system using serial and selection relaying, the effect the number of carriers has on the system, and the effect of the path loss exponent. Results show that the relaying improves the system greatly especially in the case of selection relaying and in high path loss environments.

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