A Decoding Algorithm for I-Q Space-Time Coded Systems in Fading Environments

Salam A. Zummo and Saud A. Al-Semari
Electrical Engineering Department
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
E-mail: {sazummo,semari}@kfupm.edu.sa

Abstract

This paper presents the decoding of the I-Q Space-Time (ST) codes. In this paper, the decoding problem of I-Q ST codes is formulated. The use of the super-trellis to solve the decoding problem results in an increased decoding complexity. Therefore, a simplified decoding algorithm is proposed. It is based on symbol-by-symbol detection of the Q/I components in the I/Q decoders. Simulation results showed that I-Q ST codes with the new algorithm provide coding gains over ST codes having the same complexity but with a single encoder.

1. Introduction

Wireless communication systems are required to provide services such as voice, data and multimedia applications. Since mobility is usually encountered during a mobile communication session, such systems suffer from the multipath effect and the rapidly time-varying channel characteristics [1,2]. Multipath fading causes Inter-symbol-interference (ISI), which causes a significant degradation in the system's performance and an error floor at high signal-to-noise ratio (SNR). As the transmission rate is increased, the ISI becomes worse and advanced equalization techniques are needed at the receiver. Moreover, error control codes are used to improve the system reliability.

The recently proposed Space-Time (ST) codes in [3] provide higher transmission rate, in addition to error control coding and diversity, without the need for complex equalizers. Diversity is a popular method to improve the performance of wireless systems. Transmit time diversity can be achieved by repeating the transmission of each symbol in different time slots [2]. Such a system can be viewed as a repetition code and consumes high bandwidth. Therefore, it is expected that substantial performance improvement can be achieved using more sophisticated codes, utilizing both space and time.

The concept of ST codes had appeared first in [4] as the delay diversity system, where different symbols are simultaneously transmitted via different transmit antennas. Later, ST codes were deigned explicitly for quasi-static fading channels [3]. Moreover, the performance criteria of ST codes were derived for quasi-static and rapid fading channels in [3]. Block ST coded systems were proposed in [5-7] to transmit data at the same bit rate of systems with single transmit antenna, but at a better SNR.

ST coded QPSK schemes were presented in [8] for rapid fading channels. Also, the use of the I-Q encoding technique was proposed to design a ST coded QPSK scheme. Similar techniques were applied in [9] to design ST coded 16-QAM schemes for rapid fading channels. The I-Q encoding technique was used to design ST coded systems suitable for rapid fading channels. However, the decoding problem of the resulting system was not considered. In this paper, the decoding problem of the I-Q ST codes is formulated. Then, a simplified decoding algorithm is proposed to decode I-Q ST codes.

The basic I-Q ST coded system is described in the next section. Also, a brief overview on the performance of ST codes over rapid fading channels is presented. Then, the decoding problem of I-Q ST codes is formulated. The super-trellis decoding of I-Q ST codes is examined. Then, a simplified decoding algorithm is proposed to decode the I-Q ST codes. The performance of the I-Q codes is compared with equivalent ST codes in the literature. Finally, some conclusions and suggestions for future work are discussed.

2. System Model

A ST system employing the I-Q encoding technique consists of two trellis encoders, two interleavers, two transmit antennas, M receive antennas, a deinterleaver and two decoders. In general, the number of transmit antennas is referred to as N. Figure 1 shows the structure of the I-Q encoder/decoder employing the ST concept. Each encoder encodes half the number of input bits per signaling interval. Symbols from the I-encoder contribute to the in-phase components of both signals at the two transmit antennas. Similarly, symbols from the Q-encoder contribute to the quadrature components of both signals.

The encoded ST signals are interleaved using a vector block interleaver, where each element in the interleaver is a vector containing N symbols to be transmitted via the N transmit antennas. The depth and span of the interleaver depend on the channel's fading rate ($f_D T$) and the encoder's constraint length, respectively. The interleaver is used in order to break the memory of the channel so that it approaches the behavior of independent fading channels, and hence the diversity provided by the coded system is fully utilized. The received signal at the j^{th} receive antenna is a noisy superposition of all transmitted symbols over all transmit antennas and is modeled as:

$$d_t^j = \sum_{i=1}^{N} \alpha_{ij,t} c_t^i + \eta_t^j \tag{1}$$

Where the coefficient $\alpha_{ij,t}$ is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna at time instance t. It is modeled as independent samples of a zero-mean complex Gaussian random process with variance of $1/N$. c_t^i is the transmitted symbol from the i^{th} transmit antenna, and η_t^j is additive white Gaussian noise (AWGN) sample with variance $N_o/2$. The pairwise error probability of ST coded systems over rapid Rayleigh fading channel is shown to be upper bounded as [3]:

$$P(C_l, \hat{C}_l) \leq \prod_{t \in \eta} [1 + \sum_{i=1}^{N} | c_t^i - \hat{c}_t^i |^2 (E_s/4N_o)]^{-M}, \tag{2}$$

where $\eta = \{t : \underline{c}_t \neq \underline{\hat{c}}_t\}$ and $\underline{c}_t = (c_t^1 c_t^2 \ldots . c_t^N)$ is the codeword of symbols transmitted simultaneously over all transmit antennas at time t. The parameter L, defined to be the length of the shortest error event path, is referred to as the Space-Time Minimum Time diversity (ST-MTD) of the ST code [8]. It can be visualized as the "branch-wise" Hamming distance in conventional trellis codes, by considering the whole codeword $\underline{c}_t$ as one symbol. The design rules of good ST codes over rapid fading channels state that for the system to achieve a diversity of vM, the ST-MTD of the code should be at least v [3]. Also, the code's ST Minimum Squared Product Distance (ST-MSPD), defined over the shortest error paths as:

$$d_P^2(L) = \min\{ \prod_{t=1}^{L} \sum_{i=1}^{N} | c_t^i - \hat{c}_t^i |^2 \tag{3}$$

should be maximized. Since the I-Q encoding technique increases the ST-MTD and ST-MSPD of the resultant code, it was used to design better ST codes for rapid fading channels. The I-Q ST codes were designed using the QPSK and 16-QAM signal constellations in [8] and [9], respectively. In the following, the decoding problem of the I-Q ST codes is formulated and solutions to this problem are posed.

3. Decoding I-Q ST Codes

In a typical ST coded system, the received signal at the j^{th} receive antenna has the following form:

$$d_t^j = \sum_{i=1}^{N} (\alpha_{Iij,t} + j\, \alpha_{Qij,t}).(x_t^i + jy_t^i) + \eta_t^j \tag{4}$$

Where $(\alpha_{Iij,t} + j\alpha_{Qij,t})$ is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna at time t. It is sampled from a complex Gaussian random process with zero mean and a variance of $1/N$. The symbol, $(x_t^i + jy_t^i)$ represents the complex baseband signal transmitted from the i^{th} transmit antenna at time interval t. Throughout this section, the subscript t is omitted for simplicity of notation. Equation (4) can be further manipulated as follows:

$$d^j = \sum_{i=1}^{N} \{ (\alpha_{Iij} x^i - \alpha_{Qij} y^i) + j (\alpha_{Iij} y^i + \alpha_{Qij} x^i) \} + \eta^j$$

or: $$d^j = r_I^j + jr_Q^j + \eta^j \tag{5}$$

Where the subscripts I and Q in Equation (5) denote the in-phase and quadrature components of the received signals and the fade samples. It is clear that the received signal is a function of all in-phase and quadrature components of the transmitted signals. Since the I-decoder does not know any information about the signals at the output of the Q encoder (i.e: Q components of the transmitted signals), an algorithm should be devised to detect the Q components in the I-decoder. A similar problem for detecting the I components in the Q-decoder can be stated.

3.1. Super-Trellis Decoding

In order to mitigate for this problem, the super-trellis is used to decode the I-Q ST codes. It decodes I-Q ST codes optimally by decoding the I and Q components simultaneously using sequence decoding. In this paper, the super-trellis is used to decode the 4-state QPSK and 16-QAM I-Q ST codes presented in [8] and [9], respectively. The resultant decoding complexities of the QPSK and 16-QAM codes are 32 and 64, respectively. It is clear that the decoder complexity of the 4-state I-Q QPSK ST code using super-trellis decoder is higher than that of the I-Q decoder, which is 8. It is not fair to compare the super-trellis decoder with the I-Q decoder for the QPSK I-Q ST code, because of the different complexities. However, the super-trellis decoding results are presented here for comparison with the performance of the simplified algorithm. The super-trellis complexity of the 4-state 16-QAM code is the same as the complexities of the QAM1, QAM2 codes.

3.2. A Simplified Decoding Algorithm

Unfortunately, the super-trellis decoder provides the desired gains from using the I-Q encoding technique on the cost of exponentially increasing the complexity of the decoder with the number of states of each encoder. In this paper, a simplified decoding algorithm is proposed to reduce the decoder complexity. It is based on partitioning the signal space at the output of the ST encoder and computing a metric for each subset. Set partitioning is performed such that symbols in each subset result from the same symbol at the I-encoder output. In other words, all signals in one subset have the same I component. The resultant metrics are then used in the Viterbi algorithm to decode the I components. In the following, the set partitioning is applied to the I-Q ST codes based on both the QPSK and 16-QAM constellations.

4. Decoding Examples

4.1. I-Q ST QPSK Code

Without loss of generality, the description of the algorithm is done for the I-decoder only. However, the same algorithm is extended to the Q-decoder case. Let S^{lk} denotes the set of the QPSK signal pairs that are transmitted from the I-Q ST encoder given that the bits at the output of the I-encoder are l and k. Using this notation, the bit l is the in-phase component of the signal transmitted over the first transmit antenna, while the bit k is the in-phase component of the signal transmitted via the second transmit antenna. In addition, the signal s_j^i is the QPSK signal whose label is j that is transmitted over the i^{th} transmit antenna. The QPSK signal constellation used is the Gray-mapped constellation. The 4-dimensional QPSK signal space is partitioned as follows.

$$
\begin{aligned}
S^{00} &= \{(s_0^1,s_0^2),\ (s_0^1,s_2^2),\ (s_2^1,s_0^2),\ (s_2^1,s_2^2)\}\\
{}^{10} &= \{(s_1^1,s_0^2),\ (s_1^1,s_2^2),\ (s_3^1,s_0^2),\ (s_3^1,s_2^2)\}\\
S^{01} &= \{(s_0^1,s_1^2),\ (s_0^1,s_3^2),\ (s_2^1,s_1^2),\ (s_2^1,s_3^2)\}\\
S^{11} &= \{(s_1^1,s_1^2),\ (s_1^1,s_3^2),\ (s_3^1,s_1^2),\ (s_3^1,s_3^2)\} \qquad (6)
\end{aligned}
$$

Now, four metrics corresponding to all values of l and k are computed at the I-decoder before the trellis:

$$
\begin{aligned}
M_{lk} = \min_{(s^1,s^2)\in S^{lk}} \sum_{j=1}^{M} & \Big(r_I^j - \sum_{i=1}^{N}\alpha_{Iij}x^i + \sum_{i=1}^{N}\alpha_{Qij}y^i\Big)^2 \\
& + \Big(r_Q^j - \sum_{i=1}^{N}\alpha_{Qij}x^i + \sum_{i=1}^{N}\alpha_{Iij}y^i\Big)^2 \qquad (7)
\end{aligned}
$$

Where x^i and y^i are the in-phase and quadrature components of the QPSK signal s_j^i. In each M_{lk}, four metrics are compared and the minimum is found accordingly, ending with four different metrics. Each of the remaining metrics is associated with one of the four bit combinations that may be at the output of the I-encoder. These metrics are computed before the trellis of the I-decoder and hence, they are considered to be in the detection stage.

At each state inside the trellis, the decoder compares two of the above metrics whose subscripts l and k are the same as the labels of the branches emerging at that state. The proposed decoding algorithm does need a pre-trellis metric computation, but it does not need additional computation in the trellis, since the computed metrics are needed in the Viterbi algorithm. After the metrics have been computed, the remaining work in the trellis is the comparison of the appropriate metrics.

4.2. I-Q ST 16-QAM Code

The signal space to be partitioned in this case is a 4-dimensional 16-QAM space that consists of 16^2 possible signal pairs. The partitioning is performed so that all pairs in one subset have the same in-phase component. In other words, they are caused by the same 4-AM symbol pair at the output of the I-encoder. Hence, for each 4-AM symbol pair, there are 16 possible 16-QAM signal pairs that could be transmitted from both transmit antennas.

The notation S^{lk} is the same as defined earlier with l and k denote the 4-AM symbols at the output of the I-encoder. The Gray-mapped 16-QAM signal constellation is used in this case. The set partitioning yields 16 subsets and two sets are presented for illustration:

$$
\begin{aligned}
S^{00} = \{ & (s_0^1,s_0^2), (s_0^1,s_1^2), (s_0^1,s_2^2), (s_0^1,s_3^2), (s_1^1,s_0^2),\\
& (s_1^1,s_1^2), (s_1^1,s_2^2), (s_1^1,s_3^2), (s_2^1,s_0^2), (s_2^1,s_1^2),\\
& (s_2^1,s_2^2), (s_2^1,s_3^2), (s_3^1,s_0^2), (s_3^1,s_1^2), (s_3^1,s_2^2),\\
& (s_3^1,s_3^2) \}\\
S^{12} = \{ & (s_4^1,s_8^2), (s_4^1,s_9^2), (s_4^1,s_{10}^2), (s_4^1,s_{11}^2), (s_5^1,s_8^2),\\
& (s_5^1,s_9^2), (s_5^1,s_{10}^2), (s_5^1,s_{11}^2), (s_6^1,s_8^2), (s_6^1,s_9^2),\\
& (s_6^1,s_{10}^2), (s_6^1,s_{11}^2), (s_7^1,s_8^2), (s_7^1,s_9^2), (s_7^1,s_{10}^2),\\
& (s_7^1,s_{11}^2) \} \qquad (8)
\end{aligned}
$$

Where s_j^i is the 16-QAM signal whose label is j that is transmitted over the i^{th} transmit antenna. Now, 16 metrics are computed at the I-decoder for each subset, before the trellis computation as in Equation (8) ending with 16 survivors, one for each subset. Since each encoder has two input bits, there are four possible metrics to be compared at each state in the trellis of the I-decoder. The same principle is applied to the Q-decoder case.

5. Results

Figure 2 shows the performance of the I-Q ST code compared to the ST QPSK code in [3]. It is clear that the I-Q code with the simplified decoding algorithm provides the required gain for the cases of one and two receive antennas. However, the gain is higher in the case of two receive antennas, which is counter to what is expected. This is because the simplified algorithm performs near optimum for the case of two receive antennas due to the diversity provided in detecting the Q/I components in the I/Q decoder.

Similar results for the 8-state codes are shown in Figure 3. Here, the gains are reduced due to the reduced difference between the design parameters of both the I-Q and QPSK codes. The performance of the same codes tested for correlated fading channels with proper interleaving is shown in Figure 4. A 25x16 interleaver is used. This interleaver size is enough to break the channel memory. Also, the I-Q code with the simplified algorithm provides the required gains even when non-ideal interleaving is considered.

Figure 5 shows the performance of the 16-QAM ST codes over independent fading channels. The results show that the proposed simplified decoding algorithm does not perform well if one receive antenna is used. This is because it is trying to guess for the best Q/I components from the received signals in the I/Q decoder.

In order to get the maximum performance of the I-Q ST codes, super-trellis decoding with its high decoding complexity may be used. The simplified decoding algorithm gives near the optimal performance for the case of two receive antenna with any constellation size. Also, it performs very well when a small signal constellation is used even with one receive antenna.

Similar results for the same codes are presented for the case of non-ideal interleaving. The gains are less because the correlation of the channel samples reduces the effect of the code's time diversity. However, the I-Q code with the simplified decoding algorithm shows to be the best for the two receive antenna case compared to codes with the same decoding complexity.

References

[1] Sklar, "Rayleigh Fading Channels in Mobile Digital Communication Systems Part I: Characterization," *IEEE Comm. Magazine*, September 1997.

[2] J. Poakis, *Digital Communications*. New York: McGraw-Hill, Inc., 1989.

[3] V. Tarokh, N. Seshadri and A.R. Calderbank, "Space-Time Codes For High Data Rate Wireless Communication: Performance Criterion and Code Construction," *IEEE Trans. Info. Theory,* March 98.

[4] A. Wittneben, "A New Bandwidth Efficient Transmit Antenna Modulation Diversity Scheme for Linear Digital Modulation," *Proc. IEEE ICC'93.*

[5] Siavash Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE JSAC,* Oct. 98.

[6] V. Tarokh, S. Alamouti and P. Poon, "New Detection Schemes for Transmit Diversity with no Channel Estimation," *Proc. IEEE Int. Conf. On Universal Personal Comm. '98.*

[7] S. Alamouti, V. Tarokh and P. Poon, "Trellis-Coded Modulation and Transmit Diversity: Design Criteria and Performane Evaluation," *Proc. IEEE Int. Conf. On Universal Personal Comm. '98.*

[8] S. Zummo and S. Al-Semari, "Space-Time Coded QPSK for Rapid Fading Channels," Submitted to the *ICC'2000.*

[9] S. Zummo and S. Al-Semari, "Design of 16-QAM Space-Time Codes for Rapid Rayleigh Fading Channels", Submitted to *the 10th Virginia Symposium of Wireless Personal Communications 2000,* Virginia, USA.

[10] S. Al-Semari and T. Fuja, "I-Q TCM Reliable Communication Over Rayleigh Fading Channel Close to the Cutoff Rate," *IEEE Trans. On Infor Theory,* Jan 1997.

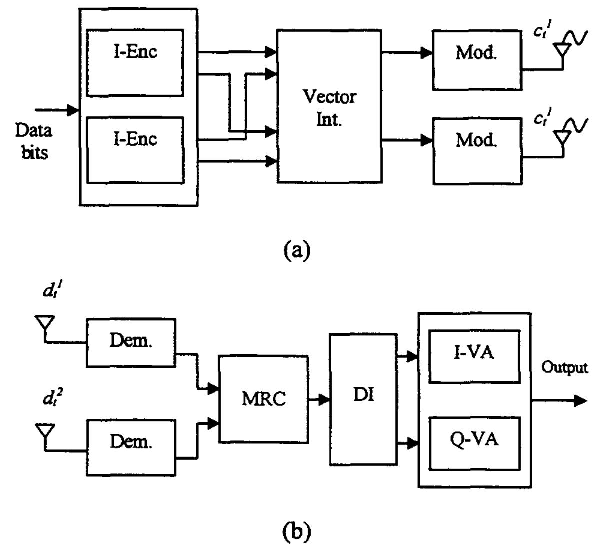

Figure 1: The structure of an I-Q ST system using two transmit antennas.

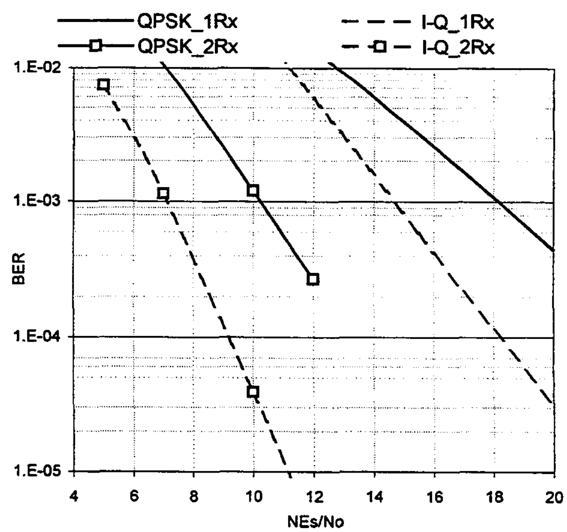

Figure 2: Performance of the 4-state QPSK codes for 1 and 2 Rx antennas over independent fading channels.

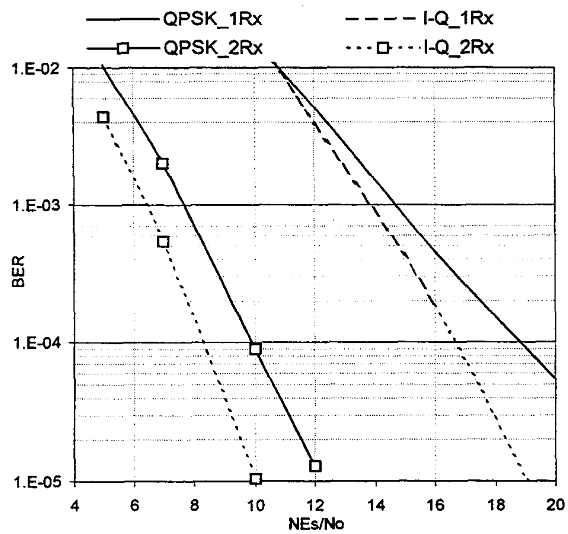

Figure 3: Performance of the 8-state QPSK codes for 1-Rx and 2-Rx antenna over rapid fading channel.

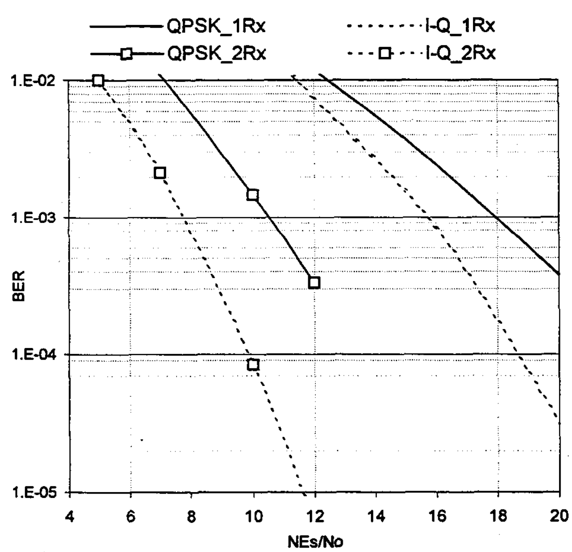

Figure 4: Performance of the 4-state QPSK codes for 1-Rx and 2-Rx antenna over correlated fading channel with f_DT=0.01.

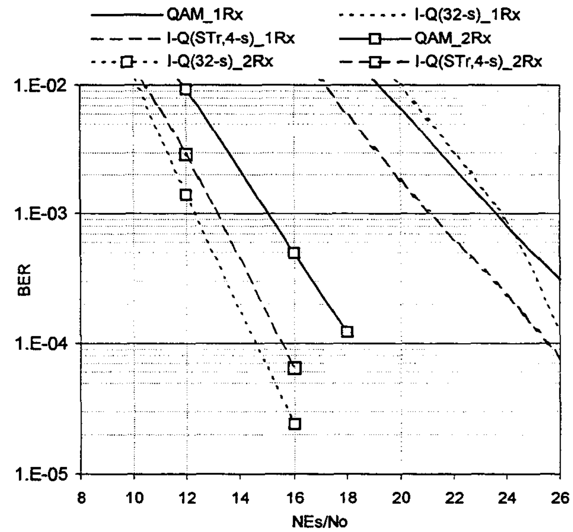

Figure 5: Performance of the 16-QAM codes for 1-Rx and 2-Rx antenna over independent fading channel. STr: Super Trellis.

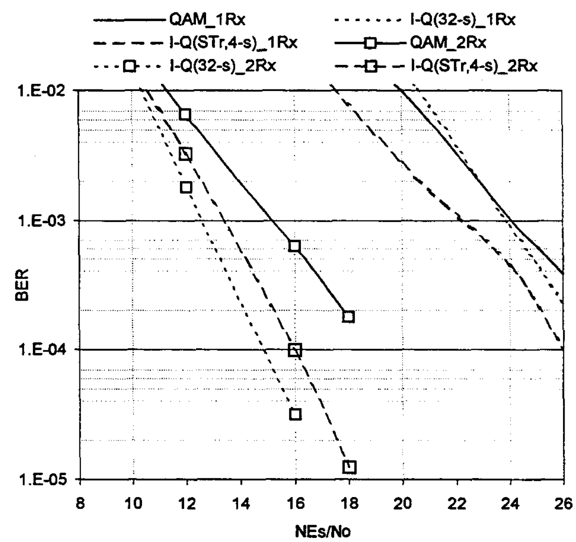

Figure 6: Performance of the 16-QAM codes for 1-Rx and 2-Rx antenna over correlated fading channel with f_DT=0.01.