ON THE PERFORMANCE OF COOPERATIVE WIRELESS NETWORKS UNDER INTERFERENCE CONDITIONS

Salam Zummo

Electrical Engineering Department
King Fahd University of Petroleum and Minerals (KFUPM)
Dhahran 31261, Saudi Arabia
E-mail: zummo@kfupm.edu.sa

ABSTRACT

User cooperation can be employed to provide transmit diversity in wireless networks . In this paper we analyze the error performance of coded cooperative diversity with multiple cooperating users over Nakagami fading channels under interference conditions. We derive the end-to-end bit error probability of coded cooperation (averaged over all cooperation scenarios). Results show that allowing more cooperating users improves the performance of the network under low loads, where two cooperating users suffice for highly-loaded networks. Furthermore, the gains obtained by increasing the number of cooperating users decreases with increasing the network load.

1. INTRODUCTION

Recently, it has been proposed to provide transmit diversity in wireless networks by employing user cooperation [1]. In user cooperation, mobile units share their antennas to achieve uplink transmit diversity as illustrated in Figure 1. Since the signal of each user undergoes an independent fading path to the base station (BS), this approach achieves spatial diversity through the partner antenna. In principle, the idea of user cooperation is based on the work of [2, 3]. In conventional user cooperation the partner repeats the received bits. Recently, *coded cooperation* was proposed [4] for two cooperating users, in which the codeword of each user is partitioned into two subframes; one subframe is transmitted by the user, and the other by the partner. The performance of coded cooperation was derived in [5] for multiple cooperating users.

In existing work on coded cooperation, interference between nodes within the network is usually neglected. However, interference exists in cooperative networks due to the use of shared resources. In [6], the performance of coded multiple-access wireless networks was analyzed under interference conditions. In this paper, we derive the error performance of coded cooperative networks with interference over Nakagami fading channels for arbitrary number of users.

The paper is organized as follows. In Section 2, the coded cooperative network is described. The end-to-end average error performance of coded cooperation is derived in Section 3. Results are presented in Section 4. The paper is concluded with main outcomes in Section 5

Thanks to KFUPM and British Council/BAE for supporting this research.

2. SYSTEM MODEL

In this paper, we consider a multiple-access wireless network of N_u users transmitting to a common base station (BS). Users are assumed to be active with probability p. The corresponding network load is $G = pN_u$. In the network, every J users (partners) cooperate in the transmission to a common BS by forming a cluster of size J. For each user in the network, a frame is formed by encoding K bits into $L = K/R$ bits, where R is the code rate. Partners within a J-user cluster cooperate by dividing their L-bit frames into J *subframes* containing $L_1, L_2, \ldots, L_J$ bits, where $L = L_1 + L_2 + \ldots + L_J$. The partitioning of the coded bits in the J subframes may be achieved using a rate-compatible punctured convolutional (RCPC) codes [7] as in [4].

During the first subframe duration, each user transmits his first subframe [4, 5] composed of $L_1 = K/R_1$ coded bits, where R_1 is the code rate of the codeword in the first subframe, obtained by puncturing the L-bit codeword into a L_1-bit punctured codeword. Clearly, $R_1 > R_J = R$. Upon the end of the first subframe, each user decodes the rate-R_1 codewords of his partners. In the remaining $J - 1$ subframes, each user transmits one subframe for each of his $J - 1$ partners in a predetermined pattern. The code rates corresponding to different cooperation levels are $R_1 > R_2 > \ldots > R_J = R$.

The modulation scheme employed is BPSK with coherent detection. The matched filter output at user k due to user l in the time interval t in the j^{th} subframe is modeled by

$$y_{l,k,j}(t) = \sqrt{E_I} a_{l,k,j} s_{l,j}(t) + z_{k,j}(t) + \sum_{i=1}^{n} a_{i,k,j} \sqrt{E_I}, \tag{1}$$

where $s_{l,j}(t)$ is the signal transmitted from user l in time instance t in the j^{th} subframe and $z_{k,j}(t)$ is an AWGN sample at user k with a Gaussian distribution given by $\mathcal{N}(0, \frac{N_0}{2})$. Here, E_I is the average received signal energy through the interuser channel. When $k = 0$, the signal

1-4244-0779-6/07/$20.00

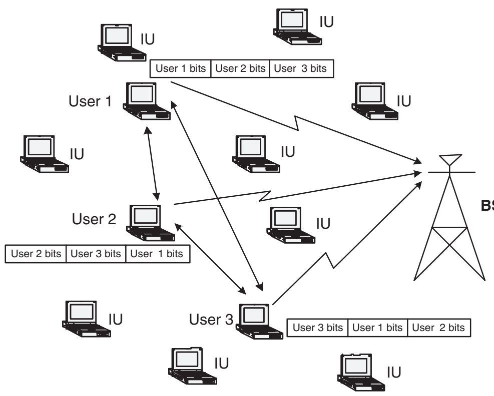

Fig. 1. Schematic diagram of a wireless network employing 3-user coded cooperation interfering users (IU).

model in (1) represents the signal from user l received at the BS through the uplink channel in the j^{th} subframe, where the average signal energy E_s replaces E_I in (1).

In (1), the coefficients $a_{l,k,j}$ and $a_{i,k,j}$ are the gains of the interuser channels between user l and user k, and between an interfering user i and user k in the j^{th} subframe, modeled as a Nakagami random variables [8]. Here, the interuser and uplink channels are assumed to be mutually independent and slow enough such that the fading process stays fixed within a subframe.

Given that n users are interfering with the signal of user l at user k, the signal-to-interference-and-noise ratio (SINR) at user k in the j^{th} subframe is written as

$$\beta_{l,k,j} = \frac{a_{l,k,j}^2 \gamma_I}{1/2 + \gamma_I \sum_{i=1}^{n} a_{i,k,j}^2}, \qquad (2)$$

where $\gamma_I = \frac{E_I}{N_0}$ is the average signal-to-noise ratio (SNR) for the interuser channels. In (2), we assumed that the desired and interfering signals have the same average received energy. Setting $k = 0$ and replacing E_I and γ_I with E_s and $\gamma_s = \frac{E_s}{N_0}$, respectively, (2) becomes the SINR at the BS due to the reception of the j^{th} subframe of user l. In the rest of the paper, the users' subscripts; namely, l and k will be removed from the SINR expression to simplify notation.

3. PERFORMANCE ANALYSIS

3.1. Average Error Probability

In a cluster of J cooperating users, there are J^2 possible cooperation scenarios. The end-to-end error probability of a user is obtained by averaging the probability of error over two random variables. The first random variable, denoted by U, indicates the number of partners who were able to decode the first subframe of the user. The second variable, denoted by V, represents the number of partners whose first subframes were decoded successfully by the user. In order to simplify analysis, we assume that the effect of duplicate reception of subframes.

The end-to-end bit error probability averaged over all

Table 1. The probabilities of *no cooperation* and *full cooperation* scenarios for a J-user cluster over Rayleigh interuser channels with a network load of $G = 0.2$ and an interference-limited interuser of an SNR γ_I.

γ_I (dB)	$p_{v,u}$	$J=2$	$J=3$	$J=4$
10	$p_{0,0}$	0.1859	0.2238	0.2951
	$p_{J-1,J-1}$	0.7964	0.7412	0.6425
∞	$p_{0,0}$	0.1402	0.1515	0.1613
	$p_{J-1,J-1}$	0.8545	0.8434	0.8298

cooperation scenarios [5] is given by

$$P_b = \sum_{v=0}^{J-1} \sum_{u=0}^{J-1} \binom{J-1}{v} \binom{J-1}{u} p_{v,u} P_b(v,u), \qquad (3)$$

where $P_b(v,u)$ is the conditional bit error probability of a user given that $U = u$ and $V = v$, and $p_{v,u}$ is the probability of such cooperation scenario given by

$$p_{v,u} = E_\beta \left\{ [1 - P_B(\beta)]^{v+u} P_B(\beta)^{2J-2-v-u} \right\}, \qquad (4)$$

where β is the SINR of the interuser channel and $P_B(\beta)$ is the frame error probability of the first subframe, which is upper bounded as in [9].

It was found out that the performance of coded cooperation is dominated by the performance of the two extreme cooperation scenarios; namely, the full cooperation with probability $p_{0,0}$, and the no cooperation with probability $p_{J-1,J-1}$. These probabilities are listed in Table I for different cluster sizes with interference-limited and different SNR values. We observe that for a fixed interuser channel quality, the probability of no cooperation increases as the cluster size increases, which causes the performance of large-size clusters to be worse than that of small-size clusters. As the uplink quality improves for a fixed interuser quality, small-size clusters are expected to outperform large-size clusters.

3.2. Conditional Error Probability

Conditioning on $U = u$ and $V = v$ has two consequences on the error performance of a user. First, the received codeword at the BS has a rate R_ξ, where $\xi = \max(J - v, u + 1)$, i.e., the rate of the received codeword is either R_{J-v} or R_{u+1}. In this case, $\{c_d\}$ used in (5) are for the rate-R_ξ code. Second, given that $U = u$, each codeword is transmitted over $u + 1$ subframes, whose lengths are $\{L_j\}_{j=1}^{u+1}$ bits. Recall that each subframe is transmitted over an independent fading channel via one of the partners in a cluster. Thus, the pairwise error probability (PEP) $P_u(v, u; d)$ is a function of the distribution of the d error bits over the $u + 1$ subframes transmitted by the $u + 1$ partners. Since the coded bits of each subframes may not be consecutive bits due to the puncturing used, this distribution is quantified assuming uniform distribution of the coded bits over the subframes [5] and is derived as follows.

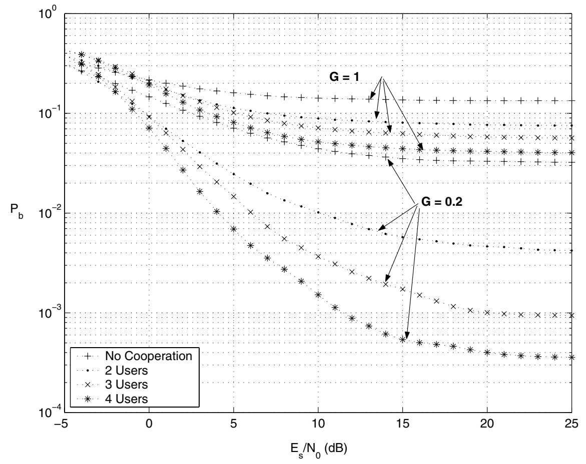

Fig. 2. Analytical bit error probability of coded cooperation in Nakagami fading with $m = 1$ (Rayleigh fading) with network loads of $G = 0.2, 1$ and perfect interuser channels.

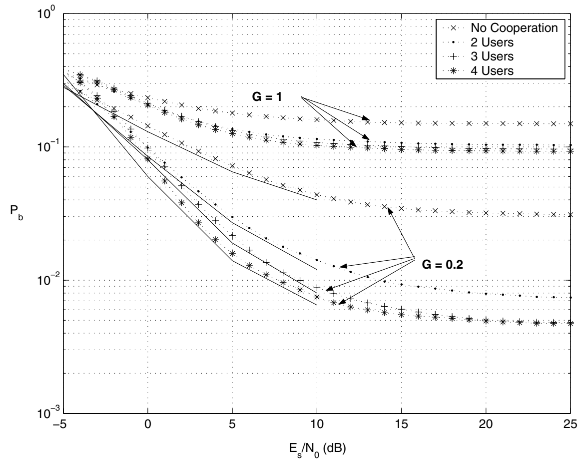

Fig. 3. Bit error probability of coded cooperation in Nakagami fading with $m = 1$ (Rayleigh fading) with network loads of $G = 0.2, 1$ and interference-limited interuser channels. (dashed: approximation, solid: simulation.)

Given $U = u$ and $V = v$ for a user in a cluster, the bit error probability of the corresponding convolutional code is upper bounded as

$$P_b(v, u) \leq \sum_{d=d_{\min}}^{L(v,u)} c_d P_u(v, u; d), \tag{5}$$

where $d_{\min}$ is the minimum distance of the code, c_d is the number of information bit errors in a codeword of weight d. In (5), $L(v, u)$ is the codeword length when $U = u$ and $V = v$ and $P_u(v, u; d)$ is the corresponding PEP for a weight-d codeword given by

$$P_u(v, u; d) = \sum_{\mathbf{w}} \frac{1}{\binom{L(v,u)}{d}} \prod_{j=1}^{u+1} \binom{L_j}{w_j} P_u(v, u; d|\mathbf{w}), \tag{6}$$

where $\mathbf{w} = \{w_j\}_{j=1}^{u+1}$ and w_j is the weight of the j^{th} subframe, and $P_c(v, u; d|\mathbf{w})$ is the conditional PEP for BPSK with coherent detection. An exact expression of the unconditional PEP can be found by using the integral expression of the Q-function, $\mathrm{Q}(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{(-x^2/2\sin^2\theta)} d\theta$ as

$$P_u(v, u; d|\mathbf{w}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{j=1}^{u+1} \Phi_\beta\left(w_j \alpha_\theta\right) d\theta, \tag{7}$$

where $\alpha_\theta = \gamma_s / \sin^2\theta$ and

$$\Phi_\beta\left(s\right) = \mathrm{E}_\beta\left[e^{-s\beta}\right], \tag{8}$$

is the moment generating function (MGF) of the random variable β and the product in (7) results from the independence of the fading processes affecting different subframes.

In order to find the MGF of β, we need to derive its pdf, which depends on the number of interfering user, which is a Binomial random variable with parameters p and N_u. Therefore, the MGF of the SINR, β is given by

$$\Phi_\beta(s) = \sum_{n=0}^{N_u} \binom{N_u}{n} p^n (1-p)^{N_u - n} \Phi_{\beta|n}(s), \tag{9}$$

where $\Phi_{\beta|n}(s)$ is the conditional MGF of the SINR, β. For integer values of the Nakagami parameter m, $\Phi_{\beta|n}(s)$ [10] is given by

$$\Phi_{\beta|n}(s) = \frac{m^m}{\Gamma(nm)} \sum_{h=0}^{m} \binom{m}{h} \frac{\Gamma(nm+h)}{m^h} \times U\left(m; m(1-n) - h + 1; 1 + \frac{m}{s}\right), \tag{10}$$

where $U(.;.;.)$ is the confluent hypergeometric function of the second kind defined in [11]. The MGF required to evaluate (7) is found by substituting (10) in (9) and expressing $U(.;.;.)$ as

$$U(a; b; x) = \frac{\pi}{\sin(\pi b)}\left[\frac{{}_1F_1(a, b; x)}{\Gamma(a - b + 1)\Gamma(b)} - \frac{x^{1-b}}{\Gamma(a)\Gamma(2-b)}\, {}_1F_1(a - b + 1, 2 - b; x)\right], \tag{11}$$

where ${}_1F_1(.,.;.)$ is the confluent hypergeometric function that is available in any numerical package. Once the MGF is evaluated, the PEP is evaluated by substituting (9) in (7). The end-to-end bit error probability is then found by substituting (7) in (6) and then in (5) and (3).

4. NUMERICAL RESULTS

For illustration, we consider coded cooperation with network loads of $G = 0.2$ and $G = 1$. Within the network, coded cooperation with cluster sizes $J = 1, 2, 3, 4$ was considered. Each user employs a RCPC code from [7] with a memory order $M = 4$, puncturing period $P = 8$ and a mother code rate $R_J = \frac{1}{4}$. In all cases, the source block is $K = 128$ information bits.

Figures 2 and 3 show the performance of coded cooperation over Rayleigh fading channels with different number of cooperating users for network loads of $G = 0.2$ and $G = 1$. We observe that the gains obtained by increasing

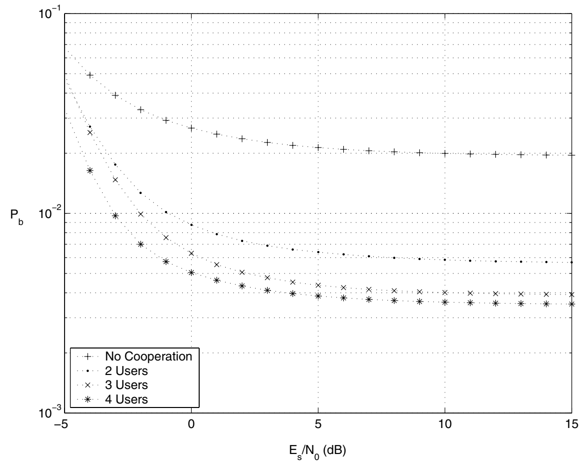

Fig. 4. Bit error probability of coded cooperation in Nakagami fading with $m = 5$ and a network load of $G = 0.2$ and interference-limited interuser channels.

the number of cooperating users decrease as the network load increases. This is expected since diversity becomes less important to the performance as the interference level increases.

At high uplink SNR the bit error probability suffers from an error floor due to interference, and the error floor decreases as the number of cooperating users increases. This applies especially for the case of perfect interuser channels as shown in Figure 2, i.e., no interference or noise in the interuser channels. Figure 3 shows the results for the case of interference-limited interuser channels. We observe that when interuser channels suffer from interference, the performance of large clusters degrades as the uplink SNR increases. This is because at high SNR the performance becomes limited by the performance of the no cooperation scenario, whose probability increases with the cluster size as shown in Table I.

Figure 4 shows the results for coded cooperation in Nakagami fading with $m = 5$ and a network load of $G = 0.2$. As discussed above, the performance gain of large clusters decreases as the interuser channels become more noisy. However, counter to the above observation, small clusters do not tend to outperform large clusters as the uplink SNR increases. This is mainly because the interuser channels are distributed according to Nakagami fading with a large Nakagami parameter, $m = 5$, which makes the probability of correct decoding of the first subframe large compared to the case of Rayleigh fading.

5. CONCLUSIONS

The performance of coded cooperation networks was analyzed in Nakagami fading under interference conditions. Results show that gains obtained by increasing the number of cooperating users decrease with increasing the network load. On the other hand, the performance of large number of cooperating users improves as the Nakagami parameter increases.

6. ACKNOWLEDGEMENTS

The author acknowledges the support provided by KFUPM and the British Council/BAE Summer Program.

7. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Auzhang, "User cooperation diversity - part i: System description," *IEEE Transactions on Communications*, vol. 51, pp. 1927–1938, January 2003.

[2] T. Cover and A. El Gamal, "Capacity theorems for the relay channels," *IEEE Transactions on Information Theory*, vol. 25, pp. 572–584, September 1979.

[3] J. Laneman and G. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Transactions on Information Theory*, vol. 49, pp. 2415 – 2425, October 2003.

[4] T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Transactions on Information Theory*, vol. 52, pp. 375–391, February 2006.

[5] S. Zummo, "Performance Analysis of Coded Cooperation Diversity in Wireless Networks," *Journal on Wireless Communications and Mobile Computing (WCMC)*, To appear, 2006.

[6] C. Leanderson and G. Caire, "The Performance of Incremental Redundancy Schemes Based on Convolutional Codes in the Block-Fading Gaussian Collision Channel," *IEEE Transactions on Wireless Communications*, vol. 3, pp. 843–854, May 2004.

[7] J. Hagenauer, "Rate Compatible Punctured Convolutional Codes (RCPC-Codes) and Their Applications," *IEEE Transactions on Communications*, vol. 36, pp. 389–400, April 1988.

[8] M. Nakagami, *"The m-Distribution- A General Formula of Intensity Distribution of Fading"*, in Statistical Methods in Radio Wave Propagation. W. C. Hoffman (ed.), Pergamon Press, NY, USA, 1960.

[9] S. Kallel and C. Leung, "Efficient ARQ Schemes With Multiple Copy Decoding," *IEEE Transactions on Communications*, vol. 40, pp. 642–650, March 1992.

[10] V. Aalo and J. Zhang, "Performance Analysis of Maximal Ratio Combining in the Presence of Multipath Equal-Power Cochannel Interferers in a Nakagami Fading Channel," *IEEE Transactions on Vehicular Technology*, vol. 50, pp. 479–503, March 2001.

[11] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press Inc., London, UK, 4th edition, 1965.