Bit Error Probability of Bit-Interleaved Coded Modulation (BICM) in Wireless Environments

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***Abstract*— In this paper a union bound on the bit error probability of bit-interleaved coded modulation (BICM) is derived. In the derivation we assume that the bit errors in a codeword are uniformly distributed over the transmitted symbols. We derive the bound for BICM systems over AWGN, Rician and Nakagami fading channels. The proposed bound is general to any signal constellation and coding scheme with a known distance spectrum.**

I. Introduction

Coded modulation [1] jointly considers the error control coding and modulation to achieve high transmission rates at good quality. In fading environments the symbol-wise Hamming distance can be increased by interleaving the coded bits prior to mapping them onto the signal constellation [2]. The random nature of distributing the bits over different symbols causes the performance analysis to be difficult. The union bound in [2], [3] was derived assuming a symbol error causes a bit error, which is not true in general. In this paper we derive a union bound on the bit error probability of BICM over AWGN and fading channels by considering the distribution of error bits over symbols in a frame.

II. The Union Bound

The bit error probability of convolutional codes is

$$P_b \leq \frac{1}{k} \sum_{d=d_{min}}^{N} w_d \; P_u(d), \tag{1}$$

where d_{min} is the minimum distance of the code, N is the codeword length, w_d is the number of codewords with output weight d, and $P_u(d)$ is the unconditional pairwise error probability (PEP) of a weight-d codeword.

In the transmitter, every m bits are mapped onto one signal point from an M-ary signal constellation, where $M = 2^m$. Here, the number of symbols in a frame is $J = \lceil \frac{N}{m} \rceil$. Hence, $P_u(d)$ is a function of the distribution of the d error bits over the J symbols in the frame. This distribution is quantified assuming uniform interleaving of the coded bits over symbols in a frame. Therefore, $P_u(d)$ is averages over the error bit distribution pattern $\mathbf{j}$ as

$$P_u(d) = \sum_{\mathbf{j}} P_u(d|\mathbf{j}) p(\mathbf{j}), \tag{2}$$

where $p(\mathbf{j})$ is obtained as in [4].

The conditional PEP with coherent detection is

$$P_c(d|\mathbf{j}) = \mathrm{E}_{d_1^2,\cdots,d_J^2} \left[\mathrm{Q} \left(\sqrt{\frac{mR_c\gamma_b}{2} \cdot \sum_{j=1}^{J} a_i^2 d_j^2} \right) \right], \tag{3}$$

where γ_b is the SNR per information bit, and d_j^2 is the squared Euclidean distance between the j^{th} component of the error codeword and that of the desired codeword. The unconditional PEP is found by averaging (3) over the fading gains and grouping symbols with the same number of error bits as

$$P_u(d|\mathbf{j}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{v=1}^{w} \left\{ \mathrm{E}_a \left[\Psi_{d_v^2}(\beta(\theta) a^2) \right] \right\}^{j_v} d\theta, \tag{4}$$

where $\beta(\theta) = \frac{mR_c\gamma_b}{4\sin^2\theta}$, $w = \min m, d$, j_v is the number of symbols with v bit errors, a is the channel random variable and $\Psi_{d_v^2}(\zeta) = \mathrm{E}_{d_v^2}\left[e^{-\zeta d_v^2}\right]$ is the characteristic function of d_v^2. In order to find the distribution of d_v^2 for any signal constellation, consider all $\binom{M}{2}$ possible distinct symbol pairs, count $\{q_{v,i}\}$ the number of symbol pairs with Hamming distance v and squared Euclidean distance $\xi_{v,i}$, $i = 1, 2, \cdots, k_v$, assuming there exists k_v possible distinct squared Euclidean distances between symbol pairs of Hamming distance v. The characteristic function of d_v^2 is then given by

$$\Psi_{d_v^2}(\zeta) = \sum_{i=1}^{k_v} p_{v,i} \; e^{-\zeta \xi_{v,i}}. \tag{5}$$

Expressions for the PEP of BICM over AWGN and Rician and Nakagami fading channels is obtain by averaging (4) over the distribution of the channel gains.

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