THROUGHPUT ANALYSIS OF ARQ PROTOCOLS IN RAYLEIGH BLOCK FADING ENVIRONMENTS

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***Abstract*— Block fading is a popular channel model that approximates the behavior of different wireless communication systems. Automatic-Repeat Request (ARQ) protocols are used to provide reliable communications in wireless networks. In this paper, the throughput of basic and hybrid ARQ protocols over Rayleigh block fading channels is derived. The throughput is computed for different modulation schemes and for a wide variety of channel memory length. Results show that longer channel memory improves the performance of basic ARQ protocols whereas it degrades the performance of hybrid ARQ protocols. This is due to the fact that the decoder in a hybrid ARQ protocol utilizes the diversity provided by the independent fading blocks to average over the channel behavior and to obtain a good performance. Furthermore, the throughput improvement obtained in basic ARQ protocols decreases with increasing the memory length.**

I. INTRODUCTION

A serious challenge to having good communication quality in wireless systems is the time-varying multipath fading environments. Error correcting codes and diversity techniques are efficient techniques to mitigate fading by providing the receiver with independent fading realizations of the channel. In order to distribute burst errors resulting from consecutive deeply faded bits in the decoder, the coded bits are interleaved prior to transmission over the channel. However, infinite interleaving is impractical in delay-sensitive applications. Therefore, channel models, such as the *block fading* channel [1], provide is an acceptable model for many wireless communication systems including frequency-hopped spread-spectrum (FH-SS), time-division multiple access (TDMA) and orthogonal-frequency division multiplexed (OFDM) systems. In this model a frame undergoes several propagation channels, where the fading is constant for signals in each block of bits and independent from a block to another.

In data networks error-free communication is essential, and this is not possible due to fading and channel noise. Frequently, error-free communication is accomplished using ARQ techniques. Basic ARQ protocols are based on error detection and retransmission [2] where no channel coding is employed. If a received packet is detected in error at the receiver, a retransmission of the same packet is initiated through a feedback channel [2]. The performance of an ARQ protocol is characterized by its throughput, which is defined as the number of packets successfully transmitted per a channel use. Selective repeat (SR) ARQ [2] is a basic ARQ protocol in which the sender retransmits only the negatively acknowledged packets. The throughput of SR ARQ is known to be the highest among the three basic ARQ protocols. The performance of basic ARQ protocols was shown to deteriorate significantly over time-varying channels such as encountered in wireless systems. Thus channel coding is used in conjunction with ARQ protocols in order to enhance their performance, leading to the hybrid ARQ protocols [3].

The throughput of ARQ and hybrid ARQ protocols have been analyzed for many communication systems [2, 4–7]. Particularly, in [4] the throughput of basic ARQ protocols over slow Rayleigh fading was derived. Chiani in [7] has derived the throughput of hybrid ARQ protocols employing short-length block codes over FH-SS systems with Rayleigh fading. The analysis requires high computational complexity for practical packet lengths used with convolutional and turbo codes. In this paper we analyze the throughput of SR ARQ protocol in block fading environments with Rayleigh distribution. The throughput of type-I hybrid ARQ over block fading channels is investigated by simulations since bounding techniques are loose for the SNR values of interest. Hybrid ARQ protocols considered in this paper are based on convolutional and turbo codes.

The outline of the paper is as follows. The system model is described in the next section. In Section III, the throughput of basic and hybrid ARQ protocols is derived. Results are discussed in Section IV and conclusions are presented in Section V.

II. SYSTEM MODEL

A. Basic ARQ System

In the basic SR ARQ the sender retransmits the packet if some error is detected in the packet. The process is repeated until the packet is successfully received [2]. A packet is composed of N bits, where each bit is modulated using a binary modulator and transmitted over the channel.

The modulation schemes considered in this paper are the coherent binary phase-shift keying (BPSK) and the noncoherent binary frequency-shift keying (BFSK). The channel we adopt is a block fading channel in which each packet is subject to F independent fading realizations, resulting in a block of length $m = \lceil \frac{N}{F} \rceil$ bits being affected by the same fading realization.

Coherent or noncoherent detection can be used at the receiver. In coherent receivers the channel phase and amplitude are available. In this case the matched filter sampled output at time l in the f^{th} fading block is given by

$$y_{f,l} = \sqrt{E_s} h_f s_{f,l} + z_{f,l}, \tag{1}$$

where E_s is the average received energy, $s_{f,l} = (-1)^{c_{f,l}}$, where $c_{f,l}$ is the corresponding information bit, and $z_{f,l}$ is an additive white noise modeled as independent zero-mean complex Gaussian random variables with variance $\frac{N_0}{2}$ per dimension, i.e., $z_{f,l} \sim \mathcal{CN}(0, N_0)$. The coefficient h_f is the channel gain in fading block f which is modeled as $\mathcal{CN}(0,1)$ and is written as $h_f = a_f \exp(j\theta_f)$, where θ_f is uniformly distributed and a_f is the amplitude that has a normalized Rayleigh distribution given by

$$f_a(a) = 2a\mathrm{e}^{-a^2}, \qquad a \geq 0. \tag{2}$$

The coherent receiver decides for the bit as 1 if $\mathrm{Re}\{y_{f,l}\} > 0$, and 0 otherwise, where $\mathrm{Re}\{.\}$ represents the real part of a complex number. Noncoherent BFSK receivers employ a square-law combining [8], whose outputs for a bit $c = 0, 1$ are represented by

$$\begin{aligned} r_{f,l}^{(I,c)} &= \sqrt{E_s} a_f \delta(c_{f,l}, c) \cos(\theta_f) + \eta_{f,l}^{(I,c)} \\ r_{f,l}^{(Q,c)} &= \sqrt{E_s} a_f \delta(c_{f,l}, c) \sin(\theta_f) + \eta_{f,l}^{(Q,c)}, \end{aligned} \tag{3}$$

where $r_{f,l}^{(I,c)}$ and $r_{f,l}^{(Q,c)}$ for $c = 0, 1$ is the correlation of the received signal with the inphase and quadrature dimensions of the signal corresponding to a coded bit c. In (3), θ_f is the unknown phase of the received signals in block f, $\delta(x,y) = 1$ if $x = y$ and $\delta(x,y) = 0$ otherwise; and $\eta_{f,l}^{(I,0)}$, $\eta_{f,l}^{(Q,0)}$ $\eta_{f,l}^{(I,1)}$ and $\eta_{f,l}^{(Q,1)}$ are independent variables with normal distribution, i.e., $\mathcal{N}(0, \frac{N_0}{2})$ distribution. The receiver chooses the bit c with the maximum $|r_{f,l}^{(I,c)}|^2 + |r_{f,l}^{(Q,c)}|^2$.

B. Hybrid ARQ System

ARQ protocols can be combined with channel coding to improve their reliability leading to the hybrid ARQ protocols [3]. In hybrid ARQ the error correction code is used to correct any errors imposed by the channel. If the error detection code still detects errors, the packet is discarded and a retransmission is requested. The transmitter in Figure 1 consists of a binary encoder (e.g., convolutional or turbo), random interleaver and a modulator. Packets are composed of N coded bits. A rate-R_c encoder maps k information bits into n coded bits, where $R_c = \frac{k}{n}$ is the code rate. Each coded bit is modulated using BPSK or BFSK. The coded bits are interleaved prior to transmission over the channel in order to spread burst errors in the decoder, which result from low instantaneous SNR at the output of the demodulator due to fading.

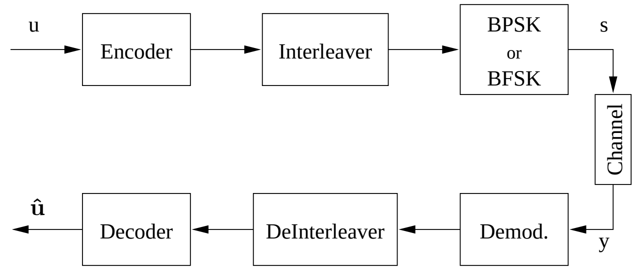

Fig. 1. Block diagram of hybrid ARQ system.

The receiver employs maximum likelihood (ML) sequence decoding which is optimal for packet error probability. If perfect SI is available at the receiver, the decoder chooses the codeword $\mathbf{S} = \{s_{f,l}, f = 1, \ldots, F, l = 1, \ldots, m\}$ that maximizes the metric

$$\mathbf{m}(\mathbf{Y}, \mathbf{S}) = \sum_{f=1}^{F} \sum_{l=1}^{m} \mathrm{Re}\{y_{f,l}^* h_f s_{f,l}\}. \tag{4}$$

The square-law detector is a suboptimal receiver with respect to minimizing the packet error probability [8]. The suboptimal decoder chooses the codeword $\mathbf{S}$ that maximizes

$$\mathbf{m}(\mathbf{R}, \mathbf{S}) = \sum_{f=1}^{F} \sum_{l=1}^{m} (r_{f,l}^{(I,c)})^2 + (r_{f,l}^{(Q,c)})^2, \tag{5}$$

where $\mathbf{R} = \{r_{f,l}^{(I,c)}, r_{f,l}^{(Q,c)}, f = 1, \ldots, F, l = 1, \ldots, m\}$. The performance of basic and hybrid SR ARQ protocols is derived below.

III. Performance Analysis

In this section the throughput of SR ARQ is computed. The performance of an ARQ protocol is characterized by its throughput η, which is defined as the ratio of the average number of packets accepted as error-free by the receiver to the total number of packets that can be transmitted [7]. In the derivation of the performance the probability of undetected errors are negligible. Under the assumptions of infinite buffer size at the receiver and noiseless feedback channel, the throughput of SR ARQ protocols is given [2] by

$$\eta = R_c(1 - P_{pe}), \tag{6}$$

where P_{pe} is the probability of packet error and R_c is the code rate of the channel code used in hybrid ARQ.

A. Basic ARQ

In basic ARQ the code rate $R_c = 1$ since no channel coding is employed. Under the block fading model a packet of length N bits undergoes F independent fading realizations, where $m = \lceil \frac{N}{F} \rceil$ is the size of each fading block. The packet error probability conditioned on the fading channel gains $\mathbf{h} = \{h_f\}_{f=1}^F$ is the probability that at least one fading block is in error, i.e.,

$$P_{pe|\mathbf{h}} = 1 - \Pr[\text{all the } F \text{ fading blocks are correct}]$$
$$= 1 - \prod_{f=1}^{F}(1 - P_{be|h_f}), \tag{7}$$

where $P_{be|h_f}$ is the probability that the f^{th} fading block is in error conditioned on the fading gain h_f. It is the probability that at least one bit in the fading block is in error

$$P_{be|h_f} = 1 - \Pr[\text{all the } m \text{ bits are correct}]$$
$$= 1 - (1 - P_{b|h_f})^m, \tag{8}$$

where $P_{b|h_f}$ is the conditional bit error probability, which is a function of the modulation scheme and the receiver employed. In the modulation schemes considered in this paper the probability of bit error is a function of the channel amplitude gain a_f, rather than the complex channel gain h_f. Let $\gamma_f = a_f^2\gamma_b$ denote the SNR value of of a bit in the f^{th} fading block, where γ_b is the SNR per information bit, then the conditional bit error probability for coherent BPSK is given by

$$P_{b|\gamma_f} = \mathrm{Q}\left(\sqrt{2\gamma_f}\right). \tag{9}$$

The conditional bit error probability for noncoherent BFSK is

$$P_{b|\gamma_f} = \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\gamma_f}. \tag{10}$$

Substituting (8) into (7), the conditional packet error probability becomes

$$P_{pe|\Gamma} = 1 - \prod_{f=1}^{F}(1 - P_{b|\gamma_f})^m, \tag{11}$$

where $\Gamma = \{\gamma_f\}_{f=1}^F$. The unconditional packet error probability is obtained by averaging (7) over the probability density function of the fading SNRs Γ. Since the fading gains affecting different fading blocks are independent, the unconditional packet error probability becomes

$$P_{pe} = 1 - \prod_{f=1}^{F}\mathrm{E}_{\gamma_f}\left[(1 - P_{b|\gamma_f})^m\right]. \tag{12}$$

Since all fading blocks are affected by independent and identical fading processes the packet error probability is

$$P_{pe} = 1 - \left\{\mathrm{E}_{\gamma}\left[(1 - P_{b|\gamma})^m\right]\right\}^F. \tag{13}$$

The packet error probability for noncoherent BFSK and coherent BPSK is found by substituting (10) and (9) in (13), respectively. Equation (13) is hard to be evaluated in a closed form, and hence numerical integration methods are used to evaluate it. The throughput of the basic SR ARQ with coherent BPSK and noncoherent is obtained by substituting (13) in (6).

B. Hybrid ARQ

For linear convolutional codes with k input bits, the bit error probability is upper bounded [9] as

$$P_b \le \frac{1}{k}\sum_{d=d_{\min}}^{N} w_d P_u(d), \tag{14}$$

where $d_{\min}$ is the minimum distance of the code, $P_u(d)$ is the unconditional pairwise error probability defined as the probability of decoding a received sequence as a weight-d codeword given that the all-zero codeword is transmitted. In (14), w_d is the number of codewords with output weight d. The weight distribution $\{w_d\}_{d=d_{\min}}^N$ is obtained directly from the weight enumerator of the code [9]. A similar union bound exists for turbo codes [10].

In block fading channels the pairwise error probability $P_u(d)$ is a function of the distribution of the d nonzero bits over the F fading blocks. In [11] $P_u(d)$ was derived for block fading channels and the derivation is summarized in the following. Denote the number of fading blocks with weight v by f_v and define $w = \min(m, d)$, then the fading blocks are distributed according to the pattern $\mathbf{f} = \{f_v\}_{v=0}^w$ if

$$F = \sum_{v=0}^{w} f_v, \qquad d = \sum_{v=1}^{w} v f_v. \tag{15}$$

Denote by $L = F - f_0$ the number of fading blocks with nonzero weights, then $P_u(d)$ is averaged over all possible fading block patterns as

$$P_u(d) = \sum_{L=\lceil d/m \rceil}^{d}\sum_{f_1=0}^{L_1}\sum_{f_2=0}^{L_2}\cdots\sum_{f_w=0}^{L_w} P_u(d|\mathbf{f})p(\mathbf{f}), \tag{16}$$

where

$$L_v = \min\left\{L - \sum_{r=1}^{v-1} f_r, \frac{d - \sum_{r=1}^{v-1} r f_r}{v}\right\}, \qquad 1 \le v \le w. \tag{17}$$

The probability of a fading block pattern $p(\mathbf{f})$ is computed using combinatorics as

$$p(\mathbf{f}) = \frac{\binom{m}{1}^{f_1}\binom{m}{2}^{f_2}\cdots\binom{m}{w}^{f_w}}{\binom{mF}{d}} \cdot \frac{F!}{f_0! f_1! \ldots f_w!}. \tag{18}$$

The union bound on the bit error probability of convolutional codes over a block fading channels is obtained by substituting (16) in (14). Substituting the bit error probability of a convolutional code in (13) results in a very complicated expression to evaluate. Furthermore, the union bound is usually loose for SNR values below the cutoff rate of the channel, which are the SNR values of interest in hybrid ARQ systems. Hence, simulation is used to get

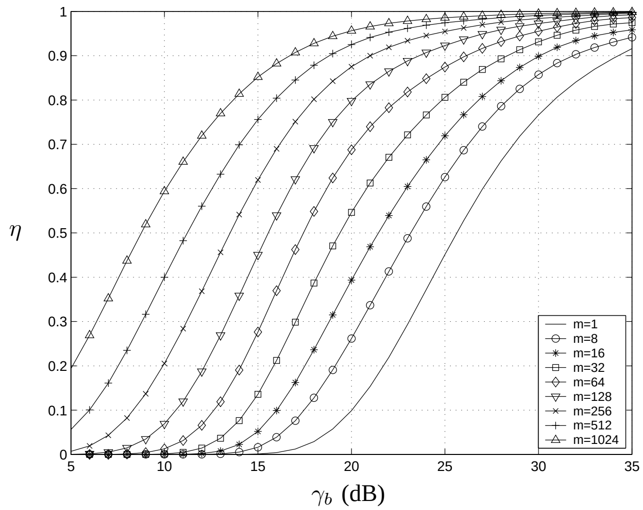

Fig. 2. Throughput of SR ARQ system employing BPSK modulation over block fading Rayleigh channels with memory length m. The packet length is $N = 1024$ bits.

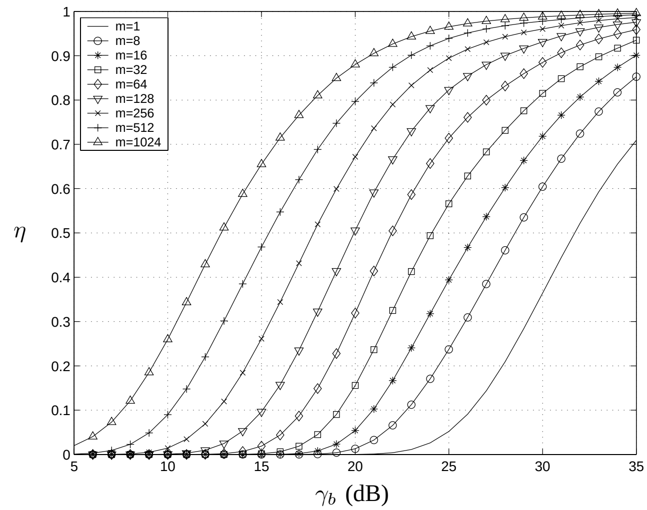

Fig. 3. Throughput of SR ARQ system employing BFSK modulation over Rayleigh block fading channels with memory length m. The packet length is $M = 1024$ bits.

some insight about the performance of hybrid ARQ protocols over block fading channels.

IV. Results

A. Basic ARQ

The throughput of the SR ARQ protocol employing BFSK and BPSK is shown in Figures 2 and 3, respectively. In the figures we see that the throughput improves as the channel memory becomes longer. Shorter channel memory results in larger number of independent fading realizations affecting a packet. As this number increases the probability that all these realizations be above the level required for successful reception of a bit decreases. Hence, the probability of error increases with shorter memory lengths and therefore the throughput degrades. From the figures, the system gains almost 2 dB for each double in the channel memory. Moreover, noncoherent BFSK encounters a SNR loss of about 4 dB compared to coherent BPSK, which is a well known result [8].

Figure 4 shows the throughput of SR ARQ with BPSK and noncoherent BFSK at SNR of 20 dB as a function of the channel memory length. It is clear that the throughput improves with longer channel memory length and the amount of improvement becomes smaller as the channel memory length increases. This means that for the same operating SNR value the throughput improves significantly in the short memory range and little improvement is achieved in the long memory range. From the figure, channel memory length greater than 200 bits provide very little throughput gain.

B. Hybrid ARQ

As discussed in Section III simulation techniques are used to illustrate the throughput of hybrid SR ARQ over block fading channel. Figure 5 shows the throughput of a rate-$\frac{1}{3}$ encoded SR ARQ protocol employing BPSK modulation. Unlike the uncoded case the throughput improves

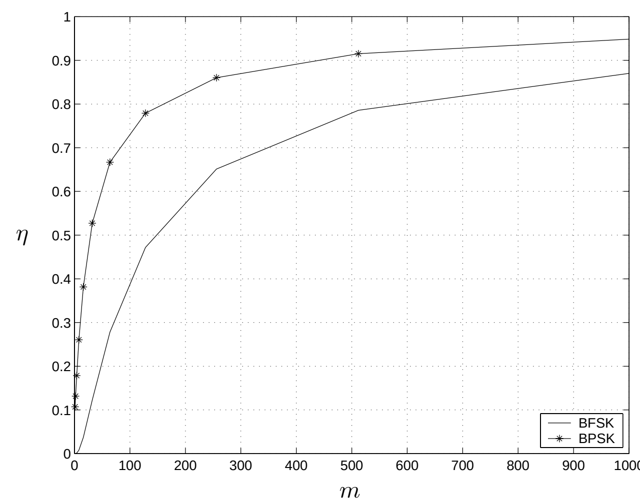

Fig. 4. Achieved throughput of SR ARQ employing coherent BPSK and BFSK modulation schemes over Rayleigh block fading channels with memory length m at SNR of 20 dB. The packet length is $N = 1024$ bits.

with decreasing the channel memory length. This is because the shorter the memory length the larger the number of independent fading realizations available at the decoder to decode a packet. As the number of fading realizations increases the decoder can use the code structure to average over the fading behavior and obtain a performance close to that obtainable performance over a non-fading channel with the average SNR. Thus shorter memory improves the throughput in hybrid ARQ protocols due to the larger diversity provided at the receiver. The same discussion is applied to the results of a rate-$\frac{1}{3}$ turbo coded SR ARQ shown in Figure 6. Again the throughput improves with shorter memory lengths. A comparison of turbo with convolutional coded SR ARQ protocols indicates that turbo codes improve the performance significantly in the presence of block fading environments, and this improvement degrades as the channel memory length increases.

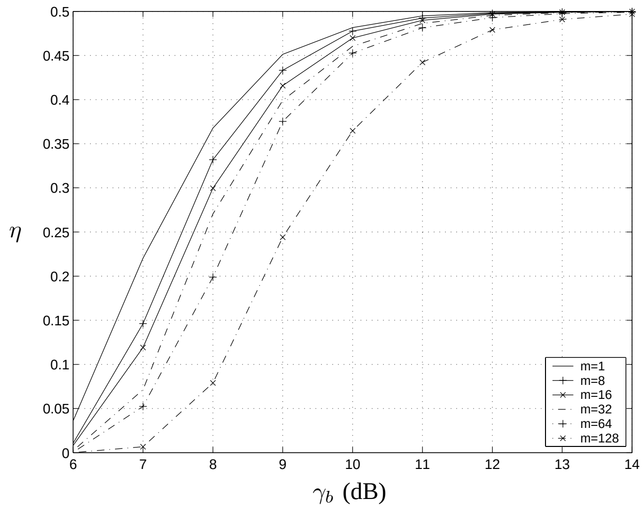

Fig. 5. Simulation of type-I hybrid ARQ system employing a rate-$\frac{1}{2}$ (5,7) convolutional code and BPSK modulation over Rayleigh block fading channels with memory length m. The packet length is $N = 1024$ coded bits.

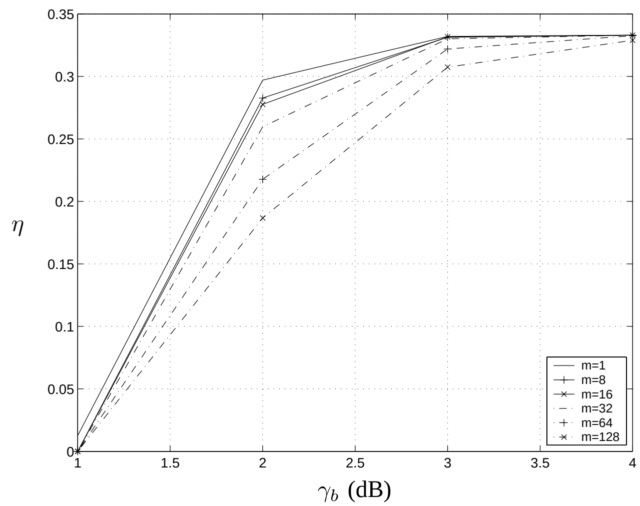

Fig. 6. Simulation of type-I hybrid ARQ system employing a rate-$\frac{1}{3}$ (1,5/7,5/7) turbo code and BPSK modulation over Rayleigh block fading channels with memory length m. The packet length is $N = 3 \times 1024$ coded bits.

V. Conclusions

In this paper we derived the throughput of basic and hybrid SR ARQ protocols over Rayleigh block fading channels. It was found out that longer channel memory improves the performance of basic ARQ protocols, whereas it degrades the performance of hybrid ARQ s. This is due to the fact that the decoder in a hybrid ARQ protocol utilizes the diversity provided by the independent fading blocks to average over the channel behavior and to obtain a good performance.

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