OUTAGE PROBABILITY OF WIRELESS NETWORKS WITH DIRECTIONAL ANTENNAS

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ABSTRACT

In this paper, we consider wireless networks with directional antennas using slotted ALOHA medium access control. The channel model incorporates the effects of propagation loss and shadowing with Rayleigh fading. By deriving the cumulative distribution function of the signal-to-interference-and-noise ratio of a certain link in the network, the outage probability of the link is analyzed. In particular, the analysis works for arbitrary beam patterns of the directional antennas used in the networks. Moreover, we propose a new parameter, the array interference factor (AIF), which fully characterizes the performance of any beam pattern in a wireless network over fading channels. Using a beam pattern with smaller AIF will give a better performance than other beam patterns with larger AIF values. The analysis of this paper enables us to find the optimal mobile density and the most efficient transmission power setting for such wireless networks.

INTRODUCTION

In wireless networks, when a mobile transmits signal to another mobile, it also causes interference to the other mobiles in the same network. This is a major factor that degrades the system performance which limits the user capacity in wireless networks. Conventionally, many systems use omnidirectional antennas which make the problem even worse; while the receiving mobile is located in a particular direction from the transmitting mobile, the omnidirectional antenna transmits signal to all directions with the same power strength. Not only does it cause interference to the other mobiles, but also the power usage is very inefficient, since the power is wasted in the directions that can not reach the receiving mobile. This give rise to the promising idea of using directional antennas in wireless networks.

By using directional antennas, both transmitting and receiving mobiles have the ability to generate beam patterns with high transmission gain and reception gain in the direction of each other, and low gain elsewhere. This makes the power usage much more efficient, which extends the battery life of the mobile. Moreover, it significantly reduces the interference problem because now only mobiles with receiving antennas pointing head-to-head to the main beam of the transmitting mobile will be interfered by the transmitting mobile. With the interference problem lightened, the system can have a higher user capacity than before.

To get the best system performance out of a wireless network, there are system parameters of the network that need to be optimized. For instance, the mobile density of the network affects the system performance to a great extent. If the density is large, mobiles experience severe interference from other mobiles which causes poor performance. On the other hand, when the mobile density is small, though the communication quality for each mobile is good, the overall system throughput is low. The network usage is thus not efficient. In order to optimally determine system parameters such as the mobile density of the network, it is necessary for us to know the system performance of the network under different settings of the system parameters. In the literature, only a few papers attempted to accomplish the goal. In [1], [2], [3], the system performance of wireless networks with omnidirectional antennas was analyzed. Later, [4] analyzed the performance of wireless networks using switched beam antennas. However, the beam pattern of the antennas used in the paper was too simplified. The result can not be applied to analyze the system performance when a realistic beam pattern is considered. [5] analyzed the performance of a receiver using a smart antenna with fixed number of interferers under Rayleigh and Rician fading. It did not analyze the performance of networks using arbitrary transmitting beam pattern either. In our work, we are proposing a method with which we can analyze the system performance using arbitrary beam pattern under a realistic channel model which includes the effect of path loss, shadowing and Rayleigh fading with slotted ALOHA [6] medium access control (MAC). Most importantly, we find that the system performance and each beam pattern are related only through one characteristic value, which we call the *antenna interference factor* (AIF), of the beam pattern. AIF can be computed very easily for any beam pattern and it enables us to do the performance comparison of various

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beam patterns very easily; any beam pattern with smaller AIF will have better performance than the others.

This paper is organized as follows. In Section II, we have a brief introduction to the channel model and the system model. In Section III, we describe how to compute the outage probability of a wireless network for a simplified beam pattern model. The analysis is then generalized to the case of an arbitrary beam pattern. In Section IV, we demonstrate and discuss various numerical examples. Finally, the conclusions are addressed in Section V.

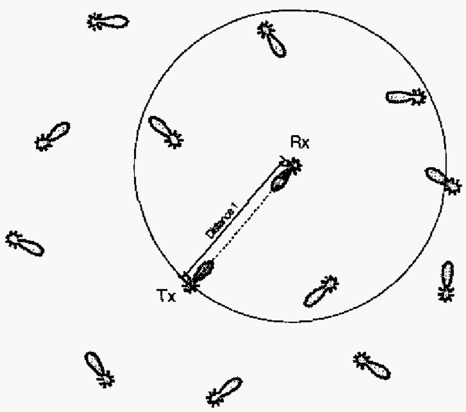

Fig. 1. Wireless network using directional antennas.

MODEL DESCRIPTION

In this paper, we consider a wireless network as shown in Fig. 1. We are interested in the performance of the communication link between two mobiles Tx (transmitting mobile) and Rx (receiving mobile). We first assume all mobiles use directional antennas for transmission and omni-directional antennas for reception. Later we will generalize the analysis to the case where directional antennas are used for both transmission and reception. We assume all mobiles use the same transmitting beam pattern. A typical beam pattern $f(\theta)$ is shown in Fig. 2, where $f(\theta)$ denotes the gain of the transmitting antenna in the direction θ. Here we assumes that Tx is always perfectly aligned to Rx, i.e., $\theta = 0$ always points to the direction of Rx. Though the beam pattern considered in this paper is in 2-D, the result can be easily generalized to 3-D beam pattern. Finally it is assumed that all mobiles have the perfect knowledge of the fading channels.

We use the same channel model as in [2], shown in Fig. 3. Assume that the distance between Tx and Rx is r_0, and Rx is in the direction of θ_0 from Tx. When Tx transmits power P_T, the received power at Rx is then $P_0 = P_T f(\theta_0)\, \zeta_0\, \xi_0^2$, where

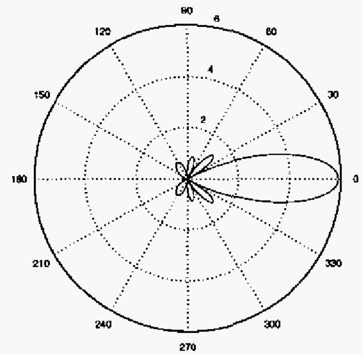

Fig. 2. Beam pattern $f(\theta)$.

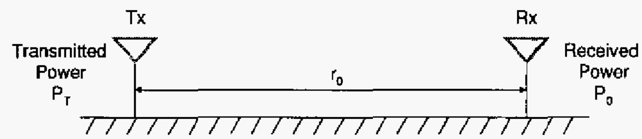

Fig. 3. Channel between Tx and Rx.

- ζ_0 accounts for the propagation loss and shadowing effect. It is log-normal distributed with conditional probability density function (pdf)

$$f(\zeta_0 \mid r_0) = \frac{1}{\sqrt{2\pi}\sigma\zeta_0}\, e^{-\frac{1}{2}\frac{\left(\log\zeta_0 - \log(K r_0^{-\eta})\right)^2}{\sigma^2}}, \qquad (1)$$

where $K r_0^{\eta}$ denotes the average attenuation level due to the propagation loss. K is a constant and η typically ranges from 3 to 4. Also note that σ is usually denoted in dB. The relation between σ and the dB value σ_{dB} is $\sigma = \frac{\sigma_{dB}\cdot\log 10}{10}$.

- ξ_0 is a Rayleigh distributed random variable which accounts for the Rayleigh fading. ξ_0^2 is exponential distributed. It is easy to see that conditioned on ζ_0, $P_0 \mid \zeta_0 \sim \mathrm{EXP}(\frac{1}{P_T f(\theta_0)\zeta_0})$ with pdf

$$f_0(P_0 \mid \zeta_0) = \frac{1}{P_T f(\theta_0)\zeta_0}\, e^{-\frac{P_0}{P_T f(\theta_0)\zeta_0}}, \qquad (2)$$

and cumulative density function (cdf)

$$F_0(P_0 \mid \zeta_0) = 1 - e^{-\frac{P_0}{P_T f(\theta_0)\zeta_0}}. \qquad (3)$$

Throughout the paper, we normalize distances to make r_0 the unit distance. The mobiles are randomly distributed on the 2-D plane as a Poisson point process. Let λ_M denote the average mobile density within a circle of radius 1 (shown in Fig. 1). For any circle of radius R on the plane, the number of mobiles in the circle $N_R \sim \mathrm{POI}(\lambda_M R^2)$, i.e., $Pr\{N_R = k\} = \frac{e^{-\lambda_M R^2}(\lambda_M R^2)^k}{k!}$. Within each mobile, data packets are generated by a Bernoulli process of rate p, i.e., packets are generated with probability p in each slot. Hence if given that there are N_R mobiles around Rx, the number of mobiles that actually interfere with Rx is randomly distributed as BIN(N_R, p). Finally for MAC, slotted ALOHA is used, every packet is transmitted at the beginning of the time slot, right after the packet is generated.

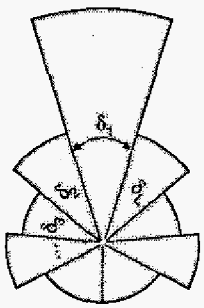

Fig. 4. Simplified beam pattern.

PERFORMANCE ANALYSIS

Simplified Beam Pattern

We first consider a simplified beam pattern shown in Fig. 4. The beam pattern has L beams, beam $1, 2, \cdots, L$, each of gain f_i and angular beam width δ_i rad. Without loss of generality, beam 1 is always the beam of the largest gain. Since most directional antenna systems use adaptive algorithm to search for the direction of Rx, we assume that beam 1 is always pointing to Rx. We will analyze the outage probability for this simplified beam pattern, and then generalize the result to an arbitrary beam pattern.

The outage probability of the link between Tx and Rx is defined by

$$\phi(b) = Pr\{\text{Outage}\} \triangleq Pr\left\{\frac{P_0}{P_I + N} < b\right\},$$

where P_0 is the received power, P_I the interference power, and N the noise power at Rx. Here b is a threshold of signal-to-interference-and-noise ratio (SINR) which depends on the channel coding used. It is assumed that packet errors occur whenever the SINR at Rx is less than the threshold b. The key concept [2] of computing $\phi(b)$ is to compute $\phi(b, a) = Pr\left\{\frac{P_0}{P_I(a)+N} < b\right\}$ first, where $P_I(a)$ only includes the interference power from those interferers located within a distance a from Rx. After limiting a to infinity, we can obtain the outage probability $\phi(b) = \lim_{a\to\infty} \phi(b, a)$.

Consider the mobiles within the circle of radius a centered at Rx (Tx excluded), we classify them into L groups. For the mobiles in the i^{th} group, they all point at Rx with their beam i. Let $\boldsymbol{K_a} = (K_1, K_2, \cdots, K_L)$ denotes the number of mobiles in each group. Since the number of mobiles in a circle of radius is of distribution POI($\lambda_M a^2$) and the probability of a mobile pointing at Rx with beam i is $\frac{\delta_i}{2\pi}$ (proportional to the angular beam width), K_is' are independent and each of distribution POI($\frac{\lambda_M a^2 \delta_i}{2\pi}$). Thus

$$P(\boldsymbol{K_a}) = \prod_{i=1}^{L} \frac{e^{\frac{-\lambda_M a^2 \delta_i}{2\pi}} (\frac{\lambda_M a^2 \delta_i}{2\pi})^{K_i}}{K_i!}.$$

Among the K_i mobiles in the i^{th} group, the number of mobiles that actually transmit packet and thus interfere with Rx, I_i, is of distribution BIN(K_i, p). Define $\boldsymbol{I_a} = (I_1, I_2, \cdots, I_L)$, we have

$$P(\boldsymbol{I_a} \mid \boldsymbol{K_a}) = \prod_{i=1}^{L} \binom{K_i}{I_i} p^{I_i} q^{K_i - I_i},$$

where $q = 1 - p$.

Number those I_i interferers in the i^{th} group from 1 to I_i, we define the following:

- r_0, ζ_0, P_0 are defined as the distance, propagation loss, received power at Rx from Tx respectively. From (3), the cdf of $P_0 \mid \zeta_0$ has the form

$$F_0(P_0 \mid \zeta_0) = 1 - e^{-\frac{P_0}{P_T f_1 \zeta_0}}.$$

- $\boldsymbol{r_i} = (r_{i,1}, r_{i,2}, \cdots, r_{i,I_i})$, $i = 1, 2, \cdots, L$. $r_{i,j}$ denotes the distances of the j^{th} interferer in the i^{th} group from Rx. Define matrix $\boldsymbol{r_a} = [\boldsymbol{r_1}^T, \boldsymbol{r_2}^T, \cdots, \boldsymbol{r_L}^T]$.

- $\boldsymbol{\zeta_i} = (\zeta_{i,1}, \zeta_{i,2}, \cdots, \zeta_{i,I_i})$, $i = 1, 2, \cdots, L$. $\zeta_{i,j}$ denotes the propagation loss at Rx experienced by the interference from the j^{th} interferer in the i^{th} group. From (1), we have

$$f(\zeta_{i,j} \mid r_{i,j}) = \frac{1}{\sqrt{2\pi}\sigma\zeta_{i,j}} e^{-\frac{1}{2}\frac{\left(\log\zeta_{i,j} - \log(K r_{i,j}^{-\eta})\right)^2}{\sigma^2}}.$$

  Define matrix $\boldsymbol{\zeta_a} = [\boldsymbol{\zeta_1}^T, \boldsymbol{\zeta_2}^T, \cdots, \boldsymbol{\zeta_L}^T]$.

- $\boldsymbol{P_i} = (P_{i,1}, P_{i,2}, \cdots, P_{i,I_i})$, $i = 1, 2, \cdots, L$. $P_{i,j}$ denotes the received interference power at Rx from the j^{th} interferer in the i^{th} group. Given that we know the value of $\zeta_{i,j}$, $P_{i,j} \mid \zeta_{i,j} \sim$ EXP($\frac{1}{P_T f_i \zeta_{i,j}}$). By (2), we have the pdf

$$f(P_{i,j} \mid \zeta_{i,j}) = \frac{1}{P_T f_i \zeta_{i,j}} e^{-\frac{P_{i,j}}{P_T f_i \zeta_{i,j}}}.$$

Now we can start to derive $\phi(a, b)$. Condition on $\boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}$, and $\boldsymbol{\zeta_a}$,

$$
\begin{aligned}
&\phi(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}) \\
&= Pr\{P_0 < bP_I(a) + bN \mid a, \boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}\} \\
&= \int_0^\infty \cdots \int_0^\infty F_0(b \sum_{i=1}^{L} \sum_{j=1}^{I_i} P_{i,j} + bN) \\
&\qquad \cdot \prod_{i=1}^{L} \prod_{j=1}^{I_i} f(P_{i,j} \mid \zeta_{i,j})\ dP_{i,j} \\
&= 1 - e^{-\frac{bN}{P_T f_1 \zeta_0}} \prod_{i=1}^{L} \prod_{j=1}^{I_i} \left[\frac{1}{1 + \frac{b f_j \zeta_{i,j}}{f_1 \zeta_0}} \right].
\end{aligned}
$$

Average over $\boldsymbol{r_a}, \boldsymbol{\zeta_a}$, we have

$$
\begin{aligned}
&\phi(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a}) \\
&= \int_0^\infty \cdots \int_0^\infty \phi(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a}, \boldsymbol{r_a}, \boldsymbol{\zeta_a}) \\
&\qquad \cdot \prod_{i=1}^{L} \prod_{j=1}^{I_i} f(\zeta_{i,j} \mid r_{i,j})\ f(r_{i,j}) dr_{i,j}\ d\zeta_{i,j} \\
&= 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{b e^{-x}}{\mathrm{SNR}_0} - \frac{x^2}{2\sigma^2}} \cdot \prod_{i=1}^{L} \left[I_a(x, \frac{f_i}{f_1}) \right]^{I_i} dx,
\end{aligned}
$$

where $\mathrm{SNR}_0 = \frac{P_T K r_0^{-\eta}}{N f_1}$ is the average power received at Rx from Tx. From [2]

$$
\begin{aligned}
&I_a(x, \psi) \\
&= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}} dy \int_0^a \frac{2r dr}{a^2 \left(1 + \psi\ b\ e^{y-x} \left(\frac{r_0}{r}\right)^\eta\right)}.
\end{aligned}
$$

Now average over $\boldsymbol{K_a}, \boldsymbol{I_a}$, we can get

$$
\begin{aligned}
&\phi(b, a) \\
&= \sum_{K_1, \ldots, K_L} P(\boldsymbol{K_a}) \sum_{I_1, \ldots, I_L} \phi(b, a \mid \boldsymbol{K_a}, \boldsymbol{I_a})\ P(\boldsymbol{I_a} \mid \boldsymbol{K_a}) \\
&= 1 - \sum_{K_1, \ldots, K_L} \prod_{i=1}^{L} \frac{e^{\frac{-\lambda_M a^2 \delta_i}{2\pi}} (\frac{\lambda_M a^2 \delta_i}{2\pi})^{K_i}}{K_i!} \cdot \frac{1}{\sqrt{2\pi}\sigma} \\
&\qquad \cdot \int_{-\infty}^{\infty} e^{-\frac{b e^{-x}}{\mathrm{SNR}_0 \cdot f_1} - \frac{x^2}{2\sigma^2}} \cdot \prod_{i=1}^{L} \left[p \cdot I_a(x, \frac{f_i}{f_1}) + q \right]^{K_i} dx \\
&= 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ \sum_{i=1}^{L} \frac{\lambda_M\ a^2\ \delta_i\ p}{2\pi} \right. \\
&\qquad \left. \cdot \left[I_a(x, \frac{f_i}{f_1}) - 1 \right] - \frac{b\ e^{-x}}{\mathrm{SNR}_0} - \frac{x^2}{2\sigma^2} \right\} dx.
\end{aligned}
$$

Limit $a \to \infty$, after a few passages we have the outage probability

$$
\begin{aligned}
&\phi(b) \\
&= 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ -\lambda_M\ p\ b^{\frac{2}{\eta}}\ e^{\frac{2\sigma^2 - 2\eta x}{\eta^2}}\ \frac{2\pi}{\eta}\ \csc\frac{2\pi}{\eta} \right. \\
&\qquad \left. \cdot \left[\sum_{i=1}^{L} \frac{\delta_i}{2\pi} \left(\frac{f_i}{f_1} \right)^{\frac{2}{\eta}} \right] - \frac{b\ e^{-x}}{\mathrm{SNR}_0} - \frac{x^2}{2\sigma^2} \right\} dx.
\end{aligned} \tag{4}
$$

Though $\phi(b)$ is not in close form, the integral can be easily computed through numerical integration. To evaluate the overall performance of the network, we are also interested in the system throughput per unit area

$$
S(b) = \frac{\lambda_M}{\pi} \cdot p \cdot (1 - \phi(b)). \tag{5}
$$

Arbitrary Beam Pattern and Array Interference Factor

Now consider a more realistic beam pattern such as the one in Fig. 2. Note that Fig. 4 will converge to Fig. 2 when $L \to \infty$ and $\delta_i \to 0$. Taking the limit of (4), we have the outage probability of the realistic beam pattern

$$
\begin{aligned}
&\lim_{L \to \infty}^{\delta_i \to 0} \phi(b) \\
&= 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ -\lambda_M\ p\ b^{\frac{2}{\eta}}\ e^{\frac{2\sigma^2 - 2\eta x}{\eta^2}}\ \frac{2\pi}{\eta}\ \csc\frac{2\pi}{\eta} \right. \\
&\qquad \left. \cdot \left[\frac{1}{2\pi} \int_0^{2\pi} \left(\frac{f(\theta)}{f(\theta_0)} \right)^{\frac{2}{\eta}} d\theta \right] - \frac{b\ e^{-x}}{\mathrm{SNR}_0} - \frac{x^2}{2\sigma^2} \right\} dx.
\end{aligned}
$$

System throughput S can be obtained simply by replacing $\phi(b)$ in (5) with $\lim_{L \to \infty}^{\delta_i \to 0} \phi(b)$. Notice that in $\phi(b)$, only $\frac{1}{2\pi} \int_0^{2\pi} \left(\frac{f(\theta)}{f(\theta_0)} \right)^{\frac{2}{\eta}} d\theta$ is related to the beam pattern $f(\theta)$. Hence we propose a new parameter, the array interference factor (AIF), for any beam pattern

$$
\mathcal{A} \triangleq \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{f(\theta)}{f(\theta_0)} \right)^{\frac{2}{\eta}} d\theta,
$$

which fully characterizes the performance of the beam pattern in the network. Any beam pattern with smaller AIF value will create (receive) less interference to (from) the system and thus have a better performance. With the introduction of AIF, now we can compare the performance of different beam patterns very easily.

For systems that use directional antennas for both transmission and reception, the outage probability can be easily generalized to

$$
\begin{aligned}
&\phi(b) \\
&= 1 - \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left\{ -\lambda_M\ p\ b^{\frac{2}{\eta}}\ e^{\frac{2\sigma^2 - 2\eta x}{\eta^2}}\ \frac{2\pi}{\eta}\ \csc\frac{2\pi}{\eta} \right. \\
&\qquad \left. \cdot [\mathcal{A}_t \cdot \mathcal{A}_r] - \frac{b\ e^{-x}}{\mathrm{SNR}_0} - \frac{x^2}{2\sigma^2} \right\} dx,
\end{aligned}
$$

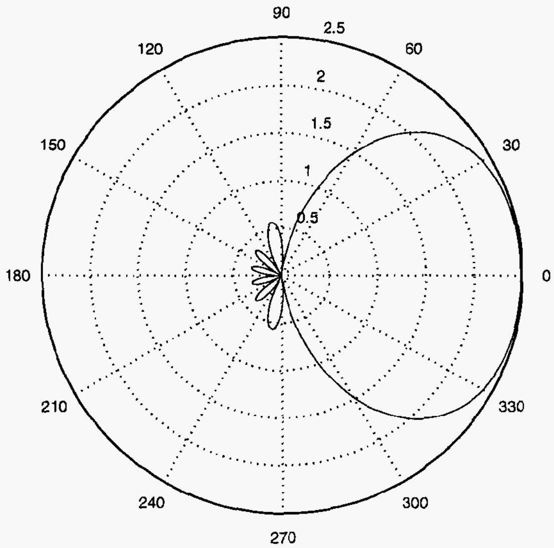

Fig. 5. Transmitting beam pattern 1 (AIF: 0.5541).

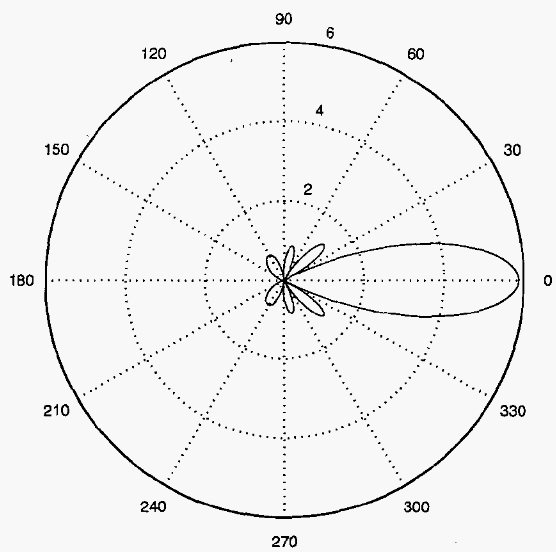

Fig. 6. Transmitting beam pattern 2 (AIF: 0.3472).

where $\mathcal{A}_t$ and $\mathcal{A}_r$ are the AIF values of the transmitting and receiving beam patterns respectively.

NUMERICAL EXAMPLES

Consider the two different transmitting beam patterns shown in Fig. 5–6. The mainlobe (AIF: 0.5541) of the first one is much wider than the second one (AIF: 0.3472). We want to compare the system performance between these two transmitting beam patterns. The receiving antenna is assumed to be omnidirectional. The SINR threshold b is set to be 5 dB, and $\sigma_{dB} = 10$ dB. The outage probability and the average system throughput is computed and plotted.

In Fig. 7, the average mobile density λ_M is fixed at

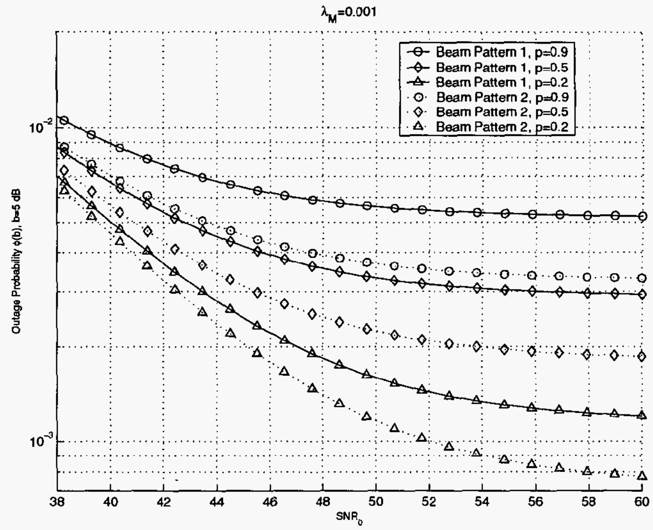

Fig. 7. Outage probability vs. received signal power average SNR_0.

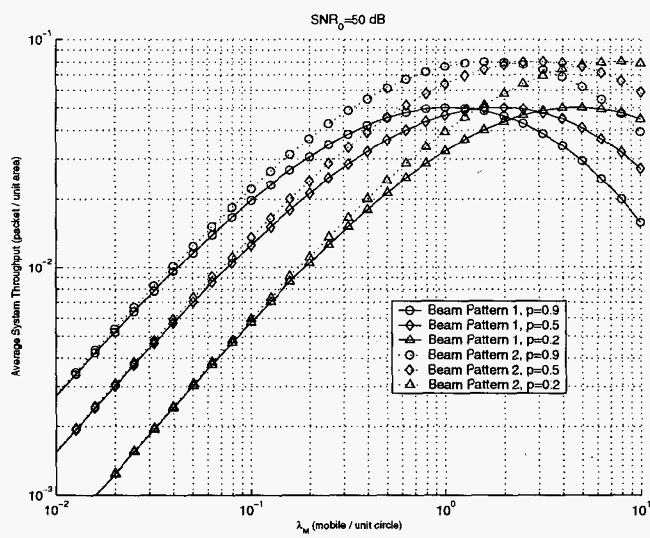

Fig. 8. Average system throughput vs. average mobile density per unit area.

0.001 per unit circle. We can see that beam pattern 2 has lower outage probability than beam patten 1 at all load p. The interference power is indeed reduced by the narrower mainlobe. We also observe that the outage probability converges as the average received signal power SNR_0 increases beyond 50 dB. When SNR_0 is large, it indicates that the noise power is small compared to the signal power. Hence the only performance degrading factor now is the interference from other mobiles. Therefore even if we increased the transmission power of all mobiles by the same factor, the SINR is almost the same and so is the outage probability. It is just a waste to increase transmission power of all mobiles beyond 50 dB under such circumstances. Our analysis can be used to determine the most efficient transmission power setting in a wireless network to save the battery life. Also note that as p increases, i.e., the mobiles generate packets with higher load, the outage probability also increases since there is more interference presented in the network.

In Fig. 8, the average system throughput is plotted. We can see that there exists an optimal mobile density for each setup. Beam pattern 2 is again having higher optimal throughput than beam pattern 1 at all load. The optimal mobile density is around $1 \sim 2$ per unit circle. Keep in mind that there are various measure for the system performance of a network. We are only providing one of them here. The optimal mobile density may not be optimal at all if other measure such as per-hop- progress is used to evaluate the system performance.

Based on the numerical results, we can conclude that the narrower the mainlobe, the better the performance is. We also see that AIF indeed determines the performance of a beam pattern, smaller AIF value gives a better performance in a wireless network. One thing to note is, we assume in this paper that Tx and Rx are always perfectly aligned to each other. As the mainlobe gets narrower, it is more difficult to keep the perfect alignment. There should be a tradeoff here. The tradeoff is further explored in our journal paper to be submitted.

CONCLUSIONS

In this paper, we analyze the performance of wireless networks using directional antennas. The beam pattern can be of any arbitrary shape. Most importantly, the characteristic value that determines the performance of a beam pattern in the network is found, which makes the comparison between beam patterns very easy. Numerical results show that our analysis can be applied to find various optimal system parameters such as transmission power and mobile density, for wireless networks.

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