Space-Time Coded QPSK for Rapid Fading Channels

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ABSTRACT

This paper presents the design of Space-Time (ST) codes suitable for rapid fading channels. The codes are designed using the QPSK signal constellation. The design of the proposed codes utilizes two different encoding methods. The first method uses a large time diversity trellis encoder, and the second one uses the I-Q encoding technique. Both methods are expected to produce ST codes that perform better than the codes presented in the literature. The proposed codes were simulated over different Rayleigh fading channels. Coding gains up to 3 dB have been observed over similar codes in the literature.

1. INTRODUCTION

Diversity is a popular method to improve the performance and throughput of wireless systems. Transmit time diversity can be achieved by repeating the transmission of each symbol in different time slots [1]. It can be viewed as a repetition code which consumes high bandwidth [2]. Therefore, substantial performance improvement can be achieved using more sophisticated codes, utilizing both space and time.

The concept of ST codes had appeared first in [3] as the delay diversity system, where different symbols are simultaneously transmitted via different transmit antennas. Later, the performance criteria of ST codes were derived in [4,5] for quasi-static and rapid fading channels. The general criteria for the design of ST codes for rapid and quasi-static fading channels were also developed. Moreover, ST codes were deigned explicitly in [4,5] for quasi-static fading channels. The ST concept was applied in [6-8] to enhance the quality of transmission at the same bit rate of systems using single transmit antenna. The same error probability can be achieved at a lower signal-to-noise ratio (SNR). Layered ST coded systems were presented in [9]. A concatenation of ST block codes with turbo codes was also proposed in [10].

Currently, all ST coded QPSK systems presented in the literature are designed for quasi-static fading channels, whose gains are constant over one frame and independent from one frame to another. This is not always the case since time-varying channels are usually encountered in mobile communications. Hence, the rapidly varying fading channels are considered in this paper.

ST coded 16-QAM schemes for rapid fading channels were presented in [11]. In this paper, two ST coded QPSK schemes are proposed. The first one is designed using a single trellis encoder with a large minimum time diversity (MTD), where the second code utilizes the I-Q encoding technique. The general ST system model is described in the next section along with the design criteria of ST codes for rapid fading channels. Then, the proposed codes are presented. Their performance is compared with corresponding ST QPSK codes in the literature. Finally, conclusions and discussion points are drawn from the obtained results.

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2. SYSTEM MODEL

A typical system that employs ST coding consists of a trellis encoder, vector block interleaver, N transmit antennas, M receive antennas, a deinterleaver and a ST decoder. In each sampling interval, the ST encoder maps the input bits onto N symbols to be transmitted (after interleaving) over the N transmit antennas at the same time. The received signal is a noisy superposition of all transmitted symbols over all transmit antennas. The signal d_t^j received at the j^{th} antenna at time t is given by:

$$d_t^j = \sum_{i=1}^{N} \alpha_{ij,t} c_t^i + \eta_t^j \qquad (1)$$

Where η_t^j is a noise added at the j^{th} receive antenna and modeled as independent samples of a zero-mean Gaussian random process with variance of $N_o/2$ per dimension. The coefficient $\alpha_{ij,t}$ is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna at time instant t. It is drawn from a complex Gaussian random variable with zero mean and variance of $1/N$. The c_t^i is the transmitted symbol from the i^{th} transmit antenna at time instant t. At the receiver, Maximal Ratio Combining (MRC) is used to combine signals at different receive antennas and the Viterbi algorithm is employed at the decoder.

The ST symbols at each antenna are interleaved using a vector block interleaver, where each element in the interleaver is a vector containing the N symbols to be transmitted via the N transmit antennas. The depth and span of the interleaver depend on the channel's fading rate (f_DT) and the encoder's constraint length, respectively. The interleaver is used in order to break the memory of the channel so that it approaches the behavior of independent fading channels, and hence the diversity provided by the coded system is fully utilized.

The performance of ST coded systems employing N transmit and M receive antennas is derived in [4] for rapid fading channels. Consider a codeword defined as $C_l = \underline{c}_1\underline{c}_2...\underline{c}_l = c_1^1c_1^2....c_1^N c_2^1c_2^2....c_2^N....c_l^1c_l^2....c_l^N$ that has been transmitted over l time intervals and was erroneously decoded as $\hat{C}_l$. The probability of deciding $\hat{C}_l$ in favor of C_l using maximum liklihood decoding is upper bounded using the Chernoff bound as [4]:

$$P(C_l,\hat{C}_l) \leq \prod_{t\in\eta} \left[1+\sum_{i=1}^{N}|c_t^i - \hat{c}_t^i|^2 \,(E_s/4N_o)\right]^{-M} \qquad (2)$$

Where $\eta = \{t : \underline{c}_t \neq \underline{\hat{c}}_t\}$ and $\underline{c}_t = (c_t^1 c_t^2 .... c_t^N)$ is the codeword of ST symbols transmitted simultaneously over all transmit antennas at time t. Define the cardinality of the set η to be $L_\eta = |\eta|$, and define $L = min\{L_\eta\}$. Therefore, L is the length of the shortest error path. It is referred to as the Space-Time Minimum Time Diversity (ST-MTD) of the code. It can be visualized as the "branch-wise" Hamming distance (HD) or the MTD in conventional trellis codes, by considering the whole codeword $\underline{c}_t$ as one symbol. The quantity multiplied by the SNR term can be referred to as the Space-Time Minimum Square Product Distance (ST-MSPD) and defined over the shortest error event path as:

$$\prod_{t\in\eta}\sum_{i=1}^{N}|c_t^i - \hat{c}_t^i|^2 \qquad (3)$$

The ST-MTD and ST-MSPD are referred to in [4] as Distance and Product criteria, respectively. So, maximizing both of them yields good ST codes suitable for rapid fading channels. The proposed ST codes are presented in the following.

3. THE PROPOSED CODES

Different ST codes were designed in [4] for the quasi-static fading channel. The ST coded QPSK scheme in [4], referred to QPSK1 here, uses a rate-2/4 trellis encoder to encode the incoming 2 bits to 4 output bits. The 4 bits at the output of the encoder are mapped onto two QPSK signals and transmitted over two antennas. The ST-MTD of the 4-state and 8-state codes is 2. The ST-MSPD's of the 4-state and 8-state codes are 4 and 16, respectively. The 4 and 8-state codes are presented here for comparison purposes. All ST codes presented in this paper provide throughput of 2 bits/s/Hz and use two transmit antennas (i.e., N=2).

The first proposed scheme, called QPSK2, also uses a rate-2/4 trellis encoder. However, it is designed so that the symbol-wise HD between branches leaving or remerging at the same encoder's state is maximized. This approach yields ST codes whose ST-MSPD is increased, and hence a better performance is expected. To be able to do this, the 4-dimensional QPSK signal space is partitioned into subsets such that the HD between signals in the same subset is increased. If this HD cannot be increased anymore, then the partitioning is performed so that the inter-subset ST squared product distance (ST-SPD) is increased. At the final stage, both the HD and the ST-SPD between pairs in the same subset are maximized. By doing this, the design criteria for rapid fading channels are satisfied. The set partitioning of multi-dimensional QPSK signal space for fading channels was presented in [12],. Since the design of trellis codes for fading channels requires maximizing the symbol-wise HD, the same set partitioning is used here to design the QPSK2 ST codes.

The set partitioning of a 4-dimensional QPSK signal space is shown in Figure 1. Signal labels of branches diverging from the same state are drawn from the same subset. Moreover, signal labels of branches remerging at the same state have HD maximized to two. The ST-MTD and ST-MSPD of the resulting code were increased by maximizing the HD and ST-SPD between the possible encoded symbols. The trellis diagrams of the 4 and 8-state codes are shown in Figure 2. The ST-MTD for both the 4-state and 8-state codes is 2. However, the ST-MSPD for both the 4-state and 8-state codes is 24, which is much higher than that of the QPSK1 codes. Since the multiplicity of the shortest error event path is reduced in the 8-state code, the 8-state code outperforms the 4-state code.

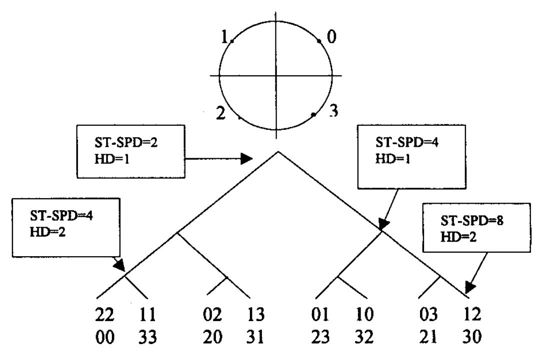

Figure 1: The 4-dimensional QPSK signal space partitioning.

Since the MTD of a trellis code is inversely proportional to the number of input bits of the encoder, then using different encoders in parallel (such as I-Q encoding) can increase the MTD. I-Q trellis codes with different throughputs were presented in [13]. These codes show significant coding gains over conventional trellis codes having the same complexity. The proposed structure of the encoder/decoder employing the ST concept is shown in Figure 3. It uses two encoders, where each one encodes half the number of input bits per signaling interval. Symbols from the I-encoder contribute to the in-phase components of both signals at the two transmit antennas. Similarly, symbols from the Q-encoder contribute to the quadrature components of both signals. This technique is used to design the I-Q QPSK ST code in the following.

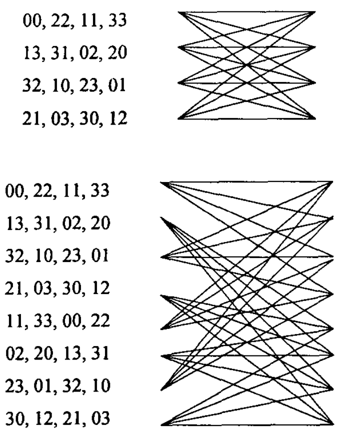

Figure 2: Trellis diagrams of QPSK2 (4 and 8-state) 2 bits/s/Hz.

The I-Q ST code uses the same encoder's trellis presented in [13] for one transmit antenna systems with a throughput of 1 bits/s/Hz. This code was designed in [13] to provide the maximum MTD possible compared to codes using single encoders. Here, it is used to design the I-Q ST code and provide higher ST-MTD and ST-MSPD than the previous two codes. This code uses the encoder/decoder shown in Figure 3, where the rate of both encoders is 1/2. Each encoder encodes one bit per signaling interval, ending with a throughput of 2 bits/s/Hz. Symbols at the output of the I-encoder constitute the I components of both signals at the two transmit antennas. Similarly, the output of the Q-encoder contribute to the Q components of both signals. The trellis diagrams of the 4-state and 8-state codes are shown in Figure 4. The ST-MTD's of the 4-state and 8-state codes are 3 and 4, respectively. The ST-MSPD of the 4-state and 8-state codes is 32. The increase in both the ST-MTD and the ST-MSPD resulting from using the I-Q encoding scheme is clear and its effect will be observed in the performance of the code.

The I-Q ST coded system uses two separate decoders at the receiver, and the transmitted signals are overlapping at the receiver. The received signal at the j^{th} receive antenna has the following form:

$$d_t^j = \sum_{i=1}^{N} (\alpha_{Iij,t}+j\,\alpha_{Qij,t})\cdot(x_t^i+jy_t^i) + \eta_t^j \tag{4}$$

Where $(\alpha_{Iij,t}+j\alpha_{Qij,t})$ is the path gain from the i^{th} transmit antenna to the j^{th} receive antenna at time t. The symbol, $(x_t^i+jy_t^i)$ represents the complex baseband signal transmitted from the i^{th} transmit antenna at time t.

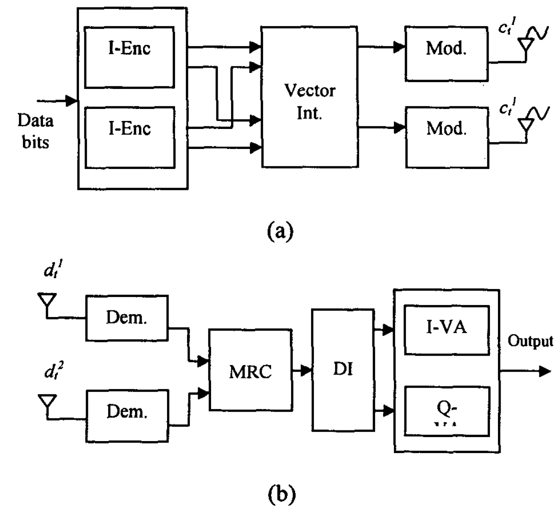

Figure 3: The structure of the I-Q ST system (a) Encoder, (b) Decoder.

In order to resolve the I and Q components from the received signal to be decoded by the I and Q decoder, a simplified decoding algorithm proposed in [14] is used. It is based on partitioning the 4-dimensional QPSK signal space at the output of the ST encoder into four subsets. Signal labels in each subset have the same I component but different Q components. At the I-decoder, four metrics are computed for each subset, ending with four surviving metrics. In the Viterbi algorithm, metrics of subsets having the same I component as branches of the trellis are compared. The same algorithm is applied in the Q-decoder.

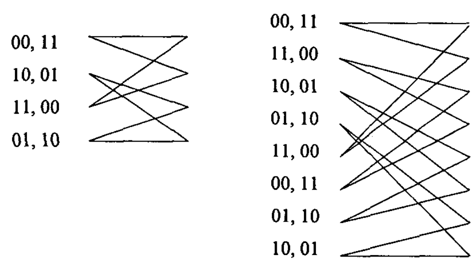

Figure 4: Trellis diagrams of the I-Q ST coded QPSK (4 and 8-state) 2 b/s/Hz.

4. PERFORMANCE COMPARISONS

The two proposed coding schemes (QPSK2 and I-Q) are compared to the QPSK1 code designed in [3]. They are tested under time-varying fading conditions in which the transmitted signals are affected by fade samples that are correlated in time. The transmit branches are assumed to be uncorrelated. For this channel, ideal interleaving is assumed at each transmit branch so that the time diversity of the code is fully utilized. This is the ideal case needed to compare the codes' theoretical performance.

Figure 5 shows the performance of the 4-state QPSK1, QPSK2 and I-Q codes over an ideally interleaved fading channel for the cases of one or two receive antennas. It is clear that the I-Q code is the best followed by QPSK2 and QPSK1. This is expected since the main controlling parameters of the code in ideally interleaved fading channels are the ST-MTD and ST-MSPD, which are higher in the I-Q codes. The I-Q code provides a coding gain of 3 dBs over QPSK1 and more than 0.5 dB over QPSK2 at bit error rate (BER) of 10^{-3}.

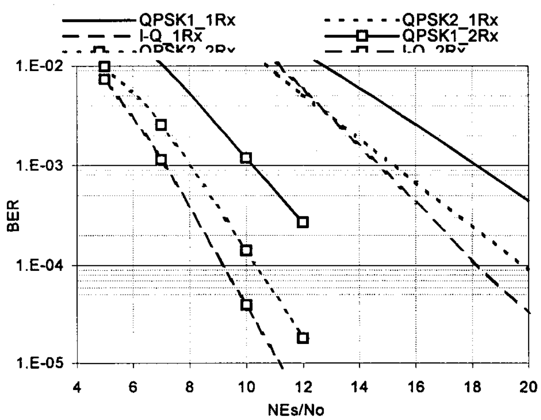

Figure 5: Performance of the 4-state QPSK codes for 1-Rx and 2-Rx antenna over rapid fading channel.

It is observed that the gain of the I-Q ST code over the other two codes is less in the case of one receive antenna. This is not in agreement with the fact that the effect of the diversity provided by the code decreases when receive diversity is used. The reason behind this disagreement is due to the suboptimal performance of the simplified decoding algorithm used to decode the I-Q ST code. This suboptimality effect is clear in the case of one receive antenna due to the unreliable output of the simplified decoder. On the other hand, the simplified algorithm is able to reliably decode the I-Q ST code in the case of two receive antennas, and hence can make use of the available diversity of the code.

Figure 6 shows the performance of the 4-state codes over correlated fading channels with a fading rate ($f_D T$) of 0.01. The transmitted symbols at each antenna are interleaved using a 25x16 vector block interleaver. This interleaver is sufficient to break the memory of the channel. It is obvious that the I-Q coding scheme is still the best followed by QPSK2 and then QPSK1. The gains of the first two codes over the last one are slightly less than that for the ideally interleaved fading channel. This reduction in gain is expected since the effect of the ST-MTD is reduced due to non-ideal interleaving.

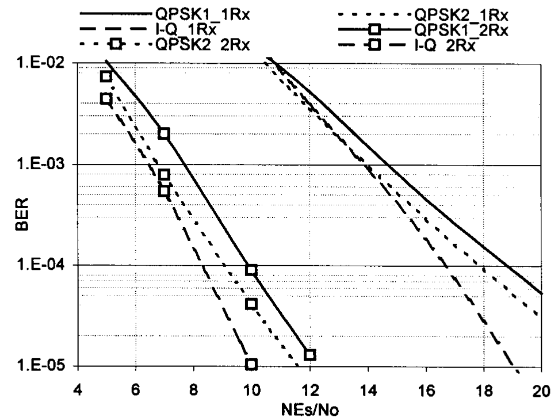

Figure 6: Performance of the 8-state QPSK codes for 1-Rx and 2-Rx antenna over rapid fading channel.

Figures 7 shows the performance of the same 4-state QPSK codes for a fading channel with a fading rate of 0.005. The interleaver used is the 25x16 interleaver and hence, is considered to be improper. The same trends observed in the ideally interleaved fading channel are observed here but with less gains for the best code over the worse ones.

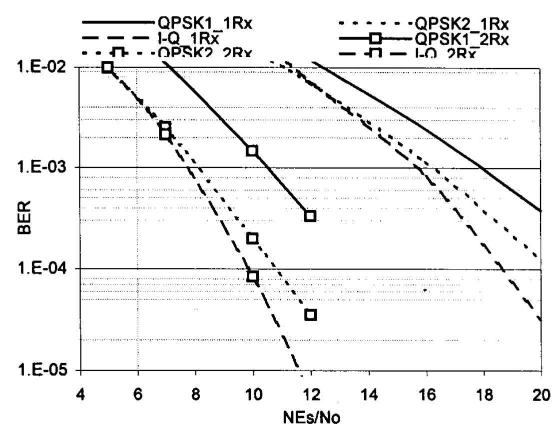

Figure 7: Performance of the 4-state QPSK codes for 1-Rx and 2-Rx antenna over correlated fading channel with $f_D T$=0.01.

5. CONCLUSIONS

Two ST coded QPSK schemes have been proposed. The first scheme is based on the conventional design of trellis codes for fading channels, while the second one utilizes the I-Q encoding technique. The proposed codes were simulated over different Rayleigh fading channels. Simulation results showed that these codes outperform the ST coded QPSK schemes presented in the literature. Coding gains as high as 3 dB were obtained.

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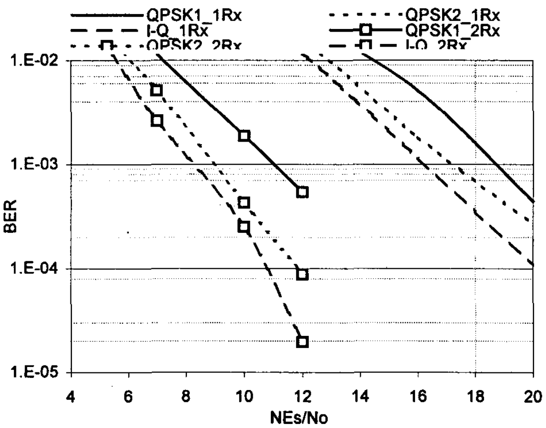

Figure 8: Performance of the 4-state QPSK codes for 1-Rx and 2-Rx antenna over correlated fading channel with $f_D T$=0.005.

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