EE 577: Wireless and Personal Communications

Small-Scale Multipath Fading

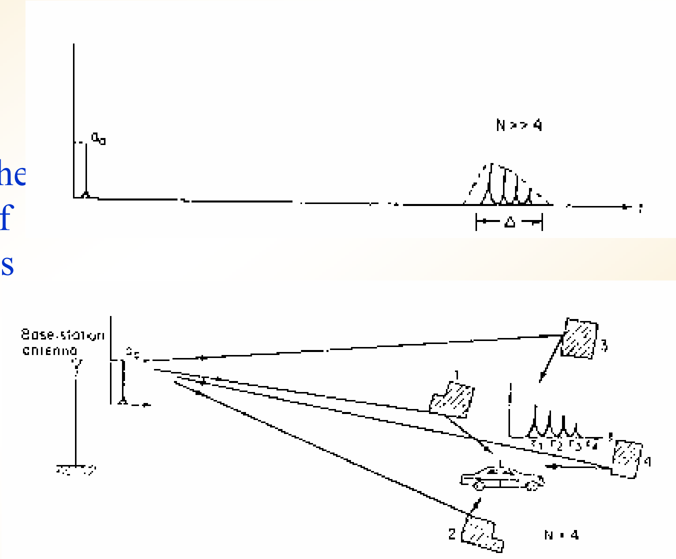
Overview

The mobile channel has small-scale propagation effects due to:

- Motion of the transmitter and/or the receiver
- Multiple scatters of the transmitted signal off buildings and structures
- Time-variable position of the receiver with respect to the environment

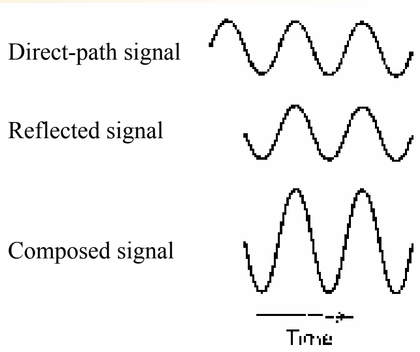
Small-Scale Fading

- Due to small changes in position
- Results from the combination of arriving signals with different delays and different attenuation.

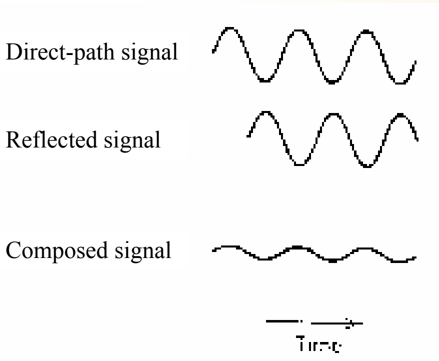


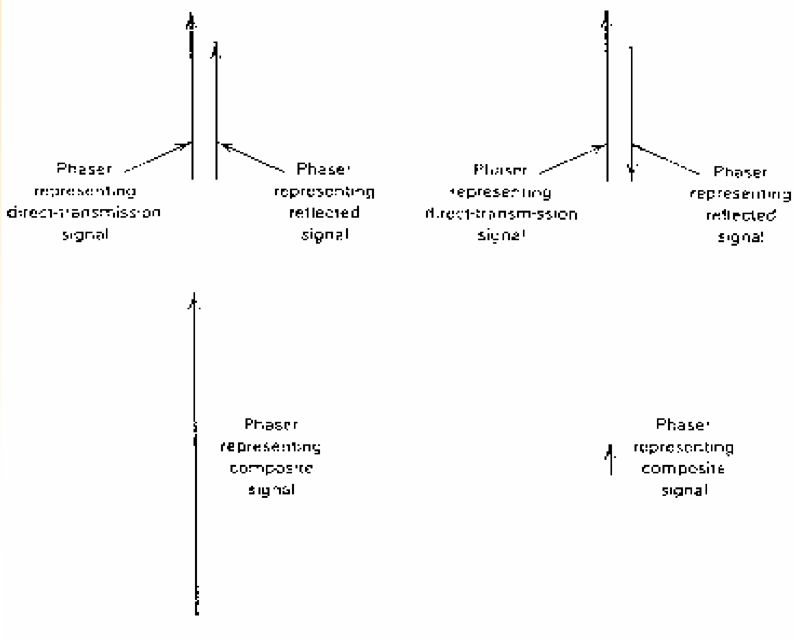
Case 1: Single-Tone, Two-Path Channel

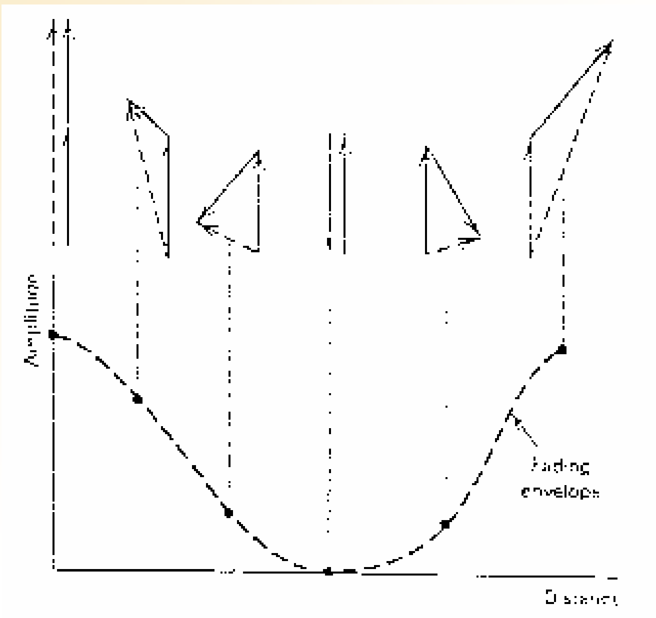
Constructive Addition



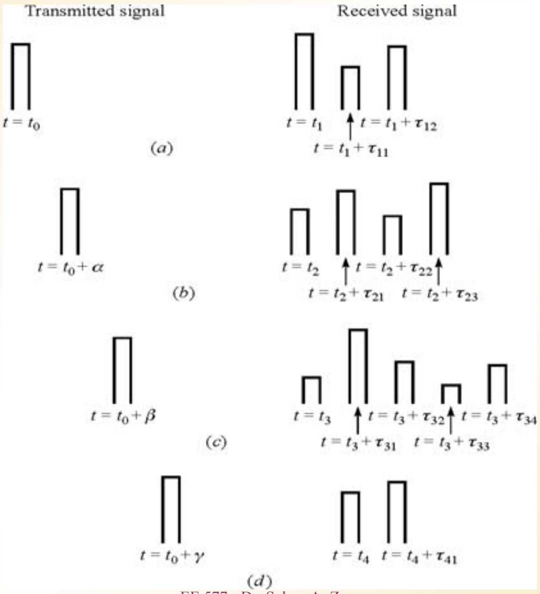
Destructive Addition



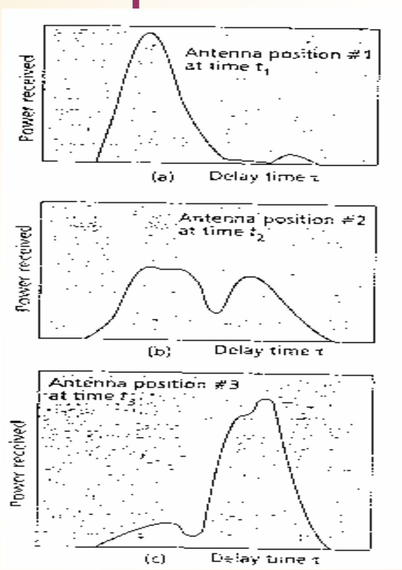

Phaser representing direct-transmission signal
Phaser representing reflected signal
Phaser representing composite signal


Amplitude
Fading envelope
Distance

Case 2: Narrow Pulse, Resolvable Multipath Channel



Case 3: Narrow pulse, Unresolvable Multipath Channel



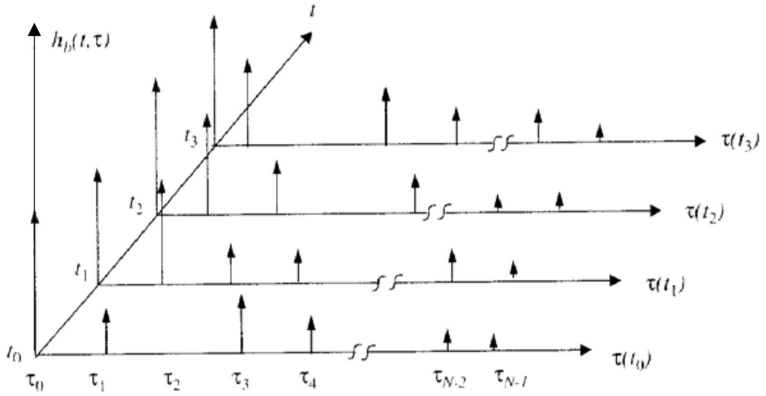
Problems of Mobile Channels

- **Multipath reception:**

  The signal is received over ***n* paths** with different delays **{τ_i's}** and different attenuation factors **{α_i's}**

- **Time variation:**

  The parameters ***n***, **{τ_i's}** and **{α_i's}** are functions of time



WSSUS Assumption

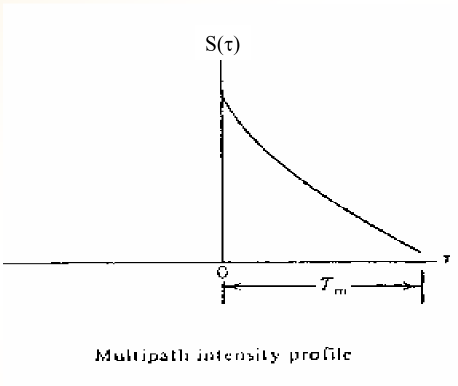
- Stands for:

  Wide Sense Stationary Uncorrelated Scattering

- Means that:
  - The random process representing each finger is WSS
  - The random processes representing different fingers (different τ_i's) are uncorrelated

Understanding Multipath: Time-Domain Analysis

Multipath Time delay Spread:



Remarks About S(τ)

- S(τ) describes how does the average received power vary as a function of excess delay τ

- **Excess delay**: the time delay from the arrival of the first echo to the last one

- The received signal usually consists of several discrete multipath components, called "*fingers*" or "bins"

Effect of Multiple Paths

The effect of multipath can be obtained by comparing values of the T_m, and the symbol time T_s:

- $T_m > T_s$: results in significant overlap among neighboring received symbol, i.e., channel-induced ISI.

- $T_m << T_s$: negligible ISI. Possible reduction in SNR due to destructive add up.

Time Dispersion Parameters

- Mean excess delay:

$$\bar{\tau} = \frac{\sum_k \alpha_k^2 \tau_k}{\sum_k \alpha_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- rms delay spread:

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k \alpha_k^2 \tau_k^2}{\sum_k \alpha_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

Time Dispersion Parameters

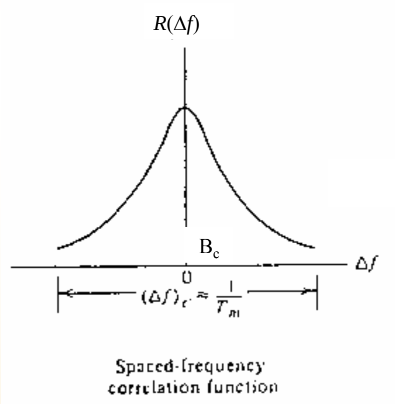
- **Maximum excess delay spread** at *X* dB is the delay at which the received power falls *X* dB below maximum finger

- **Note:** these parameters will be functions of the noise floor:

If below the noise floor, a finger will not be included and will not affect the value of the measured signal

Understanding Multipath: Frequency-Domain Analysis

- Spaced-frequency correlation function, $R(\Delta f)$
- $R(\Delta f)$ is a measure of the correlation between two frequency components spaced by Δf Hz.



Coherence Bandwidth

- **Coherence Bandwidth**, B_C, is the range of frequencies over which the channel can be considered non-distorting (equal gain and linear phase)
- This is defined as the range over which two frequencies show $\geq$ 90% correlation in their response

Coherence Bandwidth

- The **Coherence Bandwidth** is in terms of the **rms delay spread** (for 90% correlation):

$$B_C \approx 1/(50\sigma_\tau)$$

- If we are more generous (time over which the channel correlation function is > 0.5

$$B_C \approx 1/(5\sigma_\tau)$$

- Not well defined which one to use in general

Coherence Bandwidth

Example:

- rms delay for measurements is 1.37 μsec;
  B_C = 146 kHz for 50% correlation.
- If the transmission has a lower bandwidth than B_C, then no channel equalization is needed.
- If the transmission bandwidth exceeds the B_C, then equalization is needed.

Multipath Time Delay Spread

- The time delay spread can cause either:
  - flat fading
  - frequency-selective fading
- The outcome depends on whether the channel can be considered as a non-distorting filter with respect to the transmitted signal
  - very slow amplitude change
  - linear phase response

Flat Fading

- Channel acts like a relatively non-distorting filter over a channel bandwidth greater than the transmission bandwidth
- Channel does cause amplitude variations due to channel gain variations **but** the spectral content of the signal is preserved

Flat Fading

- Channel has no excess delay

- The channel delay spread, σ_τ, is related to the signal bandwidth, W, by $\sigma_\tau << 1/W$

- Such channels are sometimes called **narrowband channels** since the bandwidth is less than the coherence bandwidth ($W << B_C$)

Frequency-Selective Fading

- Channel acts like a relatively non-distorting filter over a bandwidth much less than the transmission bandwidth

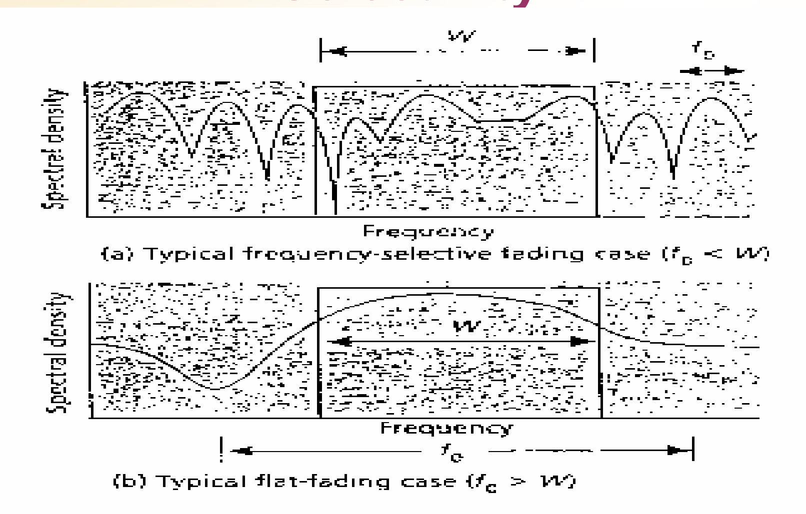
- The received signal is the sum of multiple copies of attenuated and delayed versions of the original signal

- The channel introduces Inter-symbol Interference (ISI)

Frequency-Selective Fading

- The signal bandwidth, W, and channel coherence bandwidth, B_C: $W > B_C$
- The symbol period, T_S ($T_S = 1/W$), is less than the channel delay spread: $T_S < \sigma_\tau$
- Channel is considered to be frequency selective if $\sigma_\tau > 0.1T_S$

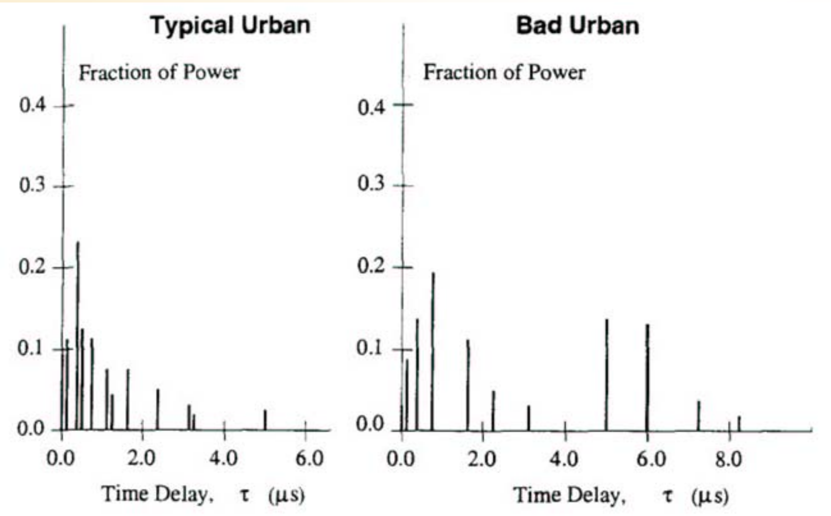
Illustration of Frequency Selectivity



Time Domain and Freq. Domain

- $R(\Delta f) \Leftrightarrow S(\tau)$ (Fourier Transform pair)
- $B_C \approx 1/T_m$ and $W \approx 1/T_s$
- **Flat fading:** when $W \ll B_C$ (or $T_s \gg T_m$) => No distortion
- **Frequency selective:** when $W > B_C$ (or $T_s < T_m$) => ISI

Typical Delay Spreads



Examples

- Example 5.4 [Rappaport 2nd ed.]
- Example 5.5 [Rappaport 2nd ed.]

Example 5.5: [Rappaport, 2nd ed.]

Resolvable multipath signals arrive with:

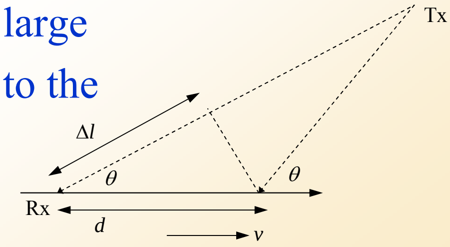
-20 dB	0 μsec
-10 dB	1 μsec
-10 dB	2 μsec
0 dB	5 μsec

Example 5.5 (cont'd)

- Mean excess delay = $\bar{\tau} = 4.38$ μs
- Mean excess delay = $\overline{\tau^2} = 21.07$ μs²
- rms delay spread = $\sigma_\tau = 1.37$ μs
- B_c (50% correlation) = $1/5\sigma_\tau = 146$ kHz
- In AMPS, $W = 30$ kHz $< B_c$ => flat fading
- In GSM, $W = 200$ kHz $> B_c$ => frequency-selective fading

Doppler Shift

- Any motion in the plane in which the electromagnetic wave is traveling towards or away from the receiver results in a shift in the carrier frequency.
- Tx-Rx distance is very large
- Motions perpendicular to the plane do not affect it.



Doppler Shift

- If the transmitter and receiver are approaching, the frequency is higher while if they are moving apart, the frequency is lower.
- The phase difference between the paths:

$$\Delta\varphi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

- The **Doppler shift** is: $f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$

Doppler Shift

- **Example:**

  Vehicle moving at 60 mph relative to a transmitter. The carrier freq. is 1850 MHz.

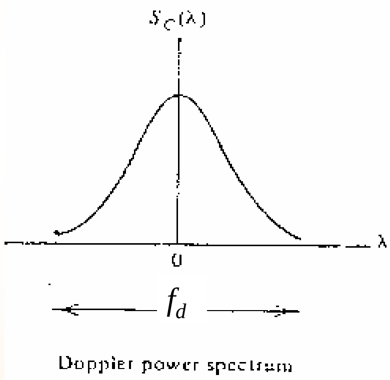
  What is the maximum Doppler shift?
- **Solution:**

  $\lambda = c/f = 0.162$ m

  f_D = 60 mph * 1609 m/mi * 1 hr/3600 sec /0.162 m = 165 Hz

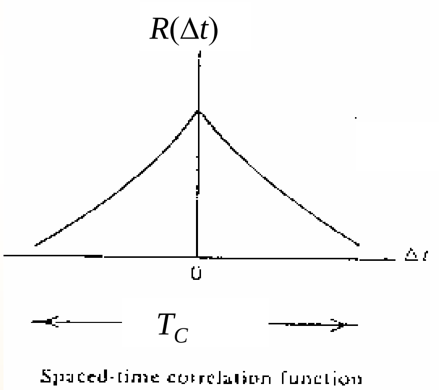
Doppler Spread Spectrum

- Shows how the spectrum of a pure sinusoid is spread by the channel
- The maximum spread in frequency is called the **Doppler Spread**, f_D



Spaced-Time Correlation Function

- It shows how the channel response is correlated at time instances separated by Δt
- $S(\lambda) \Leftrightarrow R(\Delta t)$



Coherence Time

- If a pure sinusoid is transmitted, it will be received as a range of frequencies adjacent to the sinusoid frequency
- The Doppler shift broadens the spectrum of the received signal by spreading (smearing) the basic spectrum in frequency domain
- If the signal spectrum is wide compared to this smearing, the effect is not noticeable
- **Coherence Time:** measures the time duration over which the channel characteristics can be considered to be static

Coherence Time

The channel is time-variable - but how variable is it?

- **Coherence Time** and **Doppler Spread** measure the time variability of the channel
- Doppler Spread comes from the frequency shifting of the carrier due to the Doppler shift

$$f_r = f_c \pm f_D$$

Coherence Time

- The Coherence Time, T_C is proportional to the maximum Doppler frequency f_D, (time over which the channel correlation function is larger than 0.9)

$$T_C \approx f_D^{-1}$$

- If we are more generous (time over which the channel correlation function is > 0.5):

$$T_C \approx \frac{9}{16\pi f_D}$$

Coherence Time

- The modern rule of thumb is to take the geometric average of the two:

$$T_C = \sqrt{\frac{9}{16\pi f_D^2}} = \frac{0.423}{f_D}$$

- For digital transmission, the symbol rate needs to be $R_s > 1/T_C$ for no distortion

Example - Coherence Time

- **Example (5.6)** [Rappaport 2nd ed.]**:** Travel distance of 10 m; speed of 50 m/sec, carrier at 1900 MHz.

- **Find:** The Doppler bandwidth, coherence time, the sampling time, the sample spacing, the number of samples in 10 m

Time Selectivity

- The frequency spread can cause either:
  - Fast fading
  - Slow fading

- The outcome depends on the rate of change of the transmitted signal with respect to the rate of change of the channel

Fast Fading (Time-Selective)

- The coherence time of the channel, T_C is less than the transmitted symbol period, T_S
- The Doppler bandwidth, f_D is larger than the bandwidth of the signal, W,
  => low-data rate signal
- The fast fading channel causes frequency dispersion (also called time-selective fading)

Slow Fading (Time-Nonselective)

- Channel changes at a rate much slower than the baseband signal rate:

$$T_S << T_C$$

- The Doppler spread in the frequency domain is less than the symbol bandwidth:

$$W >> f_D$$

- The speed of the mobile user relative to the data rate makes this determination

Effect of Time Variation

- Time variance is linked with the **fading rate**, $f_D T_s$:
  - $T_C > T_s$ ($f_D T_s$<1): The transmitted symbol will be subject to the same fading realization
    => Slow fading
  - $T_C < T_s$ ($f_D T_s$>1): The fading realization changes while a symbol is propagating
    => Fast fading
- Slow Fading $\Rightarrow$ Loss in SNR
- Fast fading $\Rightarrow$ Distortion and irreducible error rate.

Frequency-Nonselective, Slowly Fading Channel

- The least severe fading case:
  - No distortion.
  - No irreducible error floor.
  - Only reduction in SNR.
- **Conditions:** $f_D << W << B_c$
  $T_C >> T_s >> T_m$
- $f_D T_m$ is called the **spread factor**.
- If the spread factor is small compared to unity, i.e., $f_D T_m << 1$ (channel is under-spread), then it is possible to design a FNS, SF system.

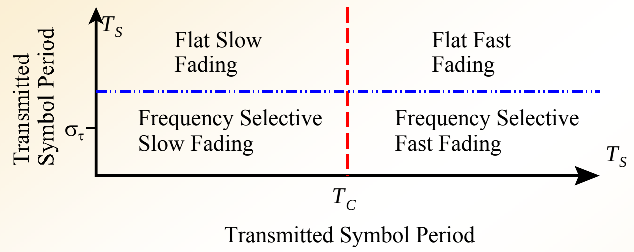
Small-Scale Fading

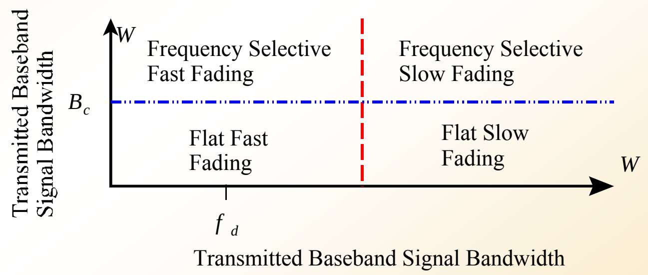
- The four fading types come from:
  - relative bandwidth of channel versus data bandwidth
  - relative delay in the channel versus data period
- Multipath delay causes **time dispersion** of the signal and **frequency-selective** fading
- Doppler shifts cause **frequency dispersion** and **time-selective** fading

Small-Scale Fading

- We can combine the four effects into a matrix of fading channel types
- We rank the fading based on:
  - Symbol duration, T_S, relative to the delay spread, σ_τ, and the coherence time, T_C
  - Symbol bandwidth, W, relative to Doppler bandwidth, f_D, and channel coherence bandwidth, B_C

Small-Scale Fading





Small-Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of Signal < BW of Channel
2. Delay spread < Symbol Period

Frequency Selective Fading
1. BW of Signal > BW of Channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol Period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Duality

$R(\Delta f)$ gives the range of frequency over which two spectral components have strong correlation.

$S(\tau)$ gives the time spreading of a transmitted pulse in the time-delay domain.

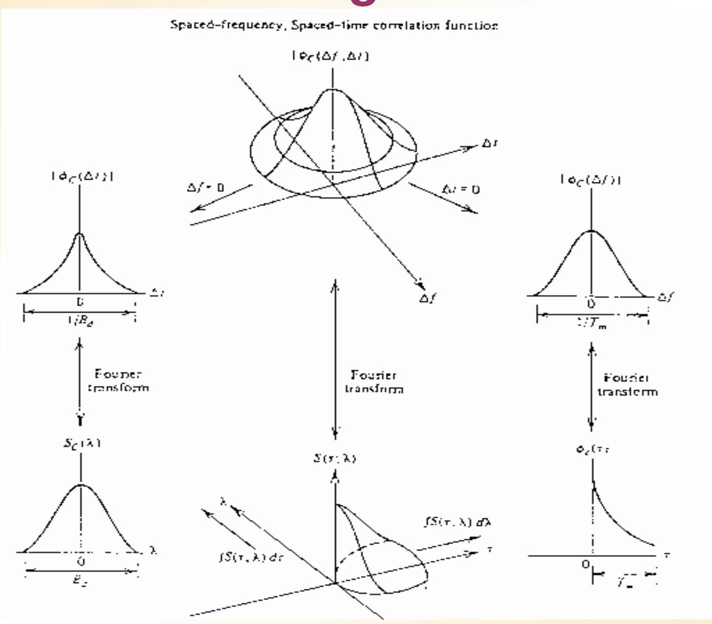
B_c sets an upper limit on the signaling rate which can be used without suffering frequency-selective fading ($W << B_c$)

$R(\Delta t)$ yields knowledge about the span of time over which two received signals have strong correlation.

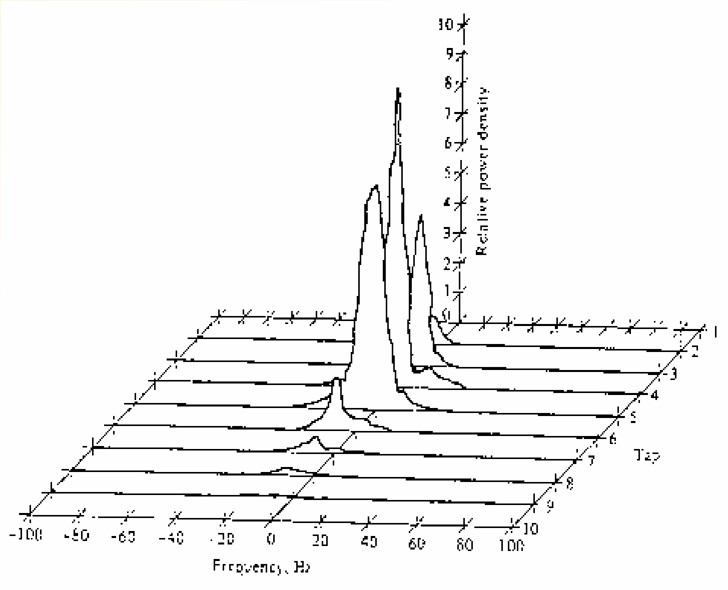
$S(\lambda)$ yields knowledge about the spectral spreading of a transmitted sinusoid in the Doppler-shift domain.

f_D sets a lower limit on the signaling rate which can be used without suffering fast-fading distortion ($W >> f_D$)

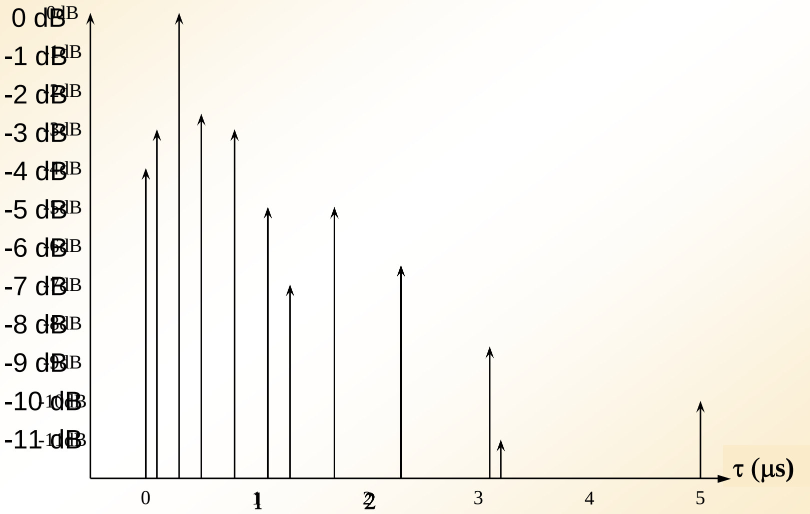
Scattering Function



Example of a Scattering Function



GSM Channel Model

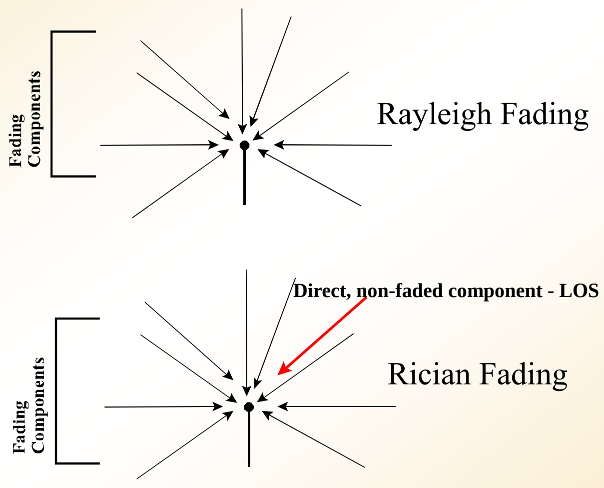


Statistical Models of Multipath

- Each resolvable path is by itself a sum of many multiple paths that have small relative delays
- The amplitude of each resolvable path is a random process (uncorrelated of other resolvable paths)
- The number of paths summing up in each resolvable path is large
- By **Central Limit Theorem**, each component (inphase and quadrature) of each resolvable path has a Gaussian distribution
- The power of resultant path is:

$$\alpha_{tot}^2 = \alpha_I^2 + \alpha_Q^2$$

Statistical Models of Multipath



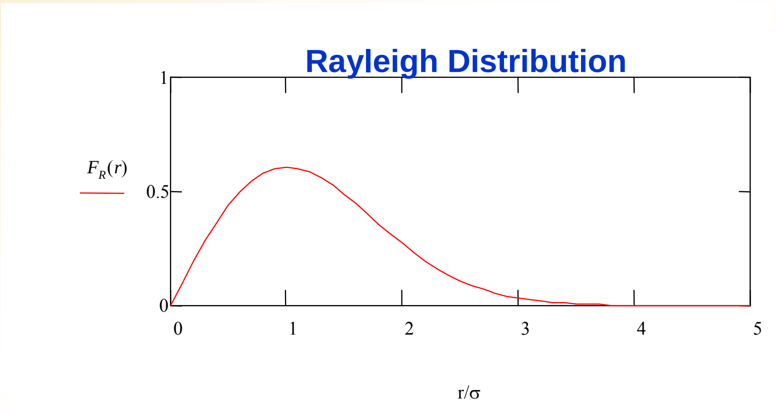
Rayleigh Fading Distribution

- This arises from having multiple incident copies of the transmitted signal with no dominant, direct incident ray (no LOS)
- The multiple copies add together into a general amplitude envelope
- From probability theory, the envelop can be modeled as a **Rayleigh** probability density function (show it!)
- The phase is **uniformly distributed** random variable (show it!)

Rayleigh Fading Distribution

- PDF: $$p(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) u(r)$$
- CDF:
$$\Pr(r \leq R) = \int_0^R p(r)dr = 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$$
- Phase PDF:
$$p(\phi) = 1/2\pi, \qquad -\pi \leq \phi \leq \pi$$

Rayleigh Fading Distribution



Rayleigh Fading Distribution

- σ is the rms value of the voltage envelope
- Mean value of the distribution is:

$$r_{mean} = 1.2533\ \sigma$$

- Median value of the distribution is:

$$r_{median} = 1.177\ \sigma$$

- AC power (variance) of the signal envelope is:

$$\sigma_r^2 = 0.4292\ \sigma$$

Rician Fading Distribution

- This arises from having multiple incident copies of the transmitted signal along with a dominant, direct incident ray (LOS)
- The multiple paths (with same delay) add together into a general amplitude envelope
- As the dominant path fades out, the PDF approaches that of a Rayleigh fading

Rician Fading Distribution

- It is the envelop of the sum of two Gaussian random variables with non-zero means given by μ_I and μ_Q
- Let $A = \mu_I^2 + \mu_Q^2$ (Non-centrality parameter)
- PDF ($A \geq 0$):

$$p(r) = \frac{r^2}{\sigma^2} \exp\left(\frac{-(r^2 + A^2)}{2\sigma^2} \right) I_0\left(\frac{rA}{\sigma^2} \right) u(r)$$

where $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind

Rician Fading Distribution

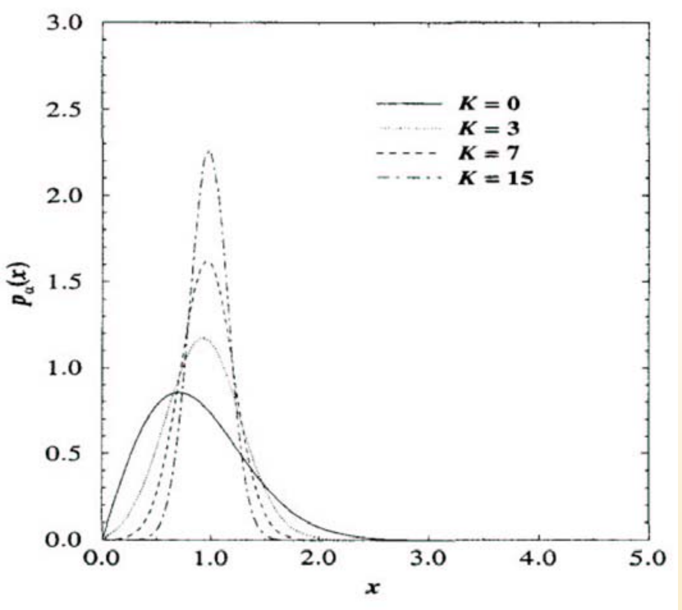
- The Rician fading is often described by the "*K*-Factor" or "Specular-to-diffuse ratio":

$$K(\text{dB}) = 10 \log [A^2/(2\sigma^2)]$$

- Physically this corresponds to the ratio of the direct-path (LOS) signal power to the variance of multipath components

- As $A \rightarrow 0$, $K \rightarrow -\infty$ dB and the LOS path fades out and the distribution becomes Rayleigh

Rician Distribution



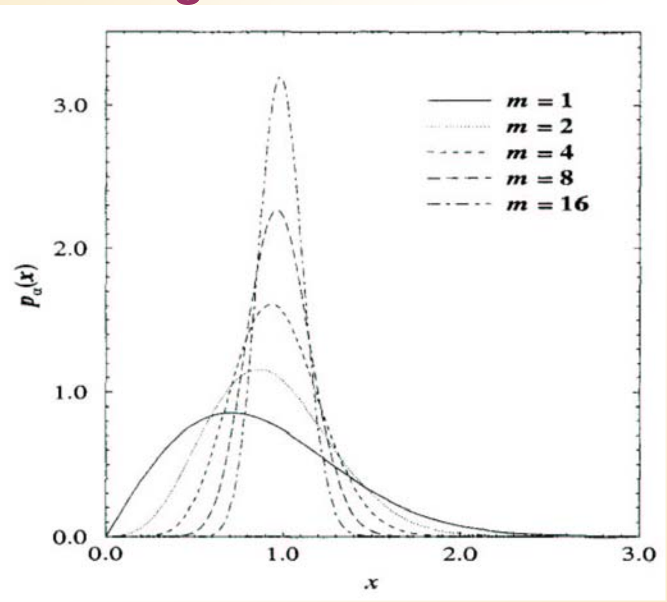
Nakagami Fading Distribution

- PDF ($m \geq 0.5$):

$$p(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega_p}\right)^m r^{2m-1} \exp\left(\frac{-m^2 r^2)}{\Omega_p{}^2}\right) u(r)$$

- Fits well to indoor mesurements
- $\Gamma(m)$ is the Gamma function
- m is the fade parameter:
  - $m = 0.5$ => single-sided Gaussian (worst)
  - $m = 1$ => Rayleigh
  - $m = \infty$ => No fading

Nakagami Distribution



Rician and Nakagami

- Nakagami distribution is analytically tractable for error performance analysis.
- Rician distribution is difficult to analyze
- Rician distribution can be approximated by Nakagami distribution for low SNR values
- Simplifies the analysis of systems undergoing Rician fading (Bessel function eliminated)

Phase Distribution

- In all fading distributions, the phase of the received signal is random
- If **coherent detection** is employed (estimates the phase), then the performance is affected by fading distribution only
- Otherwise, noncoherent detection has to be considered (phase is not estimated)

Level Crossing Rate (LCR)

- Tells how often the faded signal crosses a specified level (R) in the positive direction
- LCR is a function of the MS speed
- For **Rayleigh Fading**:

$$N_R = \sqrt{2\pi}\, f_D \rho e^{-\rho^2}$$

where $\rho = R / \Omega_P$, $\quad \Omega_P = 2\sigma^2$

- For **Rician Fading**:

$$N_R = \sqrt{2\pi(K+1)}\, f_D \rho e^{-K-(K+1)\rho^2} I_0(2\rho\sqrt{K(K+1)})$$

Average Fade Duration (AFD)

- Tells how often the faded signal stays below a specified level

$$\bar{z} = \frac{1}{N_R} \Pr[r < R]$$

- Tells how many bits will be most likely lost in a deep fade duration
- For Rayleigh Fading: $\bar{z} = \dfrac{e^{\rho^2} - 1}{\rho f_d \sqrt{2\pi}}$

Example 5.7: [Rappaport, 2nd ed.]

- Rayleigh fading with $f_D = 20$ Hz.
- Compute LCR for $\rho = 1$.
- What is the maximum velocity of the MS if $f_c = 900$ Hz?

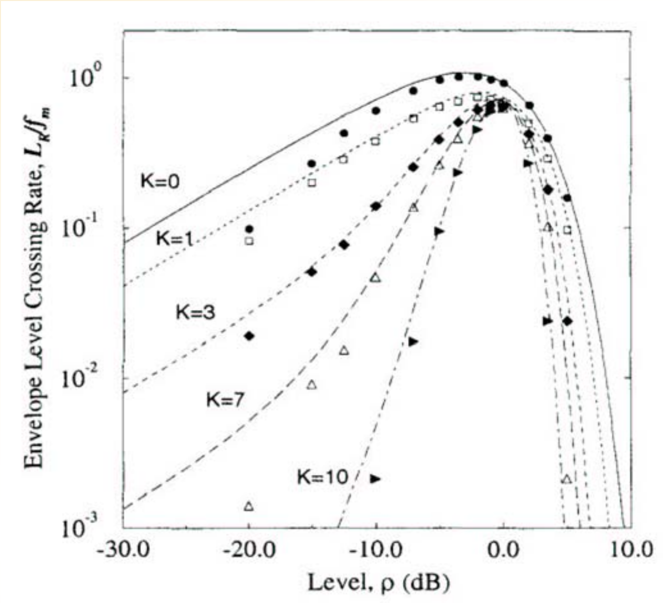
- **Solution:**
  - $N_R = 18.44$ crossings/sec
  - $v = \lambda f_D = 20*1/3 = 6.66$ m/s = 24 km/hr

Example 5.8: [Rappaport, 2nd ed.]

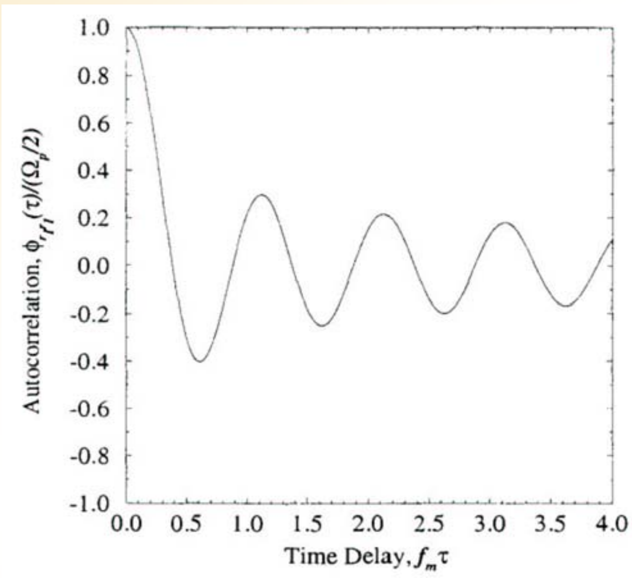
- Rayleigh fading with $f_D = 200$ Hz.
- Find AFD for $\rho = 0.01$, $\rho = 0.1$ and $\rho = 1$.

- **Solution:**
  - $\rho = 0.01$ => AFD = 19.9 μs
  - $\rho = 0.1$ => AFD = 200 μs
  - $\rho = 1$ => AFD = 3.34 ms

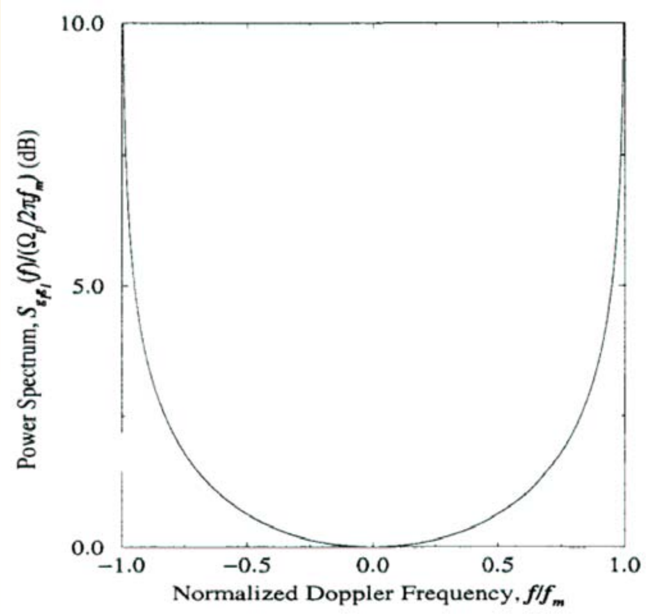
LCR vs. Level *R*



Jakes Model: Auto-correlation Function



Jakes Model: Doppler Spectrum



Two-Ray Multipath Fading Model

- Assumes only two resolvable paths. The received signal is:

  $$r(t) = [\alpha_1(t)\, e^{-j\theta 1(t)} + \alpha_2(t)\, e^{-j\theta 2(t)}]\, s(t)$$

- where:

  $s(t)$ is the transmitted signal

  $\alpha_i(t)$ is the amplitude gain for i^{th} path

  $\theta_i(t)$ is the phase for i^{th} path

  α_i's θ_i's are uncorrelated

Multi-Ray Multipath Fading Model

- Assumes N resolvable paths. The received signal is:

$$r(t) = \sum \alpha_i(t)\, e^{-j\theta_i(t)}\, s(t)$$

- where:

  $s(t)$ is the transmitted signal

  $\alpha_i(t)$ is the amplitude gain for i^{th} path

  $\theta_i(t)$ is the phase for i^{th} path

  α_i's θ_i's are uncorrelated