Small-Scale Multipath Fading

Overview

The mobile channel has small-scale propagation effects due to:

- Motion of the transmitter and/or the receiver
- Multiple scatters of the transmitted signal off buildings and structures
- Time-variable position of the receiver with respect to the environment
Small-Scale Fading

- Due to small changes in position
- Results from the combination of arriving signals with different delays and different attenuation.

Case 1: Single-Tone, Two-Path Channel

<table>
<thead>
<tr>
<th>Constructive Addition</th>
<th>Destructive Addition</th>
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</thead>
<tbody>
<tr>
<td>Direct-path signal</td>
<td>Direct-path signal</td>
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<tr>
<td>Reflected signal</td>
<td>Reflected signal</td>
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<tr>
<td>Composed signal</td>
<td>Composed signal</td>
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</tbody>
</table>
Case 2: Narrow Pulse, Resolvable Multipath Channel

Case 3: Narrow pulse, Unresolvable Multipath Channel
Problems of Mobile Channels

- **Multipath reception:**
  The signal is received over *n* paths with different delays \{τᵢ’s\} and different attenuation factors \{αᵢ’s\}

- **Time variation:**
  The parameters *n*, {τᵢ’s} and {αᵢ’s} are functions of time
WSSUS Assumption

- Stands for:
  
  **Wide Sense Stationary Uncorrelated Scattering**

- Means that:
  - The random process representing each finger is WSS
  - The random processes representing different fingers (different \( \tau_i \)’s) are uncorrelated

Understanding Multipath: Time-Domain Analysis

Multipath Time delay Spread:

\[ S(\tau) \]
Remarks About $S(\tau)$

- $S(\tau)$ describes how does the average received power vary as a function of excess delay $\tau$

- **Excess delay**: the time delay from the arrival of the first echo to the last one

- The received signal usually consists of several discrete multipath components, called “fingers” or “bins”

Effect of Multiple Paths

The effect of multipath can be obtained by comparing values of the $T_m$, and the symbol time $T_s$:

- $T_m > T_s$: results in significant overlap among neighboring received symbol, i.e., channel-induced ISI.

- $T_m \ll T_s$: negligible ISI. Possible reduction in SNR due to destructive add up.
Time Dispersion Parameters

- Mean excess delay:
  \[ \bar{\tau} = \frac{\sum_k \alpha_k^2 \tau_k}{\sum_k \alpha_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} \]

- RMS delay spread:
  \[ \sigma_{\tau} = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} \]
  \[ \frac{\sum_k \alpha_k^2 \tau_k^2}{\sum_k \alpha_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} \]

Maximum excess delay spread at \( X \) dB is the delay at which the received power falls \( X \) dB below maximum finger

Note: these parameters will be functions of the noise floor:

If below the noise floor, a finger will not be included and will not affect the value of the measured signal
Understanding Multipath: Frequency-Domain Analysis

- Spaced-frequency correlation function, $R(\Delta f)$

- $R(\Delta f)$ is a measure of the correlation between two frequency components spaced by $\Delta f$ Hz.

Coherence Bandwidth

- **Coherence Bandwidth**, $B_C$, is the range of frequencies over which the channel can be considered non-distorting (equal gain and linear phase)

- This is defined as the range over which two frequencies show $\geq 90\%$ correlation in their response
The **Coherence Bandwidth** is in terms of the **rms delay spread** (for 90% correlation):

\[ B_C \approx \frac{1}{50\sigma_t} \]

If we are more generous (time over which the channel correlation function is > 0.5)

\[ B_C \approx \frac{1}{5\sigma_t} \]

Not well defined which one to use in general

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**Example:**

- **rms delay for measurements is 1.37 µsec;**
  \[ B_C = 146 \text{ kHz for 50% correlation.} \]

- **If the transmission has a lower bandwidth than** \( B_C \), **then no channel equalization is needed.**

- **If the transmission bandwidth exceeds the** \( B_C \), **then equalization is needed.**
Multipath Time Delay Spread

- The time delay spread can cause either:
  - flat fading
  - frequency-selective fading

- The outcome depends on whether the channel can be considered as a non-distorting filter with respect to the transmitted signal:
  - very slow amplitude change
  - linear phase response

Flat Fading

- Channel acts like a relatively non-distorting filter over a channel bandwidth greater than the transmission bandwidth.

- Channel does cause amplitude variations due to channel gain variations but the spectral content of the signal is preserved.
Flat Fading

- Channel has no excess delay

- The channel delay spread, $\sigma_t$, is related to the signal bandwidth, $W$, by $\sigma_t \ll 1/W$

- Such channels are sometimes called narrowband channels since the bandwidth is less than the coherence bandwidth ($W \ll B_C$)

Frequency-Selective Fading

- Channel acts like a relatively non-distorting filter over a bandwidth much less than the transmission bandwidth

- The received signal is the sum of multiple copies of attenuated and delayed versions of the original signal

- The channel introduces Inter-symbol Interference (ISI)
Frequency-Selective Fading

- The signal bandwidth, $W$, and channel coherence bandwidth, $B_C$: $W > B_C$

- The symbol period, $T_S$ ($T_S = 1/W$), is less than the channel delay spread: $T_S < \sigma_t$

- Channel is considered to be frequency selective if $\sigma_t > 0.1 T_S$

Illustration of Frequency Selectivity

(a) Typical frequency-selective fading case ($f_c < W$)

(b) Typical flat-fading case ($f_c > W$)
Time Domain and Freq. Domain

- \( R(\Delta f) \Leftrightarrow S(\tau) \) (Fourier Transform pair)

- \( B_C \approx \frac{1}{T_m} \) and \( W \approx \frac{1}{T_s} \)

- **Flat fading:** when \( W \ll B_C \) (or \( T_s \gg T_m \))
  \( \Rightarrow \) No distortion

- **Frequency selective:** when \( W > B_C \)
  (or \( T_s < T_m \)) \( \Rightarrow \) ISI

Typical Delay Spreads

**Typical Urban**

**Bad Urban**
Examples

- Example 5.4 [Rappaport 2nd ed.]

- Example 5.5 [Rappaport 2nd ed.]

Example 5.5: [Rappaport, 2nd ed.]

Resolvable multipath signals arrive with:

- -20 dB 0 μsec
- -10 dB 1 μsec
- -10 dB 2 μsec
- 0 dB 5 μsec
Example 5.5 (cont’d)

- Mean excess delay = $\bar{\tau} = 4.38 \mu s$
- Mean excess delay = $\bar{\tau}^2 = 21.07 \mu s^2$
- rms delay spread = $\sigma_\tau = 1.37 \mu s$
- $B_c$ (50% correlation) = $1/5\sigma_\tau = 146$ kHz
- In AMPS, $W = 30$ kHz < $B_c$ => flat fading
- In GSM, $W = 200$ kHz > $B_c$ => frequency-selective fading

Doppler Shift

- Any motion in the plane in which the electromagnetic wave is traveling towards or away from the receiver results in a shift in the carrier frequency.
- Tx-Rx distance is very large
- Motions perpendicular to the plane do not affect it.
Doppler Shift

- If the transmitter and receiver are approaching, the frequency is higher while if they are moving apart, the frequency is lower.

- The phase difference between the paths:
  \[ \Delta \phi = \frac{2\pi \Delta l}{\lambda} = \frac{2\pi v \Delta t}{\lambda} \cos \theta \]

- The Doppler shift is:
  \[ f_D = \frac{1}{2\pi} \frac{\Delta \phi}{\Delta t} = \frac{v}{\lambda} \cos \theta \]

Example:
Vehicle moving at 60 mph relative to a transmitter. The carrier freq. is 1850 MHz.
What is the maximum Doppler shift?

Solution:
- \[ \lambda = c/f = 0.162 \text{ m} \]
- \[ f_D = 60 \text{ mph} \times 1609 \text{ m/mi} \times 1 \text{ hr}/3600 \text{ sec} \]
  \[ /0.162 \text{ m} = 165 \text{ Hz} \]
Doppler Spread Spectrum

- Shows how the spectrum of a pure sinusoid is spread by the channel
- The maximum spread in frequency is called the **Doppler Spread**, $f_D$

Spaced-Time Correlation Function

- It shows how the channel response is correlated at time instances separated by $\Delta t$
- $S(\lambda) \Leftrightarrow R(\Delta t)$
Coherence Time

- If a pure sinusoid is transmitted, it will be received as a range of frequencies adjacent to the sinusoid frequency.
- The Doppler shift broadens the spectrum of the received signal by spreading (smearing) the basic spectrum in frequency domain.
- If the signal spectrum is wide compared to this smearing, the effect is not noticeable.
- **Coherence Time:** measures the time duration over which the channel characteristics can be considered to be static.

Coherence Time

The channel is time-variable - but how variable is it?

- **Coherence Time** and **Doppler Spread** measure the time variability of the channel.
- Doppler Spread comes from the frequency shifting of the carrier due to the Doppler shift:

  \[ f_r = f_c \pm f_D \]
Coherence Time

The Coherence Time, $T_C$, is proportional to the maximum Doppler frequency $f_D$, (time over which the channel correlation function is larger than 0.9)

$$T_C \approx f_D^{-1}$$

If we are more generous (time over which the channel correlation function is $> 0.5$):

$$T_C \approx \frac{9}{16\pi f_D}$$

The modern rule of thumb is to take the geometric average of the two:

$$T_C = \sqrt{\frac{9}{16\pi f_D^2}} = \frac{0.423}{f_D}$$

For digital transmission, the symbol rate needs to be $R_s > 1/T_C$ for no distortion.
Example - Coherence Time

Example (5.6) [Rappaport 2nd ed.]:
Travel distance of 10 m; speed of 50 m/sec, carrier at 1900 MHz.

Find: The Doppler bandwidth, coherence time, the sampling time, the sample spacing, the number of samples in 10 m

Time Selectivity

The frequency spread can cause either:
- Fast fading
- Slow fading

The outcome depends on the rate of change of the transmitted signal with respect to the rate of change of the channel
Fast Fading (Time-Selective)

- The coherence time of the channel, $T_C$, is less than the transmitted symbol period, $T_S$
- The Doppler bandwidth, $f_D$, is larger than the bandwidth of the signal, $W$
  \[ \Rightarrow \text{low-data rate signal} \]
- The fast fading channel causes frequency dispersion (also called time-selective fading)

Slow Fading (Time-Nonselective)

- Channel changes at a rate much slower than the baseband signal rate:
  \[ T_S \ll T_C \]
- The Doppler spread in the frequency domain is less than the symbol bandwidth:
  \[ W \gg f_D \]
- The speed of the mobile user relative to the data rate makes this determination
Effect of Time Variation

- Time variance is linked with the **fading rate**, $f_D T_s$:
  - $T_C > T_s (f_D T_s < 1)$: The transmitted symbol will be subject to the same fading realization
    $\Rightarrow$ Slow fading
  - $T_C < T_s (f_D T_s > 1)$: The fading realization changes while a symbol is propagating
    $\Rightarrow$ Fast fading
- Slow Fading $\Rightarrow$ Loss in SNR
- Fast fading $\Rightarrow$ Distortion and irreducible error rate.

Frequency-Nonselective, Slowly Fading Channel

- The least severe fading case:
  - No distortion.
  - No irreducible error floor.
  - Only reduction in SNR.
- **Conditions:**
  - $f_D << W << B_c$
  - $T_C >> T_s >> T_m$
- $f_D T_m$ is called the **spread factor**.
- If the spread factor is small compared to unity, i.e., $f_D T_m << 1$ (channel is under-spread), then it is possible to design a FNS, SF system.
Small-Scale Fading

- The four fading types come from:
  - relative bandwidth of channel versus data bandwidth
  - relative delay in the channel versus data period
- Multipath delay causes time dispersion of the signal and frequency-selective fading
- Doppler shifts cause frequency dispersion and time-selective fading

We can combine the four effects into a matrix of fading channel types

We rank the fading based on:
- Symbol duration, $T_S$, relative to the delay spread, $\sigma_r$, and the coherence time, $T_C$
- Symbol bandwidth, $W$, relative to Doppler bandwidth, $f_D$, and channel coherence bandwidth, $B_C$
Small-Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

- **Flat Fading**
  1. BW of Signal < BW of Channel
  2. Delay spread < Symbol Period

- **Frequency Selective Fading**
  1. BW of Signal > BW of Channel
  2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

- **Fast Fading**
  1. High Doppler spread
  2. Coherence time < Symbol Period
  3. Channel variations faster than baseband signal variations

- **Slow Fading**
  1. Low Doppler spread
  2. Coherence time > Symbol period
  3. Channel variations slower than baseband signal variations
**Duality**

- **$R(\Delta f)$** gives the range of frequency over which two spectral components have strong correlation.

- **$S(\tau)$** gives the time spreading of a transmitted pulse in the time-delay domain.

- **$B_c$** sets an upper limit on the signaling rate which can be used without suffering frequency-selective fading ($W << B_c$).

- **$R(\Delta t)$** yields knowledge about the span of time over which two received signals have strong correlation.

- **$S(\lambda)$** yields knowledge about the spectral spreading of a transmitted sinusoid in the Doppler-shift domain.

- **$f_d$** sets a lower limit on the signaling rate which can be used without suffering fast-fading distortion ($W >> f_d$).

**Scattering Function**

[Scattering Function diagram]
Example of a Scattering Function

GSM Channel Model
Statistical Models of Multipath

- Each resolvable path is by itself a sum of many multiple paths that have small relative delays.
- The amplitude of each resolvable path is a random process (uncorrelated of other resolvable paths).
- The number of paths summing up in each resolvable path is large.
- By **Central Limit Theorem**, each component (inphase and quadrature) of each resolvable path has a Gaussian distribution.
- The power of resultant path is:
  \[ \alpha_{\text{tot}}^2 = \alpha_I^2 + \alpha_Q^2 \]

Rayleigh Fading

Rician Fading

Direct, non-faded component - LOS
Rayleigh Fading Distribution

- This arises from having multiple incident copies of the transmitted signal with no dominant, direct incident ray (no LOS).
- The multiple copies add together into a general amplitude envelope.
- From probability theory, the envelop can be modeled as a Rayleigh probability density function (show it!)
- The phase is uniformly distributed random variable (show it!)

Rayleigh Fading Distribution

- **PDF:** \( p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)u(r) \)
- **CDF:**
  \[
  \Pr(r \leq R) = \int_{0}^{R} p(r)dr = 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right)
  \]
- **Phase PDF:**
  \[ p(\phi) = \frac{1}{2\pi}, \quad -\pi \leq \phi \leq \pi \]
Rayleigh Fading Distribution

- $\sigma$ is the rms value of the voltage envelope

- Mean value of the distribution is:
  
  $$r_{\text{mean}} = 1.2533 \sigma$$

- Median value of the distribution is:
  
  $$r_{\text{median}} = 1.177 \sigma$$

- AC power (variance) of the signal envelope is:
  
  $$\sigma_r^2 = 0.4292 \sigma$$
Rician Fading Distribution

- This arises from having multiple incident copies of the transmitted signal along with a dominant, direct incident ray (LOS)

- The multiple paths (with same delay) add together into a general amplitude envelope

- As the dominant path fades out, the PDF approaches that of a Rayleigh fading

\[ p(r) = \frac{r^2}{\sigma^2} \exp\left(\frac{-(r^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right) u(r) \]

where \( I_0 (*) \) is the zeroth-order modified Bessel function of the first kind
Rician Fading Distribution

- The Rician fading is often described by the “K-Factor” or “Specular-to-diffuse ratio”:
  \[ K(\text{dB}) = 10 \log \left[ \frac{A^2}{2\sigma^2} \right] \]

- Physically this corresponds to the ratio of the direct-path (LOS) signal power to the variance of multipath components.

- As \( A \to 0, K \to -\infty \) dB and the LOS path fades out and the distribution becomes Rayleigh.
Nakagami Fading Distribution

- PDF \((m \geq 0.5)\):

\[
p(r) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega_p} \right)^m r^{2m-1} \exp\left( -\frac{m^2 r^2}{\Omega_p^2} \right) u(r)
\]

- Fits well to indoor measurements
- \(\Gamma(m)\) is the Gamma function
- \(m\) is the fade parameter:
  - \(m = 0.5\) \(\Rightarrow\) single-sided Gaussian (worst)
  - \(m = 1\) \(\Rightarrow\) Rayleigh
  - \(m = \infty\) \(\Rightarrow\) No fading

Nakagami Distribution
Rician and Nakagami

- Nakagami distribution is analytically tractable for error performance analysis.
- Rician distribution is difficult to analyze.
- Rician distribution can be approximated by Nakagami distribution for low SNR values.
- Simplifies the analysis of systems undergoing Rician fading (Bessel function eliminated).

Phase Distribution

- In all fading distributions, the phase of the received signal is random.
- If coherent detection is employed (estimates the phase), then the performance is affected by fading distribution only.
- Otherwise, noncoherent detection has to be considered (phase is not estimated).
Level Crossing Rate (LCR)

- Tells how often the faded signal crosses a specified level \((R)\) in the positive direction
- LCR is a function of the MS speed
- For Rayleigh Fading:
  \[
  N_R = \sqrt{2\pi} f_D \rho e^{-\rho^2}
  \]
  where \(\rho = R / \Omega_p\), \(\Omega_p = 2\sigma^2\)
- For Rician Fading:
  \[
  N_R = \sqrt{2\pi(K+1)} f_D \rho e^{-K(K+1)\rho^2} I_0(2\rho \sqrt{K(K+1)})
  \]

Average Fade Duration (AFD)

- Tells how often the faded signal stays below a specified level
- Tells how many bits will be most likely lost in a deep fade duration
- For Rayleigh Fading:
  \[
  z = \frac{1}{N_R} \Pr[r < R]
  \]
Example 5.7: [Rappaport, 2nd ed.]

- Rayleigh fading with $f_D = 20$ Hz.
- Compute LCR for $\rho = 1$.
- What is the maximum velocity of the MS if $f_c = 900$ Hz?

**Solution:**

- $N_R = 18.44$ crossings/sec
- $v = \lambda f_D = 20/3 = 6.66$ m/s $= 24$ km/hr

Example 5.8: [Rappaport, 2nd ed.]

- Rayleigh fading with $f_D = 200$ Hz.
- Find AFD for $\rho = 0.01$, $\rho = 0.1$ and $\rho = 1$.

**Solution:**

- $\rho = 0.01$ $=>$ AFD = 19.9 $\mu$s
- $\rho = 0.1$ $=>$ AFD = 200 $\mu$s
- $\rho = 1$ $=>$ AFD = 3.34 ms
LCR vs. Level $R$

Jakes Model: Auto-correlation Function
Two-Ray Multipath Fading Model

- Assumes only two resolvable paths. The received signal is:
  \[ r(t) = [\alpha_1(t) e^{j\theta_1(t)} + \alpha_2(t) e^{j\theta_2(t)}] s(t) \]

- where:
  - \( s(t) \) is the transmitted signal
  - \( \alpha_i(t) \) is the amplitude gain for \( i^{th} \) path
  - \( \theta_i(t) \) is the phase for \( i^{th} \) path
  - \( \alpha_i \)'s and \( \theta_i \)'s are uncorrelated
Multi-Ray Multipath Fading Model

- Assumes $N$ resolvable paths. The received signal is:
  \[ r(t) = \sum \alpha_i(t) e^{j\theta_i(t)} s(t) \]

- where:
  - $s(t)$ is the transmitted signal
  - $\alpha_i(t)$ is the amplitude gain for $i^{th}$ path
  - $\theta_i(t)$ is the phase for $i^{th}$ path
  - $\alpha_i$’s $\theta_i$’s are uncorrelated