KING FAHD UNIVERSITY OF PETROLEUM & MINERALS **ELECTRICAL ENGINEERING DEPARTMENT**

EE 575 INFORMATION THEORY **(152)**

Assignment # 1

1. Let X and Y be independent r.v.'s. Both can take on values of 0 and 1 with

$$p(x) = 0.5, \quad \forall x \qquad \text{and} \qquad p(y) = 0.5, \quad \forall y$$

   Let $Z = X \otimes Y$ (modulo-2 addition).
   (a) Make a table of all the possible probabilities $p(x,y,z)$. Show that any pair of the three r.v.'s are independent, but the three themselves are not independent.
   (b) Show that although X and Y are independent, they are not conditionally independent given Z.

2. Prove the following statements:
   (a) Show that X and Y are uncorrelated if and only if $\mathrm{cov}(X,Y) = 0$.
   (b) Show that if X and Y are independent, then they are also uncorrelated.
   (c) Show that if $X_1, X_2, \ldots, X_N$ are pairwise uncorrelated r.v.'s, then:

$$\mathrm{var}\left(\sum_{i=1}^{N} X_i\right) = \sum_{i=1}^{N} \mathrm{var}(X_i)$$

   (d) Show that the correlation coefficient satisfies $\circ_{XY} \leq 1$, with equality if and only if $Y = aX$, for some constant a.

3. For the two jointly Gaussian r.v.'s X and Y,
   (a) Find the conditional expectation of X given $Y = y$.
   (b) Show that if X and Y are uncorrelated, then they are independent.

4. The r.v.'s X and Y have the joint pdf:

$$f_{XY}(x,y) = 2e^{-(x+y)}, \qquad 0<x<y<\infty$$

   Find the pdf of $Z = X + Y$. Note: X and Y are **not independent**.

5. The received instantaneous power in a wireless communication system is an exponential r.v. Let T be the maximum of n such independent signals. Find the cdf of T.

Note: Please copy this and sign on each H.W. assignment:

I testify that I will not refer to the solutions of the assignments of EE 575 by any means and in any form and from any source, before I submit the assignment to my instructor. For programming assignments, I testify that I will not use/refer to any ready code in any means or any form throughout and until the submission of the assignment.