Home

Design of a two-slot time orthogonal line / multiple access code

A-Imam Al-sammak
Department of Electrical and Electronics Engineering, University of Bahrain,
Isa Town, Bahrain
alsammak@iee.org

R. L. Kirlin, P. F. Driessen
Electrical and Computer Engineering,
University of Victoria, BC,
Canada
peter@ece.uvic.ca
lynn@ece.uvic.ca

Abstract - In this paper, we present a new line code that allows more than one user to access the channel simultaneously. It utilizes two time slots for transmission, with aggregate transmitted bits per time slot is greater than 1 bit. 2-D constellation diagram is employed, which looks similar to the 16-point quadrature amplitude modulation (QAM), but it is different from it in three aspects. First, this scheme allows multiple users. Secondly, it employs simple addition of the same signal type in each time slot. Thirdly, it utilizes time orthogonal rather than phase orthogonal signals used in QAM, and hence have less stringent synchronization requirements.

I Introduction

Communication channel bandwidth is a scarce commodity, so efficient ways of utilizing it is always being searched for. One technique is to allow more than one user to access the same channel. There are several ways of achieving this goal, among them are: time division multiplexing (TDM), frequency division multiplexing (FDM), spread spectrum and collaborative coding [2]. In this paper we present a scheme, which allows more bits per channel use than standard time division multiplexing scheme. This code will be of interest to modem designers especially for applications such as cable or point-to-point communications.

II Problem description:

U communication channel users each independently select one of M symbols coded as 2 bits (binary) from his independent alphabet A_u, of M symbols, as indicated in figure 1. The U symbols might be added decimally either a) in the transmitter or b) at the receiver. We specifically are concerned with a), addition in the transmitter, as in cable or point-to-point radio. For example, the sum of the first (or second, or…) bits of the U symbols may result in a number 0 through U. The vector V whose B elements are the individual bit sums must uniquely specify each user's symbol that contributed to the sum.

III Definitions

Let

U = number of users.
S_b = sum of user bits in time slot b, b = 1,2.
V = the base 10, length 2 vector whose elements are the sums of each users' 2-bit words: $\mathbf{V} = S_1 10^1 + S_2 10^2$.
w = a length 2U vector containing the U concatenated 2-bit symbols.
M = number of 2-bit symbols used by each user
$\mathbf{m}_u$ = vector whose elements are the 2 bits selected by user u.
m_{ub} = b^{th} element in $\mathbf{m}_u$
u_i = original (uncoded) users' symbols (i = 1,2, ..U)

Then:

- The maximum number of possible values of each S_b is U.
- The number of possible unique values of **V** is $N_{Vmax} = (U+1)^2$.
- The maximum number of unique symbols available to all users is $M_{max} = 2^2 = 4$, but not all may be used by all users if unique decodability is imposed.
- The number possible values of **w** is N_w**.** However for decoding (from V) uniqueness, the actual N_w words used must be less than or equal to N_V. That implies a smaller number M symbols / user might be used. The possible number of codewords is thus equal to M^U

From the above discussion, the following constraint can be formulated:

$$N_w \le N_{Vmax} \quad \text{i.e.}$$

$$M^U \le (U+1)^2 \qquad (1)$$

We can use this constraint two ways: for fixed U we can select an actual M small enough that $M^U \le (U+1)^2$, or for a fixed M we may increase U so that $M^U \le (U+1)^2$.

Based on the above constraint, we will answer the questions:

1. How many sum vectors V can there be?
2. What are the maximum number U of users that can use M symbols; alternately, given U users, what is the minimum number M of bits that is required.
3. How can we uniquely determine the user symbols associated with each V?

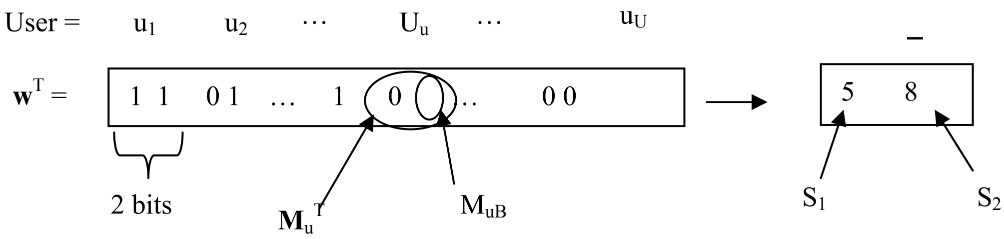

Figure 1: Makeup of U users, 2-bit binary symbols m_j constituting their concatenated source symbol words, w, along with the associated sum vector **V** composed of 2 bit sums $\mathbf{S}_b$ from U symbols.

The answer to question 1 is simply $(U+1)^2$, assuming that each user can select his word using a binary scheme, i.e. '0' or '1' for transmitting each bit! Note that a single user's bit is coded as a combination of 2 bits for transmission!

IV Code Design

A communication scheme can be designed in such a way that for any combination of the U-users data, a unique point from a sum vector constellation diagram is sent. As an example, consider the 4-user (U=4), 2-bit code. If M = 2 symbols/user, this code needs to use only 16 points from the 25 possible sums in the 2-dimensional space, i.e. from the grid of size $(U+1)^2 = 5 \times 5 = (U+1)\bullet(U+1)$. Results are shown in the constellation maps in figure 2. There are many possibilities (15 out of 25 = 2042975) to select a constellation of 16 points out of the available 25, but a constellation that gives minimum probability of error has to be chosen. Probability of error is inversely proportional to the minimum distance of the constellation, and hence one with largest average distance will be selected. Two of such possibilities are shown in figures 3 and 4 respectively, they are named here as X(cross)-type and QAM-type constellation respectively. The QAM-like gives minimum probability of error, as will be shown in section V, and hence is recommended.

At each transmission time U users' selected symbols are assigned by lookup table to the 2-bit sums according to a mapping such as in figure 5. The receiver reverses the lookup and delivers the user's messages to U recipients. The dashed blocks in figure 4 are needed only in the case wherein there is only one user and his data stream is broken into U symbols in sequence, and these are encoded the same way.

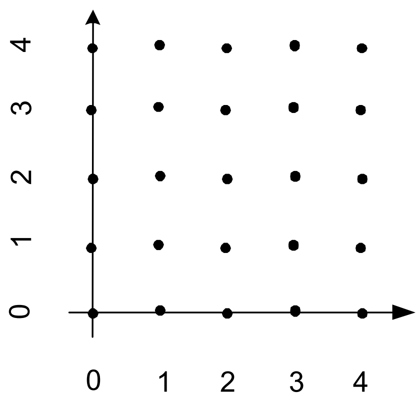

Figure 2: Two dimensional signal space for U = 4 users, 2 time slots per symbol (dimensions $x = t_1$ and $y = t_2$) and M = 2 symbols/user (**unipolar** 0 or 1); all possible sum voltage levels.

Top

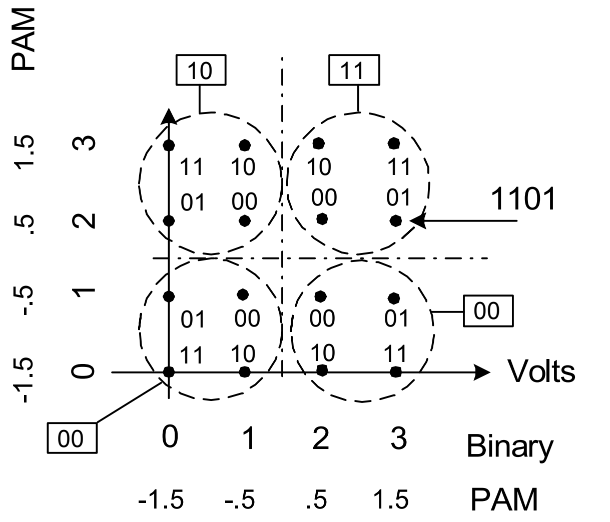

Figure 3: Two dimensional signal space for U = 4 users, 2 time slots and M=2 symbols / user, utilizing QAM-like constellation.

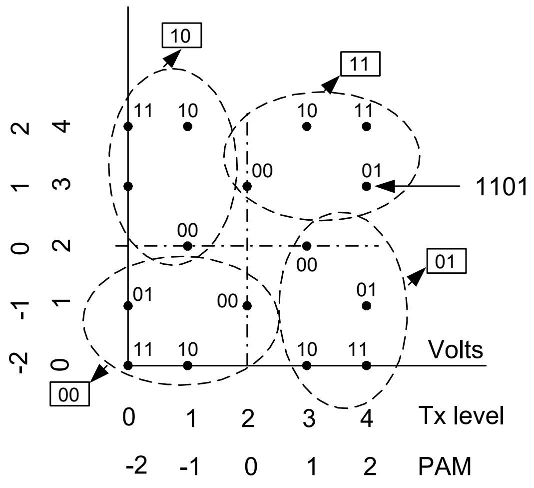

Figure 4: Constellation diagram for four users (U=4), two bits/symbol (B=2) code, M = 2 symbols/user illustrating both direct transmit level coding and PAM transmission.

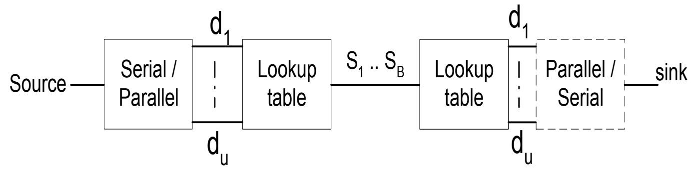

Figure 5: U users' sy mbol sums are assigned by lookup table. The receiver reverses the lookup and delivers the user's messages to U recipients, dashed blocks are needed only for the single user case, where coding is applied to one user's data.

To ease encoding, the number of symbols/user can be chosen to be 2, 4, 8, …, 2^B, and to minimize bit error rate the codewords are Gray coded to the extent possible. The constellation diagrams for the 4-user code is given in figures 3 and 4. for both schemes, two sets of constellation axes are given, one is for all-positive level symbols only (Direct transmission, direct Tx), while the second uses 0-centered PAM (pulse amplitude modulation). In real life applications, the PAM is preferred as it has no dc component.

V Comparison between QAM and the X-like

Because of similarities between this code and B-dimensional signaling (e.g. quadrature amplitude modulation (QAM for B=2), it is of interest to compare the bit error rates for an example design. In this section we concentrate mainly on comparison between the X-type constellations with the QAM, in terms of minimum distance (d_{min}), and hence probability of error. The QAM-type constellation is a replica of the QAM constellation, and hence is expected to have the same d_{min}. Our example has M = 2 and B = 2. The energy of any constellation point in a QAM is calculated as

$$E_i = (T/2)(a^2 + b^2) \qquad (2)$$

For the new code as per figure 4 with 5 possible levels –2,-1,-,1,2, the average energy for equally likely signals is given by:

$$E_{ave} = \frac{1}{16}\left\{4\left[(0^2+1^2)+(1^2+2^2)+(1^2+2^2)+(2^2+2^2)\right]\right\} = \frac{19T}{16} = 1.1875T \qquad (3)$$

If the *same voltage span* is used for both schemes, then we must multiply the result of our scheme by a factor of $((6/4)^2 = 2.25)$. Therefore:

Top

$E_{ave} = 2.25 * (19/16)\ T = 2.6719\ T$ (4)

The calculation of bit error rate (BER) is a function of the minimum Euclidean distance *d* on the constellation. Measuring this distance directly from the figures, we find that for QAM-16 $d = 2$; while for our code $d = 1$. To compare the QAM with X-like scheme, there is one of two normalizing options:

1. Same voltage span: since the voltage span is 4 for our code but 6 for QAM, then the 'equivalent distance' factor is 1.5.
2. Same average bit energy: scaling our voltage levels by α and equating the two energies/bit, we get 3.75 T = (19/16) Tα^2 or $\alpha = 1.777$; i.e. $d = 1.777$.

It is found that the minimum distance of X-like code is slightly inferior to the QAM (.75 (1.5/2) and .889 (1.777/2) respectively).

We expect the deterioration in BER to be not as poor as the factor of 0.889, due to the fact that only 4 adjacent signal pairs in the constellation have this minimum distance while the rest have greater.

The minimum distance is the same if we choose the 16 points as shown in figure 3, QAM-like scheme, since it is equivalent to the 16 points QAM constellation.

Based on the above discussion, we recommend the use of QAM-like constellation in our new line code for 4-users. This suggests that higher throughputs can be achieved by going for larger QAM constellations, at the expense of reduced minimum distance. By adopting this (time orthogonal) QAM technique, further advantages can utilized such as error detection and correction.

VI Conclusions

A new 2 time slot orthogonal code was presented in this paper. Two schemes were presented: the x-like scheme and the PAM scheme. Although the x-like shows greater distances between most of the constellation points, the corner points have a smaller distance. Since the error performance is governed by the minimum distance of the constellation, a QAM like constellation is recommended. This suggests that higher throughputs can be achieved by going for larger QAM constellations, at the expense of reduced minimum distance. By adopting (time

orthogonal) QAM technique, further advantages can utilized such as error detection and correction. The principle of having QAM constellation in time can be extended to more than two bits, and it is the subject of further research work.

References:

1. A. Al-Sammak, R. l. Kirlin and P.F. Driessen, Design of a new Line Code", International Symposium on Advanced Radio Technologies (ISART 2002), Boulder, Colorado, USA, 4-6 March 2002.
2. A. Al-Sammak, "Five user collaborative code with rate 1.67", IEE Electronics Letters, vol. 37, No 3, pp. 183-184, 1st February 2001.