

# PERFORMANCE OF MULTI-SPLIT DECISION FEEDBACK EQUALIZER

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**Abstract** - In this paper, the idea of multi-split adaptive filtering is applied on DFE. The feed-forward and feedback sections in the DFE are divided into parallel sub-filters by imposing separately the symmetry and antisymmetry conditions on the impulse responses of filters by using appropriate and distinct sets of linear constraints. The purpose of this is to obtain a new structure that gives better performance than the conventional structure. The performance of multi-split DFE is tested with different algorithms (LMS, RLS, DCT-LMS) and compared to that of multi-split linear transversal Equalizer.

**Keywords** – *Equalization, Multi-split, Decision-feedback, RLS, LMS, DCT-LMS Algorithms.*

## I. INTRODUCTION

Equalization plays an important role in a communication system. In some applications, the channel characteristics are not known in advance. Moreover, they may change from time to time, specially in a wireless communication system [1]. It is, therefore, important to continuously track the channel characteristics. To do so, adaptive equalization techniques are used, where an adaptive algorithm is used to adjust the filter coefficients. At the receiver an *equalizer*, as shown in Figure 1, is used in order to reduce the effect of Inter-Symbol Interference (ISI) to a minimum and hence maximize the probability of correct decisions.

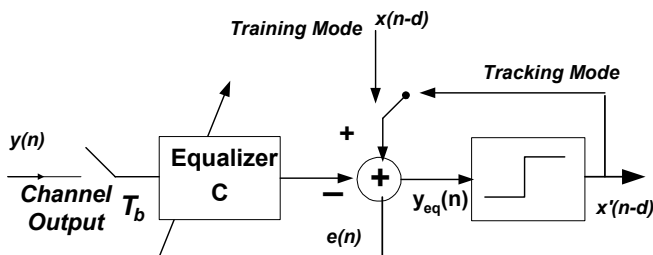


Fig. 1. Adaptive equalization

The convergence of the equalizer taps to the optimum value is an important issue. Faster convergence is needed to track rapid channel changes. It is found that faster

convergence and complexity of the equalizer are two conflicting parameters.

In order to improve the convergence rate with a small increase in complexity, the split adaptive filter is a good choice [4-6]. Split processing technique guarantees stability and give quicker convergence at the expense of a moderate increase in computation.

In adaptive equalization, the convergence behavior is governed by the eigenvalue spread; a lower spread increases the convergence rate. It has been shown that whitening the equalizer input using some transformations such as Karhunen-Loeve transform and discrete cosine transform can significantly reduce the eigenvalue spread [4].

The split processing has gained an increasing attention in recent years. Delsarte and Genin [2] worked on a split version of the Levinson algorithm for real Toeplitz matrices. The work in [2] extended in [3] to classical algorithms in linear prediction theory. The performance of a split-path LMS adaptive filter for autoregressive modeling was analyzed in [4] in which the authors showed that the new model can provide a much faster rate of convergence with a small increase in computation complexity. In [6-7], the authors suggested a new structure for the split transversal filter [6] and they used the continuously split procedure to introduce the multi-split adaptive filter [7].

In this work, the idea of multi-split (MS) adaptive filtering is applied to the decision-feedback equalizer (DFE). This is done by continuously splitting both the forward filter and the feedback filter leading to a new structure and is called Multi-split DFE (MS-DFE). Each split imposes separately symmetry and anti-symmetry conditions on the impulse responses of filters connected in parallel by using appropriate and distinct sets of linear constraints [6].

Adaptive algorithms such as the LMS, LMS-DCT and RLS algorithms are used to update the tap coefficients of the equalizer. In the case of multi-splitting, only MS-LMS and MS-RLS algorithms are used.

The next section begins with a brief description of the principle of split and multi-split FIR filters. This principle is applied to both Linear Transversal Equalizer (LTE) and DFE to produce MS-LTE and MS-DFE. Simulation results

in section 3 illustrate the performance of MS-LTE and MS-DFE. Finally, the paper ends with conclusions.

## II. SPLIT AND MULTI-SPLIT FIR FILTER

A sequence in an FIR filter can be expressed as the sum of a *conjugate-symmetric* sequence and a *conjugate-anti-symmetric* sequence. The *conjugate-symmetric* (*conjugate-anti-symmetric*) sequence is given by half the sum (*difference*) of the original sequence and its conjugate and backward version [6].

To start, let us represent the tap-weight vector of FIR transversal filter in the following form

$$\mathbf{w} = [w_1(n), w_2(n), \dots, w_N(n)]^T \quad (1)$$

Let  $\mathbf{w}_a$  and  $\mathbf{w}_s$  denote the vectors of the conjugate-symmetric and conjugate-anti-symmetric, respectively, of  $\mathbf{w}$ , then

$$\mathbf{w} = \mathbf{w}_s + \mathbf{w}_a \quad (2)$$

where

$$\begin{aligned} \mathbf{w}_s &= \frac{1}{2}(\mathbf{w} + \mathbf{J}\mathbf{w}^*) \\ \mathbf{w}_a &= \frac{1}{2}(\mathbf{w} - \mathbf{J}\mathbf{w}^*) \end{aligned} \quad (3)$$

\* represents the complex conjugate operation and  $\mathbf{J}$  is the reflection matrix or exchange matrix which has unit elements along the cross diagonal and zeros elsewhere, that is,

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

Pre-multiplying any vector with  $\mathbf{J}$  will flip the vector from top to bottom. Figure 2 illustrates the splitting technique.

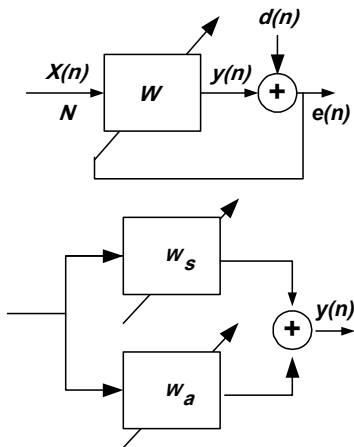


Fig. 2. Adaptive split transversal filtering

The principle of a linearly constrained transversal filter is to minimize the estimated error  $e(n)$  subject to a set of linear equations defined by:

$$\mathbf{C}_{K \times N}^T \mathbf{w}_{N \times 1} = \mathbf{f}_{K \times 1} \quad (5)$$

where  $\mathbf{C}$  is the constraint matrix and  $\mathbf{f}$  is the element response vector. Applying this principle, the symmetry and antisymmetry conditions of  $\mathbf{w}_a$  and  $\mathbf{w}_s$  can be easily introduced through a linearly constrained approach [6-7] where two cases are considered for  $\mathbf{C}_s$ ,  $\mathbf{C}_a$ ,  $\mathbf{f}_a$  and  $\mathbf{f}_s$ .

- Case 1:  $N$  odd ( $K=(N-1)/2$ ):

$$\mathbf{C}_s = \begin{bmatrix} \mathbf{I}_{K \times K} \\ \mathbf{0}_{1 \times K} \\ -\mathbf{J}_{K \times K} \end{bmatrix}_{N \times K}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{I}_{K \times K} & \mathbf{0}_{K \times 1} \\ \mathbf{0}_{1 \times K} & \sqrt{2} \\ \mathbf{J}_{K \times K} & \mathbf{0}_{K \times 1} \end{bmatrix}_{N \times (K+1)} \quad (6)$$

$$\mathbf{f}_s = \mathbf{0}_{K \times 1}, \quad \mathbf{f}_a = \mathbf{0}_{(K+1) \times 1}$$

- Case 2:  $N$  even ( $K=N/2$ ):

$$\mathbf{C}_s = \begin{bmatrix} \mathbf{I}_{K \times K} \\ -\mathbf{J}_{K \times K} \end{bmatrix}_{K \times K}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{I}_{K \times K} \\ \mathbf{J}_{K \times K} \end{bmatrix}_{K \times K} \quad (7)$$

$$\mathbf{f}_s = \mathbf{f}_a = \mathbf{0}_{K \times 1}$$

In both cases the following are imposed in the constrained optimization process:

$$\mathbf{C}_s^T \mathbf{w}_s = \mathbf{f}_s \quad \text{and} \quad \mathbf{C}_a^T \mathbf{w}_a = \mathbf{f}_a \quad (8)$$

Using the generalized sidelobe canceller structure (GSC) [6] and the above constraint matrices we get the split-filtering structure shown in Figure 3. In the case of  $N$  even, the first  $N/2$  coefficients of vectors  $\mathbf{w}_s$  and  $\mathbf{w}_a$  compose  $\mathbf{w}_{\perp s}$  and  $\mathbf{w}_{\perp a}$ , respectively. Also, note that multiplying  $\mathbf{w}_{\perp s}$  by  $\mathbf{C}_a$  gives  $\mathbf{w}_s$  and  $\mathbf{w}_{\perp a}$  by  $\mathbf{C}_s$  gives  $\mathbf{w}_a$  [6-7]. According to [6], this simple structure comes from two facts. First, since  $\mathbf{f}_s = \mathbf{f}_a = \mathbf{0}_{K \times 1}$  (even case), the filters of the GSC structure which satisfy the symmetric and anti-symmetric constraints are equal to zero. The second fact is that the antisymmetry constraint matrix  $\mathbf{C}_a$  is one of the possible signal blocking matrices of the symmetric part.

Now by considering each branch in Figure 3 separately, both  $\mathbf{w}_{\perp s}$  and  $\mathbf{w}_{\perp a}$  can also be divided into their symmetric and anti-symmetric parts. Continuous split process will lead to multi-split filter [7]. Figure 4 shows the multi-split adaptive filter. This scheme can be viewed as a linear transformation of  $\mathbf{x}(n)$  denoted by

$$\mathbf{x}_{\perp}(n) = \mathbf{T}^T \mathbf{x}(n) \quad (9)$$

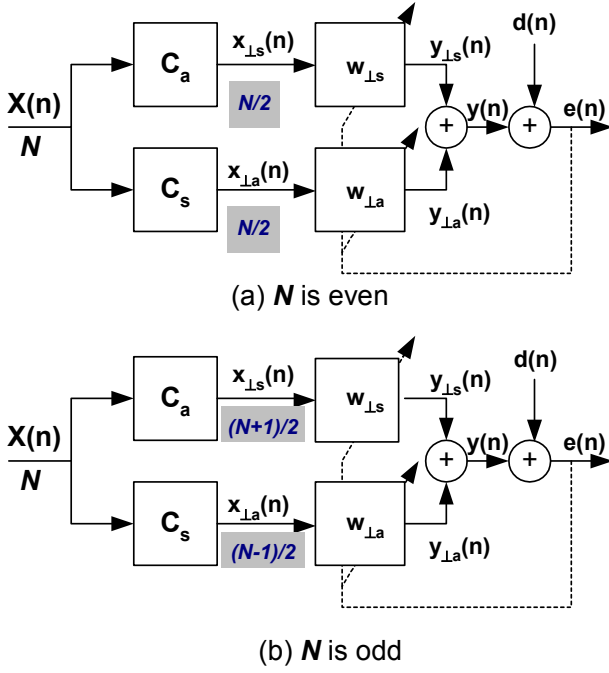


Fig. 3. GSC implementation of the split filter

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{C}_{aM}^T & \mathbf{C}_{aM-1}^T & \cdots & \mathbf{C}_{a1}^T \\ \mathbf{C}_{sM}^T & \mathbf{C}_{sM-1}^T & \cdots & \mathbf{C}_{s1}^T \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{C}_{sM}^T & \mathbf{C}_{sM-1}^T & \cdots & \mathbf{C}_{s1}^T \end{bmatrix}_{N \times N} \quad (10)$$

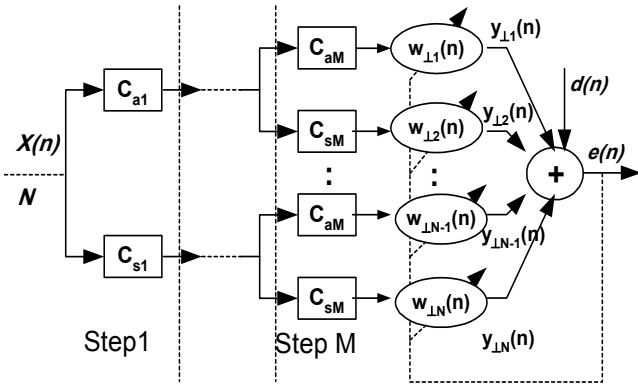


Fig. 4. Multi-split adaptive filter

In the case of  $N=2^M$ , which is a special case,  $\mathbf{T}$  consists of +1 and -1 giving a well-known matrix form called the

Hadamard matrix  $\mathbf{H}$ . As a result, a compact form of the multi-split scheme is shown in Figure 5.

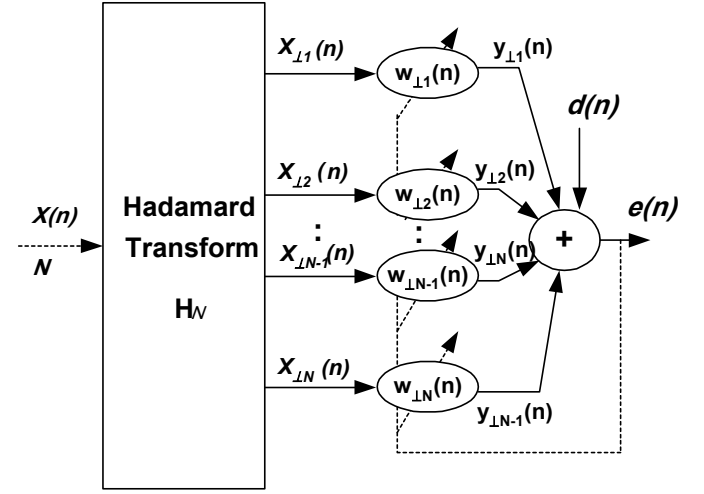


Fig. 5. Hadamard transform of the input  $x(n)$

The use of Hadamard is possible if and only if the input vector  $\mathbf{x}(n)$  is a power of two ( $N=2^M$ ). If this is not the case,  $\mathbf{T}$  should be used instead of  $\mathbf{H}$ .

In this paper the principle of multi-split filtering is applied on both LTE and DFE. The MS-LMS and MS-RLS algorithms are used to update the tap coefficients.

### III. SIMULATION RESULTS

Both MS-LTE and MS-DFE are simulated. The adaptive equalizer scheme used in the simulation is shown in Figure 6. BPSK is used to form the input sequence, which is  $\pm 1$ . A total of 12 taps in both LTE and DFE are used. In the DFE, 8 taps are used for the forward filter and 4 taps for the feedback filter in order to use Hadamard transform since the taps are power of 2.

The channel impulse response is described by the raised cosine:

$$h(n) = \begin{cases} \frac{1}{2} (1 + \cos(\frac{2\pi}{S}(n-2))), & n = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

where  $S$  controls the eigenvalue spread  $\chi(\mathbf{R})$  of the correlation matrix of the input vector to the equalizer. Two eigenvalue spread are used in simulations,  $\chi(\mathbf{R}) = 6.0782$  for  $S=2.9$  and  $\chi(\mathbf{R}) = 46.8216$  for  $S = 3.5$ . However, only those results corresponding to  $\chi(\mathbf{R}) = 46.8216$  are shown. The noise  $z(n)$  is additive white Gaussian noise with zero mean and variance equal to 0.001.

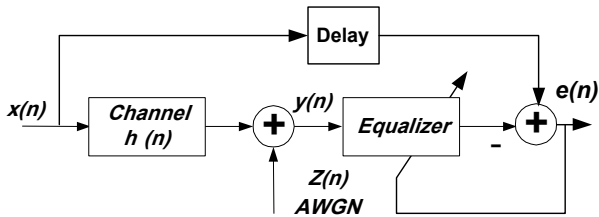


Fig. 6. Adaptive equalizer

Figures 7 and 8 show the results for MSE obtained over 300 independent trials. The figures show the learning curves of LTE-LMS (RLS/DCT-LMS), MS-LTE-LMS (RLS), DFE-LMS (RLS/DCT-LMS) and MS-DFE-LMS (RLS) for eigenvalue spreads  $\chi(\mathbf{R}) = 46.8216$ . It can be noted that the performance of DFE is better than the LTE in term of convergence rate regardless the algorithm type. In both types of equalizers (DFE & LTE) the MS-LMS gives better result than the standard LMS algorithm.

An interesting observation is that in case of LTE, the MS-LMS is somewhat sensitive to variations in eigenvalue spread more than the DCT-LMS and this was also noted in [7]. This explains the good performance of DCT-LMS over MS-LMS. However, this is not the case with the DFE. Both MS-LMS and DCT-LMS give the same performance and same rate of convergence when the eigenvalue spread is low. Figure 9 shows the learning curves of MS-DFE-LMS, DFE-DCT-LMS and DFE-LMS for the same step size, which is  $1/2N$  and  $\chi(\mathbf{R}) = 6.0782$ . This means, that at least for this channel, we get the same performance with less computation by using DFE-MS-LMS since the computational complexity in DCT-LMS is more than that in MS-LMS. However, as the eigenvalue spread increases, the situation is different in which the performance of DCT-DFE-LMS is better as it can be noted in Figure 10. In this figure, the step size of all curves is the same ( $1/2N$ ) and  $\chi(\mathbf{R}) = 46.8216$ . In Figure 11, the DFE-DCT-LMS learning curve was brought to the same level of MS-DFE-LMS by using a step size of  $1/3N$ . It can be seen that the performance for both are close.

In the case of RLS algorithm, we did not gain much by doing multi-split. In both type of equalizer, RLS and MS-RLS gave the same rate of convergence and performance. Figure 12 shows the BER performance of the different structure and algorithms and for  $\chi(\mathbf{R}) = 46.8216$ .

In the previous simulation results, both forward and feedback filters were fully "splitted" in the case of MS-DFE-LMS and MS-DFE-RLS. However, the performance of MS-DFE-LMS in which the feedback filter wasn't "splitted" was tested and the performance was almost identical in both convergence and BER, which means that complexity is reduced.

#### IV. CONCLUSIONS

In this paper, the idea of multi-split adaptive filter was applied on both LTE and DFE. In each equalizer, the FIR

filters (forward and feedback in the case of DFE) were divided into parallel sub-filters by imposing separately the symmetry and antisymmetry conditions on the impulse responses of filters by using appropriate and distinct sets of linear constraints. The purpose of this is to obtain a new structure that gives better performance than the conventional structure. Multi-splitting resulted in MS-LTE and MS-DFE. The algorithms used in these new structure and MS-LMS and MS-RLS algorithms were used to update the tap coefficients of both MS-LTE and MS-DFE. In both types of equalizers (DFE & LTE) the MS-LMS gave better result than the standard LMS algorithm. On the other hand, MS-RLS did not give any improvement over standard RLS. Finally, for the raised cosine channel the same performance of DFE-DCT-LMS was obtained using MS-DFE-LMS with lower computation complexity. This is valid only for low eigenvalue spread. However, this is not the case with LTE. Also, the performance of MS-DFE-LMS in which the feedback filter wasn't "splitted" was almost the same as if the feedback filter was "splitted".

#### ACKNOWLEDGEMENT

The authors acknowledge the support of KFUPM.

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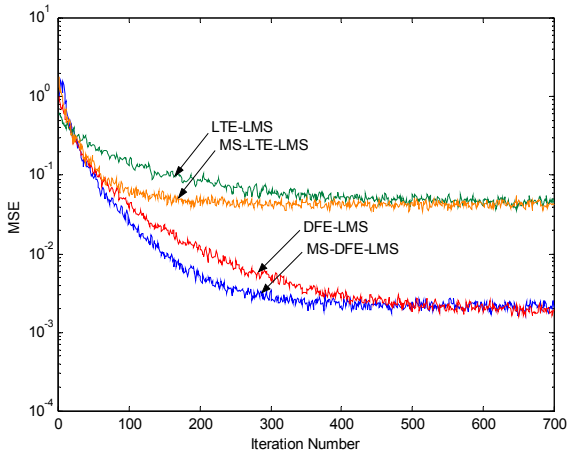


Fig. 7. Learning curves for different algorithms and  $\chi(R) = 46.8216$  for the same steady state error.

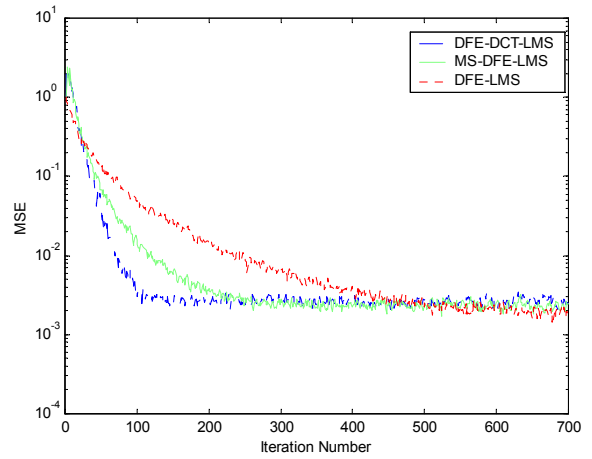


Fig. 10. Learning curves of MS-DFE-LMS, DFE-DCT-LMS and DFE-LMS for the same step size  $(1/2N)$  and  $\chi(R) = 46.8216$ .

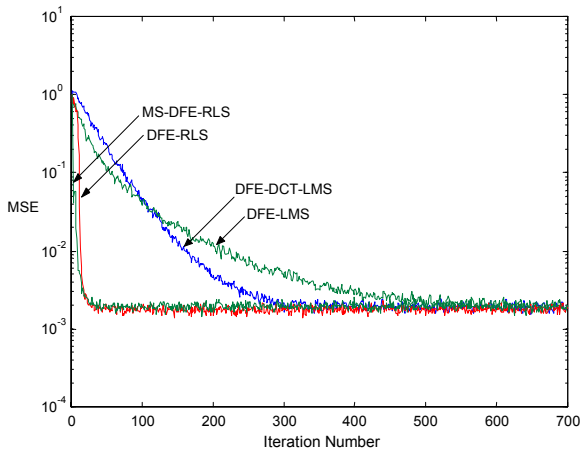


Fig. 8. Learning curves for different algorithms and  $\chi(R) = 46.8216$  for the same steady state error.

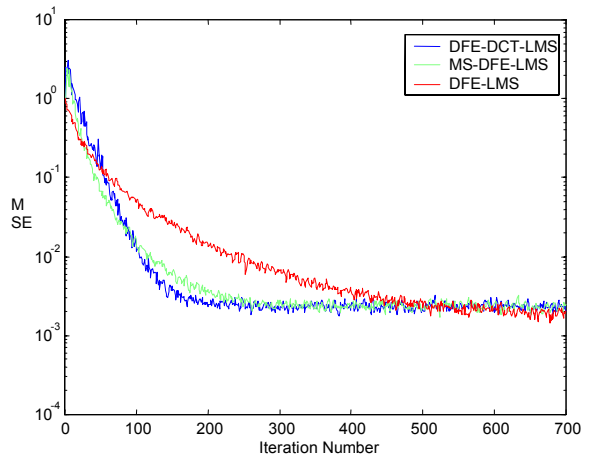


Fig. 11. Learning curves of MS-DFE-LMS and DFE-LMS for step size equal  $(1/2N)$  and DFE-DCT-LMS for step size equal  $(1/3N)$  and  $\chi(R) = 46.8216$ .

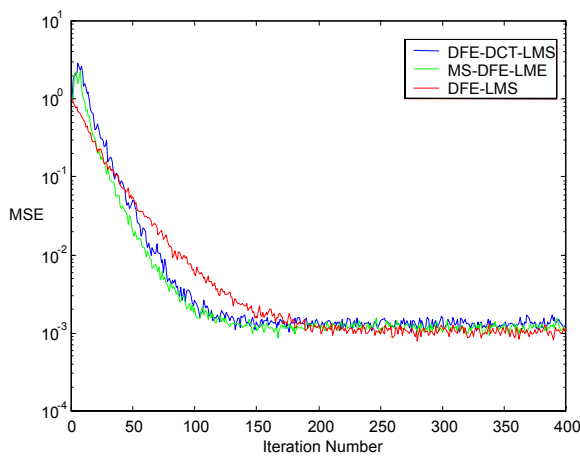


Fig. 9. Learning curves of MS-DFE-LMS, DFE-DCT-LMS and DFE-LMS for the same step size  $(1/2N)$  and  $\chi(R) = 6.0782$ .

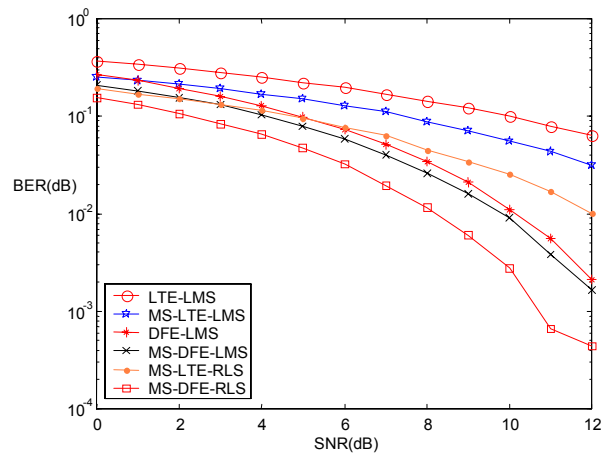


Fig. 12. BER Performance for different algorithms and  $\chi(R) = 46.8216$ .