Charging of Spherical Particles in Unipolar Ionized Electric Fields

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Abstract: - This paper is aimed at calculating the electric field inside and outside a spherical particle positioned in a uniform dc electric field. This makes it possible to assess the polarization surface charge. With the flow of corona ions along the dc field, particle charging takes place and the electric field inside and outside the particle charge until the particle is charged to the saturation value. The dependency of the surface charge density due to corona ions on the applied dc field is determined and correlated to the Panthenier and Moreau-Hanot limit.

I. Introduction

Electrical charging of small particles is an essential process in many electrostatic applications [1], such as precipitation of dust, deposition of powers and separation of granular materials. Motion of a charged particle can be easily controlled by the electric force, which depends on the charging level. Therefore, it is desirable to charge the particle to as high value as possible.

Despite some inconveniences–as compared with other techniques, for example, tribocharging – corona charging is preferred in many applications due to its reliability and high charge values. A neutral, dielectric or conducting, particle placed in the ionic flow (generated most often by corona discharge) is bombarded by ions and charged with the same polarity as that of the ions.

This paper is aimed at calculating the electric field inside and outside the particle in absence of the corona ions. Subsequently, the interfacial surface charge (polarization surface charge) is assessed. With the progress of particle charging by corona ions, the electric field inside and outside the particle changes until the particle is charged to the saturation value. The method of charge simulation [1, 2] is used for field calculation before and during particle charging.

II. Method of Analysis

Figure 1 shows a spherical particle of radius R, permittivity ε_2 and ohmic conductivity σ_2 placed in a uniform positive corona ion space-charge of density ρ. A uniform dc electric field E_0 along Z-direction is applied at infinity in the surrounding medium of permitivity ε_1. Thus, the particle is positioned in a medium of conductivity σ_1 (=ρk), where k is the ion mobility (=$1.5\text{x}10^{-4} m^2/s.V.$).

The electric field E_0 is produced by two infinite plates; one is charged positively with a density (=ε_1 E_0) and the other with a negative charge density (=-ε_1 E_0). The spacing (L) between the two sheets is infinitely large (one hundred times the particle radius). Thus, the positive and negative plates are positioned at z = -L/2 and z = +L/2 while the particle is at z = 0. The respective potentials of the plates are $+V_0/2$ and $-V_0/2$ where $V_0 = E_0L$.

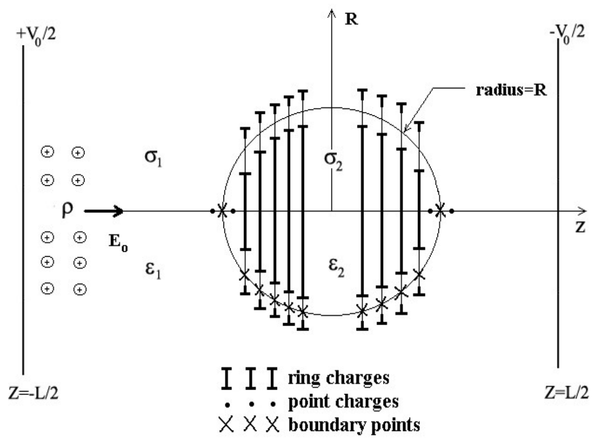

Fig.1. Charge representation of a particle in a monopolar ionized field.

II.1. Particle Before Charging

The electric field is responsible for polarization charge in the particle which is more or less conducting, the negative move against electric field and positive ones with the electric field. The polarization surface charge on the spherical particle is simulated by two sets of ring charges, with centers along Z-axis. Fig.1. One set of unknown ring charges q_j, j = 1, 2.….N inside the particle, the other set is outside the particle with unknown ring charges q_j, j = N + 1, N + 2, ….2N. The Z-coordinates of the ring charges inside the particle are the same as those of the rings outside the particle. The radius of the inner rings is a fraction β of the particle radius at the same Z-level. However, the radius of the outer ring is γ times the particle radius at the same Z-level with γ > 1. In addition to the ring charges, two sets of point charges are positioned on the Z-axis as shown in Fig. 1. Each set has two point charges one inside the particle and the other outside the particle. Thus, the total number of unknowns is 2N + 4.

To determine the values of the unknown charges q_j, j = 1, 2, ….2N + 4, N boundary points are selected around the particle surface. The boundary points have the same Z-coordinates as those of the simulation ring charges, Fig. 1. This is in addition to two boundary points on the Z-axis as shown in Fig. 1. Thus, the total number of boundary points is N + 2.

At each boundary point, two boundary conditions are satisfied, namely; the continuity of potential (Dirichlet condition) and the continuity of the electric flux.

$$\varphi_1 (r_b, z_b) = \varphi_2 (r_b, z_b) \qquad (1)$$

The potential φ_1 (r_b, z_b) at the point (r_b, z_b) is the algebraic sum of potentials at the point due to the charged plates and the inner simulation charges (N rings and 2 point charges) if the point is seen from the medium side. If the boundary point is seen from the particle side, the potential φ_2 (r_b, z_b) is the algebraic sum of the potentials due to the charged plates and the outer simulation charges (N rings and 2 point charges). The potential at the boundary point due to the applied field E_o is expressed as:

$$\phi (r_b, z_b) = V_0 z_b / L \qquad (2)$$

The continuity of the electric flux at a boundary point (r_b, z_b) is expressed as:

$$\varepsilon_1 E_{n1} (r_b, z_b) = \varepsilon_2 E_{n2} (r_b, z_b) \qquad (3)$$

If the point is seen from the medium side, the component E_{n1} (r_b, z_b) of the electric field normal to the particle surface at the point (r_b, z_b) is the vectorial sum of the normal component of the applied field E_0 and the normal field components due to the inner simulation charges, all calculated at this point.

If the boundary point is seen from the particle size, the normal component E_{n2} (r_b, z_b) of the electric field is the vectorial sum of the normal field component of the applied field E_0 and the normal field components due to the outer simulation charges, all calculated at this point.

Applying the two boundary conditions, namely; the continuity of potential and electric flux, at all boundary points (of number = 2N + 2) formulates 2N + 4 equations into 2N + 4 unknowns. Simultaneous solution of these equations determines the unknown simulation charges. Once the unknown charges are determined, the electric field can be calculated inside and outside the particle. The interface charge or the polarization surface charge σ_{sp} at a point (r_b, z_b) on the particle surface is determined as equal to the change of the normal electric flux at the point when seen from the medium and the particle sides, i.e.

$$\sigma_{sp} = \varepsilon_2 E_{n2} (r_b, z_b) - \varepsilon_1 E_{n1} (r_b, z_b) \qquad (4)$$

where ε_1 and ε_2 are, respectively, the permittivity of the surrounding medium and the particle.

II.2. Particle During Charging

With the flow of the space charge along the applied field, the particle is charged and the boundary conditions determining the unknown charges are different from those before particle charging. The Dirichlet boundary condition is still valid. The continuity condition of electric flux is substituted by the continuity of the current density at each boundary point.

The continuity of the current density at a boundary point (r_b, z_b) is expressed as:

$$J_{n1} = J_{n2}$$

Hence; $$\sigma_1 E_{n1} (r_b, z_b) = \sigma_2 E_{n2} (r_b, z_b) \qquad (5)$$

where σ_1 and σ_2 are respectively the conductivity of the surrounding medium and the particle. J_{n1} and J_{n2} are the current density at the boundary point when seen from the medium and particle sides.

Corresponding to E_{n1} and E_{n1}, the electric-flux density values are:

$$D_{n1} = \varepsilon_1 E_{n1} = \varepsilon_1 \frac{J_{n1}}{\sigma_1}$$

$$D_{n2} = \varepsilon_2 E_{n2} = \varepsilon_2 \frac{J_{n2}}{\sigma_2} = \varepsilon_2 \frac{J_{n1}}{\sigma_2}$$

The surface charge density

$$\sigma_s = D_{n2} - D_{n1} = J_{n1}\left(\frac{\varepsilon_2}{\sigma_2} - \frac{\varepsilon_1}{\sigma_1}\right) = \sigma_1 E_{n1}\left(\frac{\varepsilon_2}{\sigma_2} - \frac{\varepsilon_1}{\sigma_1}\right) \qquad (6)$$

The surface charge density σ_{sc} due to particle charging by corona ions is expressed as

$$\sigma_{sc} = \sigma_s - \sigma_{sp} \qquad (7)$$

III. Results and Discussion

For an applied field E_o of 1 V/m, Fig.2 shows how the electric field E_2 inside the particle decreases with the increase of the particle conductivity σ_2 for the same conductivity value of the surrounding medium (σ_1 =10^{-9}Ù$^{-1}$.m^{-1}). The field E_2 approaches the zero value at high σ_2 values (σ_2 10^{-7}Ù$^{-1}$.m^{-1}) where the particle behaves as a conducting sphere in a uniform field. On the other hand, the value of E_2 approaches the limiting value of 1.5 times the applied field E_o at low σ_2 values (σ_2 10^{-12}Ù$^{-1}$.m^{-1}). This conforms with the field values of the case of a perfect insulating sphere positioned in a uniform field.

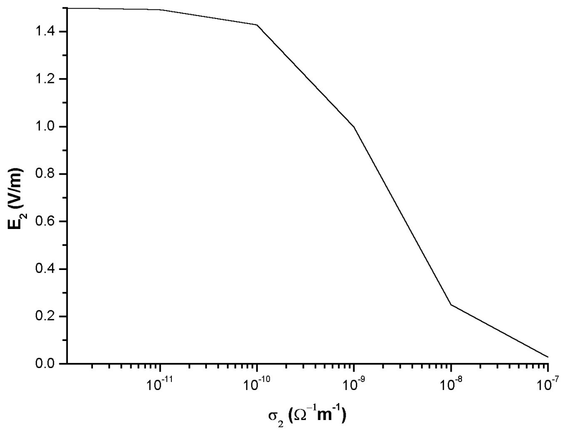

Fig. 2. Electric field E_2 inside the particle against the particle conductivity σ_2 for the same conductivity value of the surrounding medium (σ_1 = 10^{-9} Ω^{-1}.m^{-1}, E_0 = 1 V/m).

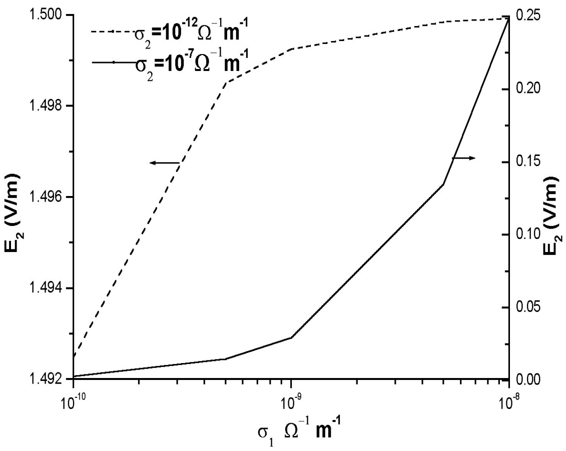

Fig. 3. Electric field E_2 inside the particle against the conductivity σ_1 of the surrounding medium for two different values of the particle conductivity σ_2 (E_0 = 1 V/m)

For an applied field E_o of 1 V/m, Fig.3 shows how the electric field E_2 inside the particle depends on the conductivity σ_1 of the surrounding medium for two different values of the particle conductivity σ_2 (10^{-12} and 10^{-7}Ù$^{-1}$.m^{-1}). As long as $\sigma_2 << \sigma_1$, the particle behaves a an insulating sphere and the field E_2 inside the particle approaches the limiting value 1.5 E_o. This conforms with Fig.3 for σ_2=10^{-12} Ù$^{-1}$.m^{-1}, where the value of E_2 increases from 1.492 E_o to reach the limiting value 1.5 E_o as the ratio σ_2/σ_1, decreases from 10^{-2} to 10^{-4}. On the other hand, the value of E_2 approaches the zero value when $\sigma_2 >> \sigma_1$, where the particle behaves as a conducting sphere. This conforms with Fig.3 for σ_2 =10^{-7}Ù$^{-1}$.m^{-1}, where the value of E_2 decreases from 0.25 to about 0.01 as the ratio σ_2 /σ_1, increases from 10 to 1000.

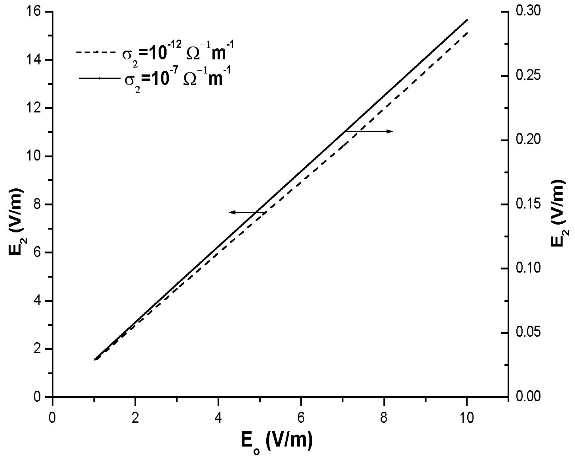

Fig. 4. Dependency of the electric field E_2 inside the particle on the applied field E_0 for two different values of the particle conductivity σ_2 (σ_1 = 10^{-9} Ω^{-1}.m^{-1}).

For a conductivity value σ_1 of 10^{-9}Ù$^{-1}$.m^{-1}, Fig.4 shows how the electric field E_2 inside the particle depends on the applied field E_o for two different values of the particle conductivity σ_2 (10^{-12} and 10^{-7}Ù$^{-1}$.m^{-1}). For σ_2 =10^{-12}Ù$^{-1}$.m^{-1}, the ratio σ_2 /σ_1 equals 10^{-3} and the particle behaves as an insulating sphere where the field E_2 is1.5 times E_o . This explains the linear relationship shown in Fig.4 between E_2 and E_o with slope equals 1.5 for σ_2 = 10^{-12}Ù$^{-1}$.m^{-1}, On the other hand, the ratio σ_2 /σ_1 equals 10^2 for σ_2 =10^{-7} Ù$^{-1}$.m^{-1}and the particle behaves approximately as a conducting sphere where the field E_2 inside the particle assumes low values with respect to applied field E_o . Therefore, the linear relationship between E_2 and E_o for σ_2 = 10^{-7}Ù$^{-1}$.m^{-1} dictates a value of E_o around 0.29 V/m against 10 V/m for the applied field E_o, Fig.4.

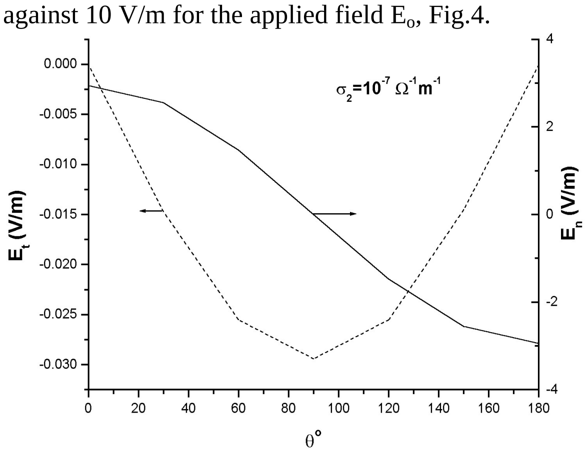

Fig. 5a. Variation of the normal E_n and tangential E_t components of the electric field over the particle surface (0 ≤ è ≤ 180°) (σ_1 = 10^{-9} Ω^{-1}.m^{-1}, E_0 = 1 V/m).

For a conductivity value σ_1 of 10^{-9}Ù$^{-1}$.m^{-1} of the surrounding medium, Fig.5a shows the variation of the normal and tangential components of the field E_1 over the surface of the particle with conductivity σ_2 =10^{-7}Ù$^{-1}$.m^{-1}. A conducting sphere positioned in a uniform field E_o distorts the field which reaches a limiting value of 3 E_o where the field meets the sphere, Fig.1. At è=0, the normal component E_n of the surface field E_1 is equal to $3E_o$ providing that E_o extends along the Z-axis, Fig.1. At è= ð, E_n = $-3E_o$. Subsequently, the tangential component E_t of the surface field E_1 is zero at è=0 and è= ð. With the decreases of è, the normal component E_n decrease from the value 3 E_o reaching the zero value at è= ð/2 and $-3E_o$ at è= ð. This conforms with Fig.5a where the normal component E_n changes between the limits $\pm 2.9E_o$,with E_o =1V/m.

On the contrary, the tangential field E_t increases with the increase of è starting from è=0 reaching its maximum value at è= ð/2. As the particle behaves as a conducting particle, the maximum value of the tangential field is small with respect to the maximum value of the normal component. This is quite clear from the respective values of the normal and tangential field components of Fig.5a.

For a conductivity value σ_1 of 10^{-9}Ù$^{-1}$.m^{-1} of the surrounding medium, Fig.5b shows the variation of the normal E_n and tangential E_t components of the field E1 over the surface of a particle with conductivity σ_2 =10^{-12}Ù$^{-1}$.m^{-1}. As $\sigma_2 << \sigma_1$, the particle behaves oppositely to that of the conducting sphere and the maximum value of the tangential field is significantly high with respect to the maximum value of the normal component. However, the trend of variation of the normal and tangential components of the field over the particle surface as shown in fig. 5b is the same as that for the conducting particle in Fig. 5a.

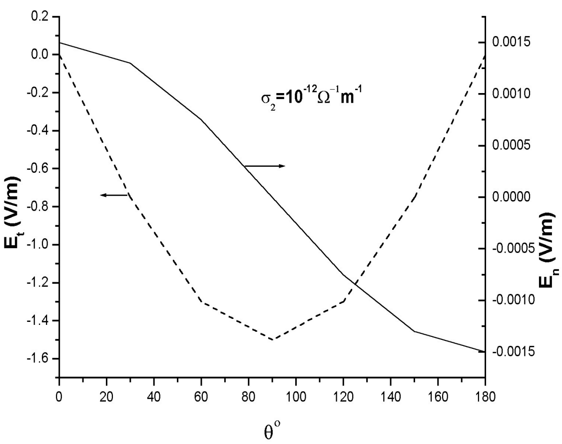

Fig. 5b. Variation of the normal E_n and tangential E_t components of the electric field over the particle surface ($0 \leq \theta \leq 180°$) ($\sigma_1 = 10^{-9}\ \Omega^{-1}.m^{-1}$, E_0 = 1 V/m).

For an applied field E_o of 1V/m and conductivity value σ_1 of 10^{-9}Ù$^{-1}$.m^{-1}, Fig.6 shows the distribution of the polarization charge density σ_{sp} over the particle surface for two different values of the particle conductivity σ_2 (10^{-12} and 10^{-7}Ù$^{-1}$.m^{-1}). The surface charge density σ_{sp} depends not only on the conductivity values σ_2 and σ_1 of the particle and the surrounding medium, but also on the respective values of the permittivity å$_2$ and å$_1$. In Fig.6, å$_1$ is that of the surrounding gas (=å$_o$) and å$_2$ of the particle permitivity is 3å$_o$, where å$_o$ is the permittivity of free space. For σ_2 =10^{-7}Ù$^{-1}$.m^{-1}, the particle behaves as a conducting sphere and a surface charge appears on the particle to account for diminishing the field inside. The surface charge is of negative sign on the incident part of the sphere and of positive sign on the backward part. The surface charge density σ_{sp} changes from maximum positive at è=0 to maximum negative at è= ð.The charge density σ_{sp} takes zero value at è= ±ð/2 as shown in Fig.6.It is worthy to mention that total charge on the entire particle is zero.

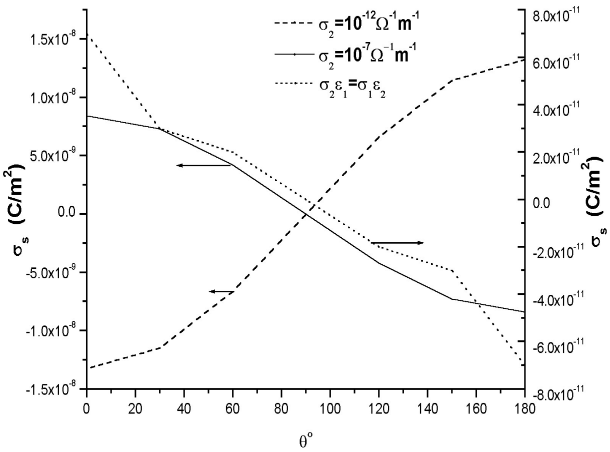

Fig. 6. Variation of the interfacial surface charge density σ_s over the particle surface ($0 \leq \theta \leq 180°$) for two different values of the particle conductivity σ_2 and for the condition $\sigma_2\ \varepsilon_1 = \sigma_1\ \varepsilon_2$ ($\sigma_1 = 10^{-9}\ \Omega^{-1}.m^{-1}$, E_0 = 1 V/m).

For σ_2 =10^{-12}Ù$^{-1}$.m^{-1}, the particle behaves as an insulating sphere and an interfacial surface charge appears to conduct the electric field E_2 inside the particle. The interfacial charge is of positive sign on the incident part of the sphere and of negative sign the backward part. The interfacial charge density changes from maximum negative at è=0 to maximum positive at è= ð with a zero value at è= ±ð/2 as shown in Fig.6. Also, the total charge on the entire particle is zero, the same as the case of a conducting particle of σ_2 =10^{-7}Ù$^{-1}$.m^{-1}. Fig.6 shows also the surface charge density σ_{sp} over the sphere for the condition $\varepsilon_1\sigma_2 = \varepsilon_2\ \sigma_1$ ($\varepsilon_1 = \varepsilon_o$ and $\varepsilon_2=100\varepsilon_o$; σ_1=10^{-9}Ù$^{-1}$.m^{-1} and σ_2=10^{-7}Ù$^{-1}$.m^{-1}), which is negligible in comparison with the discussed case of

$\sigma_2 = 10^{-7}$Ù$^{-1}$.m^{-1}. This condition corresponds to equality of the relaxation time in both the particle and the surrounding medium, where there is no chance for surface charge to appear in conformity with the charge density values reported in Fig.6 for the case of $\sigma_2 = 10^{-7}$Ù$^{-1}$.m^{-1}. Theoretically speaking, the charge density at the condition $\varepsilon_1\sigma_2 = \varepsilon_2\ \sigma_1$ is zero as depicted by eqn. (6).

The positive ion flow of density ñ ($6.6x10^{-6}$ C/m^3) takes place along the field lines that start far away (at Z =-L/2, Fig.1) and approach the particle to charge it where the radial electric field is negative. The charge density over the particle surface is expressed by eqn.(6). For an interfacial charge to appear on the particle surface under the conditions: $\varepsilon_1 = \varepsilon_o$, $\sigma_1=10^{-9}$Ù$^{-1}$.m^{-1}, $\sigma_2 = 10^{-7}$Ù$^{-1}$.m^{-1}, the permitivity ε_2 of the particle should exceed $100\varepsilon_o$. The charge density σ_{sc} due to corona ion charging calculated according to eqn. (7) vary around the particle as shown in Fig.7 for $\varepsilon_2 = 1000\varepsilon_1$ and two different values of the applied field E_o. It is quite clear that the charge density σ_{sc} is almost constant around the particle and increases linearly with the value of the applied field. The almost uniform distribution of the charge (σ_{sc} is almost constant) over the particle surface, Fig 7, is explained by the fact that the particle behaves as a conducting sphere. This is because the particle conductivity σ_2 is much larger than σ_1 of the surrounding medium and the particle permitivity is1000 times ε_o of the surrounding medium. Therefore, the charge received by the particle from the ions distributes itself uniformly on the surface and increases with the applied field up to the saturation value Q_{max} calculated by Pauthenier and Moreau-Hanot classical formula [5]:

$$Q_{max} = 12\ \text{ð}\ \varepsilon_1 E_o\ R^2 \qquad (8)$$

where R is the particle radius**.**

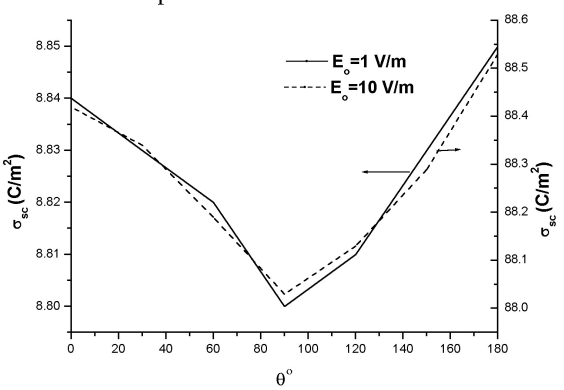

Fig. 7 Variation of the charge density σ_{sc} due to corona charging over the particle surface (0 è 180 °) for two different values of the applied field E_o. ($\sigma_1=10^{-9}$Ù$^{-1}$.m^{-1}, $\sigma_2 = 10^{-7}$Ù$^{-1}$ m^{-1}, $\varepsilon_1 = \varepsilon_o$ and $\varepsilon_2 = 1000\ \varepsilon_o$)

IV. Conclusions

On the basis of the present analysis, one may draw the following conclusions for spherical particles:

(1) In absence of corona ions, the conductivity value of the particle determines how high is the electric field inside the particle. For perfect insulating particles, the inside field reaches the limit of about 1.5 times the applied dc field. On the other hand, the inside field diminishes down to zero for conducting particles.

(2) In absence of corona ions, the conductivity value σ_1 of the surrounding medium also determines the field inside the particle. For $\sigma_2 >> \sigma_1$, the particle behaves as a conducting sphere. On the other hand, the particle behaves as an insulating sphere for $\sigma_2 << \sigma_1$.

(3) In absence of corona ions, the field inside the particle is directly related to the applied dc field irrespective of the value of the particle conductivity.

(4) In absence of corona ions, the normal component of the field over the surface of a conducting particle is dominating with respect to the tangential component. On the contrary, the tangential component for an insulating particle is dominating with respect to the normal component.

(5) With particle charging by corona ions, the charge density distribution over the particle surface is almost uniform for conducting particles. The value of the charge density increases linearly with the value of the applied dc field.

V. References

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