Iterative Spatial Sequence Estimator for
Multi-Group Space Time Trellis Coded Systems

Samir Al-Ghadhban, Maruf Mohammad, and B. Woerner
Mobile and Portable Radio Research Group
Virginia Tech
E-Mail: {maruf, samir}@vt.edu

Abstract: **In this paper, we will present and evaluate a new detection algorithm for decoding multi-group space time trellis coded systems. These systems consider a single user who transmits simultaneously through *K* parallel space time trellis encoders. Therefore, they can provide high spectral efficiencies similar to the BLAST architecture plus transmit diversity advantages and coding gains. The developed detector is called maximum a posteriori spatial sequence estimator and it has the flexibility of trading complexity with diversity advantage. The algorithm could provide higher receive diversity with higher complexity. The simulation result shows a performance improvement of this new algorithm over interference nulling and cancellation techniques.**

keywords- Multi-layered STTC; Spatial multiplexing and transmit diversity; SSE

I. INTRODUCTION

Multiple-input multiple-output (MIMO) fading channels can boost up information capacities by order of magnitudes. Communication schemes over MIMO channels can be classified under two categories: spatial multiplexing and transmit diversity. V-BLAST is a spatial multiplexing scheme that provides full multiplexing gain and no transmit diversity gain. On the other hand, space time coded schemes provide full transmit diversity and no multiplexing gains. The trade off between spatial multiplexing and diversity is studied in [1]. In this paper, we will consider a high data rate scheme that combines spatial multiplexing, transmit diversity and coding gains [2][3]. We will refer to it as multi-group space time trellis coded (MGSTTC) system. For *K* groups and full-rate full-rank STTC, the rate of this scheme will be *K* symbols/sec with a transmit diversity order equals to the transmit diversity of each STTC.

Joint and group nulling and interference cancellation algorithms for MGSTTC systems were studied in [2][3]. The algorithms differ in their complexity, error propagation and receive diversity order. Joint detectors provide full receive diversity and no error propagation but with exponential complexity. On the other hand, group nulling and interference cancellation algorithms have cubic complexity per group but suffer from error propagation, due to cancellation, and reduced receive diversity, due to nulling.

A novel spatial sequence estimator (SSE) for V-BLAST is proposed in [4]. The algorithm combines group interference nulling and joint detection. It has the flexibility to tradeoff complexity with receive diversity order and it doesn't suffer from error propagations. Also, it can work with number of receive antennas fewer than what V-BLAST algorithm requires. By applying reduced state sequence estimation, complexity can be further reduced with little performance degradation [4].

In this paper, we will apply SSE with soft-input soft-output (SISO) maximum a posteriori (MAP) algorithm as the detection stage in the multi-group receiver. It will iterate and share soft information with the soft decoding stage.

The remainder of this paper is organized as follow. The multi-group system model is described in Section 2. A brief overview of previous decoding algorithms for MGSTTC systems is presented in Section 3. The MAP-SSE algorithm is described in Section 4. Simulation results and discussion are provided in Section 5.

II. SYSTEM MODEL

A. Encoder

The encoder is divided into K parallel groups $\{G_i\text{: } i=1, 2, \ldots, K\}$. Each group is a space time trellis encoder (STTE) with n_i transmit antennas. The total number of transmit antennas is N_T. The input stream is divided into K blocks $\{B_i\text{: } i=1, 2, \ldots, K\}$. Each G_i encodes B_i bits into the STTC (C_i) which will be transmitted through n_i antennas. All groups operate simultaneously and the transmission is assumed synchronous. Figure 1 shows the system architecture. The n_i transmitted symbols from G_i at time t are represented by the column vector $\mathbf{x}_i^t = \left([s_i[1], s_i[2], \ldots, s_i[n_i]]\right)^T$, where $(\bullet)^T$ denotes the transpose of the original quantity and $s_i[n]$ is the symbol transmitted from group i at antenna n. All the symbols are drawn from the signal set $\mathcal{A}$ with $M = |\mathcal{A}|$. The transmitted vector from all groups at time t will be:

$$\mathbf{x}^t = \left[(\mathbf{x}_1^t)^T \quad \cdots \quad (\mathbf{x}_K^t)^T \right] \tag{1}$$

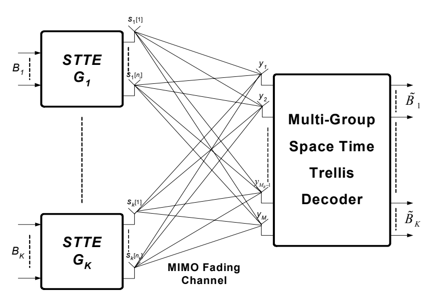

Figure 1: Block diagram of the multi-group space time trellis coded system

B. *Discrete Channel Model*

The channel is an $N_R \times N_T$ MIMO flat fading channel, where N_R is the number of receive antennas. Each path is a quasi-static fading channel for which the fading coefficient is constant over one frame and independent from one frame to another. Each coefficient is a complex Gaussian random variable with mean zero and variance 0.5 per dimension. It has a Rayleigh distributed envelope and a uniformly distributed phase. Let h_{mn} be the path fade from antenna n to antenna m, where $n=1,2,\ldots,N_T$ and $m=1,2,\ldots,N_R$. The channel coefficient matrix is given by

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1n_1} & h_{1(n_1+1)} & \cdots & h_{1N_T} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R n_1} & h_{N_R(n_1+1)} & \cdots & h_{N_R N_T} \end{bmatrix}_{N_R \times N_T} \tag{2}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_K \end{bmatrix}_{N_R \times N_T}$$

where $\mathbf{H}_i$ is the channel coefficient matrix of the MIMO channel from group G_i to the receiver.
The received vector at time t is:

$$\mathbf{y}^t = \mathbf{H}\mathbf{x}^t + \boldsymbol{\eta}^t \tag{3}$$

where $\boldsymbol{\eta}^t$ is the additive white Gaussian noise (AWGN) vector. Each element has a zero mean and a variance of $N_0/2$ per dimension.

III. DECODING ALGORITHMS FOR MGSTTC SYSTEMS

Several decoding algorithms for MGSTTC systems were proposed in [2][3]. The multi-group receivers were classified under two categories: joint detection/decoding and group interference nulling/cancellation (GINC).

The optimum joint space time trellis detector (OJSTTD) is based on constructing the super trellis that describes the behavior of the K parallel groups. In order to simplify the huge exponential complexity of the optimum detector, the receiver was broken into two stages: detection followed by a decoding stage. Soft information can be shared iteratively between the two stages. This suboptimum algorithm used a MAP detector followed by a MAP-STTD for each group. The joint detectors provide full receive diversity advantages and don't suffer from error propagation. Although the decoding stage has linear complexity per group, the detection stage is still exponential.

Less complex receivers were based on serial and parallel group interference nulling/cancellation algorithms (SGINC and PGINC). The serial processing, which was a generalization of the V-BLAST algorithm, was first proposed in [2]. The serial algorithm starts by nulling out all interferers expect the first group. Then the first group will be decoded by the STTD. After that, its contribution will be subtracted from the received vector (interference cancellation). Then, the interference nulling and cancellation will be repeated serially. The nulling operation has cubic complexity per group while the decoding and cancellation has linear complexity per group. However, it suffers from error propagation and unequal receive diversity advantage for each group. To overcome these drawbacks, we applied in [3] an iterative parallel processing. The iteration should reduce the error propagation effects and the parallel processing should theoretically provide full receive diversity per group. However, in practice, it never achieves full receive diversity and the error propagation severely degrades performance [3].

IV. MAP-SSE ALGORITHM

To avoid cancellation and keep the advantages of joint detection, a novel spatial sequence estimation (SSE) based algorithm is developed in [3]. A SISO version based on MAP algorithm is developed in this paper. The MAP-SSE will replace the MAP-detector stage in the suboptimum iterative joint decoder in [1]. The block diagram for two groups is shown in Figure 2. Before explaining the algorithm, we will state its main advantages. First, it provides the receiver with the flexibility of trading of complexity with receive diversity order and hence the performance. Another advantage is that it doesn't suffer from error propagation. Furthermore, unlike GINC algorithms, it can be applied even when the number of receive antenna is fewer than the number of transmit antenna. Also, the trellis description of the SSE is a tail-biting trellis. So by employing iterative detection within the estimator, further performance improvements are possible.

In this algorithm, a trellis over space is built for different groups of transmit antennas. The grouping of transmit antennas doesn't have to coincide with the groups of the space time encoder. Actually it is independent of the encoder. The trellis formulation is explained as follows, let's consider a group of L transmit antennas with $L \le N_T$. The state at the k-th stage, σ_k, describes all possible values taken on by $\mu = L-1$ transmit antennas. Drawing an analogy to the ISI channel, μ corresponds to the memory of the channel while L is the length of the channel. Since the antenna grouping is independent of the encoder, we will drop the sub-encoder index i in (1) and the time index for simplicity. Thus, the transmitted vector from all antennas will be $\mathbf{x} = \left([s[1], s[2], \ldots, s[N_T]]\right)^T$. The first stage of the trellis may be initialized as

$$\sigma_1 \triangleq \left(s[\mu], \ldots, s[2], s[1]\right), \quad 0 < \mu < N_T \tag{4}$$

The subsequent stages can be derived according to

$$\sigma_{k+1} \triangleq \left(s[t], \sigma_k[1:\mu-1]\right), \quad 1 \le k < N_T \tag{5}$$

where

$$t = \mathrm{mod}(\mu + k - 1, N_T + 1) + \left\lfloor (\mu + k - 1)/(N_T + 1) \right\rfloor \tag{6}$$

Thus, the number of states at each stage is M^{L-1}, where M is the cardinality of the signal set. For successive stages, transmit antennas are grouped in such a way that there is a valid transition between states. Like in any other trellis, for a valid transition from state σ_k to σ_{k+1}, the first $\mu-1$ elements of state σ_k should match the last $\mu-1$ elements of state σ_{k+1}. This is ensured by (5). An example trellis is depicted in Figure 3. Note that the trellis starts and terminates at the same state description, i.e. the trellis wraps around upon itself. This type of trellis is called *tail-biting* trellis.

Once the trellis is formulated, the received signal component corresponding to each stage is constructed by employing group interference nulling (GIN) technique [2]. This technique maintains the desired group of signals while suppresses the effect of interfering signals. At any stage of the trellis, all the transmitting antennas except those present in the group are nulled out.

To illustrate GIN method, let us assume that the first stage, σ_1, is formed according to (5). Since transmitting antennas 1,2,…,L are present in the group, the remaining transmit antennas L+1,L+2,…,N_T are nulled out to calculate the corresponding received vector and modified channel matrix of the first stage.

Let $\mathcal{N}(\sigma_1)$ be the null space of $\mathbf{\Lambda}(\sigma_1)$ where

$$\mathbf{\Lambda}(\sigma_1)_{N_R\times N_T-L} = \begin{bmatrix} h_{1,L+1} & h_{1,L+2} & \cdots & h_{1,N_T} \\ h_{2,L+1} & h_{2,L+2} & \cdots & h_{2,N_T} \\ \cdots & \vdots & \ddots & \vdots \\ h_{N_R,L+1} & h_{N_R,L+2} & \cdots & h_{N_R,N_T} \end{bmatrix} \quad (7)$$

Since the dimension of $\mathcal{N}(\sigma_1)$ is $(N_R - N_T + L)$, an $(N_R - N_T + L)\times N_R$ matrix, $\mathbf{\Theta}(\sigma_1)$, whose rows form a set or orthonormal vectors in $\mathcal{N}(\sigma_1)$ can be obtained. By left multiplying (3) by $\mathbf{\Theta}(\sigma_1)$, the decision statistics for the first stage is found.

$$\tilde{\mathbf{r}}_1 = \mathbf{\Theta}(\sigma_1)\mathbf{y} = \tilde{\mathbf{h}}_1\tilde{\mathbf{a}}_1 + \tilde{\boldsymbol{\eta}}_1 \quad (8)$$

where $\tilde{\mathbf{h}}_1$ is the $(N_R - N_T + L)\times L$ modified channel matrix, $\tilde{\boldsymbol{\eta}}_1$ is the $(N_R - N_T + L)\times 1$ noise vector, and $\tilde{\mathbf{a}}_1$ is the $L\times 1$ transmitted symbol vector, $\tilde{\mathbf{a}}_1 = ([s[1], s[2], \ldots, s[L])^T$. Note that due to nulling, the receive diversity gain is reduced to $(N_R - N_T + L)$. Using the same technique, the received signal components and modified channel matrices for the subsequent stages are found. Since the performance of STTDs and trellis codes in general degrades with hard detection, MAP algorithm similar to the one presented in [5] is built on the SSE to iterate soft information with the STTDs. The MAP-SSE takes in the received vector, channels state information and the priori probabilities from the decoders and calculates new posteriori probabilities of the transmitted symbols. Let the spatial sequence of received signals over the N_T+1 stages be $\mathbf{r} = (\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_2, \cdots, \tilde{\mathbf{r}}_{N_T+1})$, then the branch metric of the state transition from state σ_{k-1} to σ_k is:

$$\gamma(\sigma_{k-1} \to \sigma_k) = \exp\left(\frac{\left\|\tilde{\mathbf{r}}_k - \tilde{\mathbf{h}}_k\tilde{\mathbf{a}}\right\|^2}{N_0}\right)\prod_{l=1}^{L} P_e^{dec}\left[s[l]\right] \quad (9)$$

the forward recursion is:

$$\alpha(\sigma_k) = \sum_{\{\sigma_{k-1}\}} \alpha(\sigma_k)\gamma(\sigma_{k-1} \to \sigma_k) \quad (10)$$

where $\{\sigma_{k-1}\}$ is the set of all states at the k-1 stage which have valid transitions to σ_k. In other words, the summation is performed over all transitions arriving at σ_k.

the backward recursion is:

$$\beta(\sigma_k) = \sum_{\{\sigma_{k+1}\}} \beta(\sigma_{k+1})\gamma(\sigma_k \to \sigma_{k+1}) \quad (11)$$

the summation is performed over all transitions leaving σ_k.

The posteriori probability that the transmitted symbol from antenna n at stage k is equal to signal x is:

$$p_e^{\det}[s_k[n]=x] = C_k \sum_{\{\sigma_{k-1}\to\sigma_k | s_k[n]=x\}} \alpha(\sigma_{k-1})\beta(\sigma_k)\gamma(\sigma_{k-1}\to\sigma_k) \quad (12)$$

these probabilities will be used as the priori probabilities at the MAP-STTD.

At the decoding stage, there are K STTD. The MAP algorithm for the i^{th} STTD has the following branch metric:

$$\gamma(\sigma_t \to \sigma_{t+1}) = P[\sigma_{t+1} | \sigma_t]\prod_{n=1}^{n_i} P_e^{\det}[s_i[n] = x_n \mid \mathbf{Y}] \quad (13)$$

where t=1,2,…,L_c, where L_c is the frame length of the STTC. Also, $\mathbf{Y} = (\mathbf{y}^1, \mathbf{y}^2, \cdots, \mathbf{y}^{L_c})$ is the sequence of received vectors. Furthermore, $P[\sigma_{t+1} | \sigma_t]$ is the message (information input) probability which is assumed to be drawn from i.i.d source. Therefore, $P[\sigma_{t+1} | \sigma_t] = 1/2^{|B_i|}$ (where $|B_i|$ is the number of information bits encoded by the i^{th} group).

Then, new extrinsic probabilities of the symbols transmitted form antenna n of group i at time t is:

$$P_e^{dec}(s_i^t[n]=x) = C_t \sum_{\{\sigma_{t-1}\to\sigma_t | s_i[n]=x\}} \alpha(\sigma_{t-1})\beta(\sigma_t)P[\sigma_t \mid \sigma_{t-1}] \quad (14)$$

where the summation is performed over all valid transitions arriving at σ_t such that the transmitted symbol from antenna n is equal to x.

Furthermore, the a posteriori probabilities of the information bits will be:

$$P(b_i^t = u) = c \sum_{\{\sigma_{t-1}\to\sigma_t | b_i^t = u\}} \alpha(\sigma_{t-1})\gamma(\sigma_{t-1} \to \sigma_t)\beta(\sigma_t) \quad (15)$$

Where the summation is performed aver all state transitions at time t that correspond to a bit u. The new extrinsic probabilities will be fed back to the MAP-SSE.

The design parameter of the proposed algorithm is the number of elements in the antenna group, L, which sets the trade-off between complexity and performance. Also this parameter determines the receive diversity achieved by each space time code. All codes achieve equal receive diversity level of $N_R - N_T + L$. The two extreme cases are

a) full receive diversity of N_R. This corresponds to maximum likelihood detection with $L = N_T$.
b) receive diversity of 2. This is the minimum receive diversity achieved by SSE. Note that the receive diversity of 1 corresponds to Zero Forcing (ZF) based detection.

Cleary as more transmit antennas are grouped to from a state, i.e., L increases, the diversity advantage increases resulting performance improvement. However, the number of states in the trellis, M^{L-1}, and thus the complexity also increases exponentially.

V. SIMULATION RESULTS

To evaluate the performance of the new algorithm, a simulation study is done for two-group STTC system. Each encoder uses an 8-states QPSK STTC [6] with rank two and two transmit antennas. Thus, the transmit diversity of the system is two and the spectral efficiency is 4 bps/Hz. However, the receive diversity depends on the decoding algorithm. Figure 4 shows the performance of the 2-group STTC system with MAP-SSE with different receive diversity order (RxDiv). Recall that the transmit symbols from each antenna is called a layer. Nulling two layers results in a RxDiv=2 and the number of states in the SSE trellis is 4. Furthermore, nulling one layer greatly improves the performance since RxDiv=3 while the number of states will increase to 16, which is still considered low. Since MAP-SSE does not have known states for initialization and termination, most of the errors occur at the trellis edges. By exploiting the tail-biting feature of the trellis, the forward and backward variables in the MAP-SSE can be updated using iteration. We call this iterative spatial sequence estimator (ISSE). Figure 5 shows that 0.5 to 1 dB gain is achieved by ISSE over SSE after one iteration.

The result of comparing the SSE performance with the other multi-group decoders is shown in Figure 6. The result shows that the SSE-STTD algorithm outperforms the group interference nulling/ cancellation (GINC) techniques. That is because the SSE algorithm eliminates the error propagation problem. For instant, the PGINC algorithm should theoretically provide full receive diversity (RxDiv=4). However, due to error propagation, it never achieves that. On the other hand, the ISSE gained around 2.5dB at RxDiv=3. Furthermore, the performance of the SSE is lower bounded by the joint detection/decoding algorithms which correspond to nulling zero layers and providing full receive diversity.

VI. CONCLUSION

In this paper, a new soft-input soft-output spatial sequence estimator is proposed. It is used in the detector stage for decoding multi-group space time trellis coded systems. The multi-group system combines spatial multiplexing (high data rate), transmit diversity (mitigate fading) and coding gains. The spatial sequence estimator algorithm has the flexibility to tradeoff complexity with receive diversity order. Also, it doesn't suffer from error propagation. The algorithm outperforms group interference nulling and cancellation algorithms.

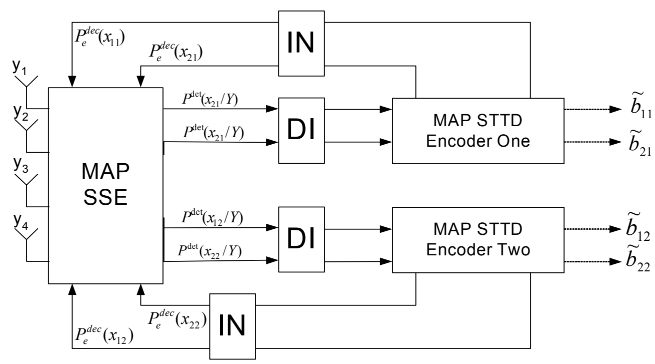

Figure 2: Block diagram of iterative SSE-STTD receiver for 2GSTTC system

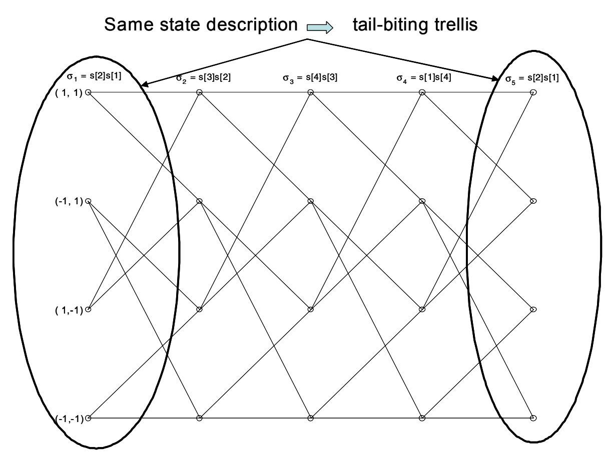

Figure 3: SSE trellis diagram for BPSK, N_T=4, L=3.

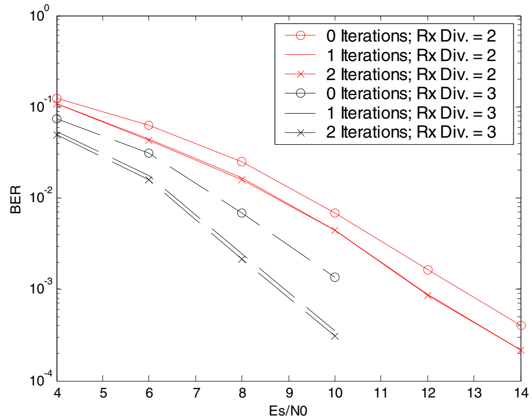

Figure 4: Two GSTTC system performance with Iterative MAPSSE-MAPSTTD over 4x4 quasi-static fading channels

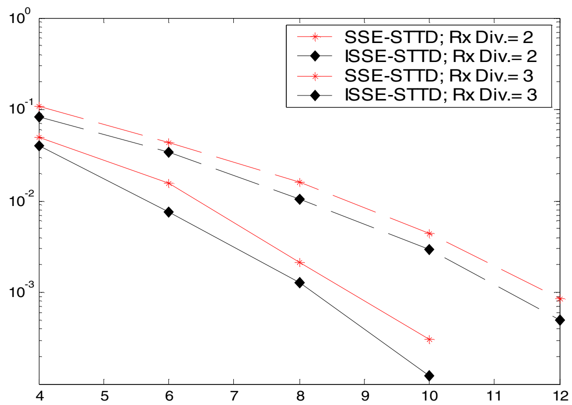

Figure 5: Comparison of the performance of 2GSTTC system with MAP-SSE and MAP-ISSE detection stage

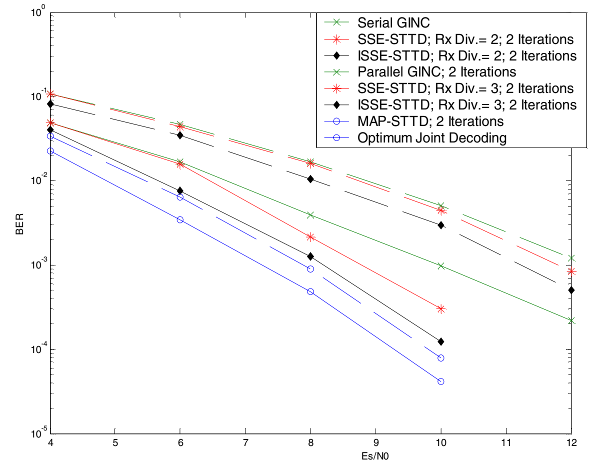

Figure 6: Performance comparison of decoding algorithms for a 2GSTTC system over 4x4 quasi-static fading channels.

REFERENCES

[1] L. Zheng and D. Tse. "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channala," IEEE Trans. on Information Theory, VOL. 49, NO. 5, May 2003

[2] Tarokh, V.; Naguib, A.; Seshadri, N.; Calderbank, A.R. "Combined array processing and space-time coding ", *Information Theory, IEEE Transactions on ,* Volume: 45 Issue: 4 , May 1999 Page(s): 1121 -1128

[3] Al-Ghadhban, Samir and B. Woerner. "Iterative Joint and Interference Nulling/ Cancellation Decoding Algorithms for Multi-Group Space Time Trellis Coded Systems." *IEEE WCNC04.* March 04.

[4] M. Mohammad, S. Al-Ghadhban, B. Woerner, and W. Tranter "Spatial Sequence Estimation Based Decoding Algorithm for V-BLAST" will be presented in IEEE VTC Fall 04.

[5] Magniez, P and Duhamel, "Turbo-Equalization applied to Trellis-Coded-Modulation," *VTC 1999-Fall. IEEE VTS Soth*. VOL. 5, PP, 2556-2560

[6] V. Tarokh, N. Seshadri and A.R. Calderbank, "Space-Time Codes for High Data Rates Wireless Communications: Performance Criterion and Code Construction," *IEEE Transactions on Information Theory,* Vol 44, Number 2, pp. 744-765, Mar. 1998.