Comparing Decoding Algorithms for Multi-Layer Space-Time Block Codes

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Abstract

In this paper we consider a multiple input multiple output multi-group space-time coding system that is a combination of V-BLAST and space-time block codes. Decoding order is an important issue for serial group interference cancellation (SGIC) algorithm. Different ordering schemes for SGIC are compared. While layers in SGIC suffer from unequal receive diversity, parallel group interference cancellation (PGIC) has the potential to achieve full receive diversity for all the layers. Performance of both the algorithms are analyzed and compared. Finally we show that the optimal performance of layered block codes could be achieved by sphere decoding (SD) which approaches the maximum likelihood detection performance with lower complexity.

Keywords: Multi-layer system, space-time block codes, sphere decoding, ordering scheme, interference cancellation.

1. Introduction

Information studies have shown that rich-scattering wireless channels have huge theoretical capacity [1],[2]. Using multiple input and multiple output (MIMO) antennas is a viable way of achieving this capacity. One of the MIMO techniques that received considerable attention in recent times is Vertical – Bell-Labs LAyered Space-Time Architecture (V-BLAST) [3],[4]. For a single user, this architecture provides tremendous spectral efficiency without increasing the total transmitting power or bandwidth. However, V-BLAST does not exploit transmit diversity and also it suffers from error propagation problem resulting in a lower information capacity.

Recently, in [5] Space-Time Block Code (STBC) has been proposed as a simple transmit diversity scheme. The combination of V-BLAST and STBC is referred to as multi-layer space-time block codes (MLSTBC) in this paper. The basic idea of this scheme is to partition the transmit antennas into different groups and assign each group to a layer of V-BLAST. Within each group, the signals are space-time block coded. So the transmit diversity of the layered architecture increases. Also, exploiting the orthogonal nature of STBC, the number of received antennas can be reduced compared to traditional V-BLAST. V-BLAST with space-time coding has been proposed before in [6],[7],[8]. In [6], space-time trellis code is used in each layer with different transmission power. In other words, the decoding order is pre-determined based on the power level. In [7], space-time block code is associated with each layer of V-BLAST as a way of improving energy efficiency. At the receiver, a reduced number of antennas are used to take advantage of the delay structure of STBCs. Performance of layered STBC with power allocation and pre-determined detection order is compared with equal power allocation schemes in [8]. In this paper, two sub-optimal interference cancellation (IC) algorithms are compared. The first one is serial group interference cancellation (SGIC). This technique serially decodes each layer by successive nulling of previously decoded layers. Error propagation and unequal diversity advantages for each group are two of the most common problems of SGIC that severely affect its performance. The other IC method is parallel group interference cancellation (PGIC) algorithm. This algorithm consists of two stages: parallel nulling followed by parallel interference cancellation and detection of all layers. Since the algorithm does not null out layers in the second stage, it has the potential to achieve full receive diversity for each group. However, both these IC algorithms suffer from error propagation.

As a joint detection algorithm, sphere decoding (SD) [12-14] is applied to MLSTBC to decode the individual layers. It is found that this joint detection schemes outperforms both PGIC and SGIC. Also it approaches the limit of Maximum Likelihood (ML) decoding with reasonable complexity.

The MLSTBC system will have the same transmit diversity advantage as each STBC while the receive diversity advantage will depend on the applied decoding algorithm.

The remainder of this paper is organized as follow. The system model of MLSTBC is briefly described in Section 2. Group interference cancellation techniques along with different detection ordering schemes are discussed in Section 3. In Section 4, the concept of sphere decoding as applied to MLSTBC is introduced. Simulation results are presented in Section 5. Finally, conclusions are drawn in Section 6.

2. System model

We consider a MIMO system that has n_T transmitting and n_R receiving antennas and is denoted by a (n_T, n_R) system. Throughout this paper, we assume that all the transmitters are synchronized. Figure 1 shows the system architecture of MLSTBC. A block of B input bits is sent to the vector encoder of V-BLAST that produces q bit streams (layers) of length $B_1, B_2, \ldots, B_q$ with $B = B_1 + B_2 + \ldots + B_q$. The transmit antennas n_T are partitioned into q groups of $n_1, n_2, \ldots, n_q$, with $n_T = n_1 + n_2 + \ldots + n_q$, where n_1 is the number of antennas in the first group and so on. Each bit stream, B_i, $1 \le i \le q$, is then sent through the corresponding space-time block encoder (STBE$_i$). The output of the encoder is a $n_i \times l$ codeword, $\mathbf{c}_i$, over l time intervals. The coded outputs from all the layers are transmitted simultaneously over the wireless channel. The transmit antennas of all the groups are allocated equal power and the total transmission power is fixed. At any time instant, the transmitted symbols can be written as

$$\mathbf{C} = \left[\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_q\right]^T \tag{1}$$

The channel is assumed to be quasi-static Rayleigh flat fading, i.e. the channel variation is assumed to be negligible over two consecutive symbols. Each fade coefficient is a complex Gaussian random variable with zero mean and 0.5 variance per dimension. The channel state information (CSI) is assumed perfectly known at the receiver. However, the transmitter has no knowledge of the channel. Let h_{mn} be the path gain from the transmitting antenna n to the receiving antenna m, with $n = 1, 2, \ldots n_T$ and $m = 1, 2, \ldots, n_R$. The channel matrix is defined as

$$\mathbf{H}_{n_R \times n_T} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_1} & h_{1,n_1+1} & \cdots & h_{1,n_T} \\ h_{2,1} & \cdots & h_{2,n_1} & h_{2,n_1+1} & \cdots & h_{2,n_T} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1} & \cdots & h_{n_R,n_1} & h_{n_R,n_1+1} & \cdots & h_{n_R,n_T} \end{bmatrix} \tag{2}$$

Partitioning $\mathbf{H}$ into groups corresponding to each layer, the channel matrix can be rewritten as

$$\mathbf{H} = \left[\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_q\right] \tag{3}$$

where

$$\underset{n_R \times n_1}{\mathbf{h}_1} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_1} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \cdots & h_{n_R,n_1} \end{bmatrix} \tag{4}$$

is the fade coefficients of the first layer.

At each receiving antenna, the received signal is the superposition of n_T transmitted signals corrupted by Rayleigh fading and noise. The received signal over l time intervals can be written as

$$\mathbf{r}_1 = \mathbf{HC} + \mathbf{v} \tag{5}$$

where $\mathbf{r}_1$ is original received signal matrix of dimension $n_R \times l$, $\mathbf{v}$ is the $n_R \times l$ noise matrix with independent complex Gaussian variables of zero mean and $N_0/2$ variance per dimension.

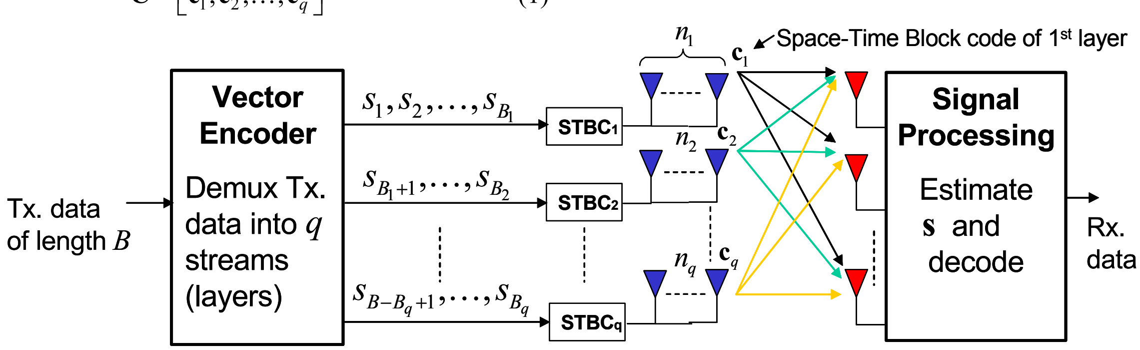

Figure 1: System model of multi-layered space-time block coding architecture.

3. Group interference cancellation and ordering schemes

Group interference cancellation technique [6] can maintain the desired group of signals while suppressing the effect of interfering group of signals. First, the decoder nulls out all the interfering groups except the desired one. After the nulling operation, the desired group is decoded and the interference due to this group is reconstructed. Once the interference is calculated it can be subtracted out from the received signal either serially or in parallel.

To illustrate group interference cancellation method, let us assume the desired group is i. Let $N\ (\mathbf{c}_i)$ be the null space of $\mathbf{\Lambda}(\mathbf{c}_i)$ where

$$\mathbf{\Lambda}(\mathbf{c}_i) = \left[\mathbf{h}_1, \ldots, \mathbf{h}_{i-1}, \mathbf{h}_{i+1} \ldots, \mathbf{h}_q\right] \tag{6}$$

Since the dimension of $N\ (\mathbf{c}_i)$ is $(n_R - n_T + n_i)$, an $(n_R - n_T + n_i) \times n_R$ matrix, $\mathbf{\Theta}(\mathbf{c}_i)$, whose rows form a set or orthonormal vectors in $N\ (\mathbf{c}_i)$ can be obtained. Left multiplying (5) by $\mathbf{\Theta}(\mathbf{c}_i)$, the decision statistics for $\mathbf{c}_i$ is found.

$$\tilde{\mathbf{r}}_1 = \mathbf{\Theta}(\mathbf{c}_i)\mathbf{r}_1 = \tilde{\mathbf{h}}_i \mathbf{c}_i + \tilde{\mathbf{v}} \tag{7}$$

where $\tilde{\mathbf{h}}_i$ is the $(n_R - n_T + n_i) \times n_i$ modified channel matrix to decode $\mathbf{c}_i$, and $\tilde{\mathbf{v}}$ is the $(n_R - n_T + n_i) \times l$ noise matrix.

Note that due to nulling, $\mathbf{c}_i$ has a diversity gain of $n_i \times (n_R - n_T + n_i)$. Once $\mathbf{c}_i$ is decoded, its contribution is subtracted out from the original received signal.

$$\mathbf{r}_2 = \mathbf{r}_1 - \tilde{\mathbf{h}}_i \hat{\mathbf{c}}_i \tag{8}$$

where $\hat{\mathbf{c}}_i$ is the estimated code word of layer i.

The nulling and cancellation procedure in (7) and (8), respectively is repeated serially until all the layers are decoded. We refer this decoding algorithm as serial group interference cancellation (SGIC). An important aspect of SGIC is the decoding order. In this paper we compare two ordering schemes. It is intuitive to decode the group with the maximum SNR first to optimize performance. In the first scheme, we select the group from the channel matrix, $\mathbf{H}$, that has the highest Frobenius norm. The detection order k is given by

$$k = \arg\max_i \left\{ \|\mathbf{h}_i\|_F \right\}, \quad 1 \le i \le q \tag{9}$$

In the second scheme, the ordering is based on the highest singular value (SV) of the modified channel matrix, $\tilde{\mathbf{h}}_i$, and is given by

$$k = \arg\max_i \left\{ \rho_i \right\}, \qquad 1 \le i \le q \tag{10}$$

where ρ_i is the singular value of $\tilde{\mathbf{h}}_i$.

The performance of SGIC is affected by error propagation. The other disadvantage of this method is that it can not exploit the full receive diversity advantage. The earlier decoded layers have less receive diversity than the later and it impacts the overall system performance.

In parallel group interference cancellation (PGIC), the contribution of the desired layer is calculated the same way as in SGIC. However, in the second stage, the contributions from all the layers except the desired one is subtracted out from the received signal to form the decision statistics. The decision statistics of i-th layer is given by

$$\tilde{\mathbf{r}}_i = \mathbf{r}_1 - \sum_{\substack{j=1 \\ j \ne i}}^{q} \tilde{\mathbf{h}}_j \hat{\mathbf{c}}_j, \qquad 1 \le i \le q \tag{11}$$

Note that unlike SGIC, each group in PGIC has full receive diversity, since it does not null out interference. PGIC has been applied in [9] for decoding multi-layer space-time trellis codes.

4. Sphere decoding

Sphere decoding (SD) algorithm received a lot of attention recently because it can achieve maximum likelihood (ML) detection performance with moderate complexity. It was originally proposed in [10] to solve the problem of finding the shortest vector in a lattice and further analyzed in [11]. Viterbo and Boutros, in [12], introduced the algorithm to communication systems over fading channels. Over MIMO fading channels, several researchers adapted the SD due to its great performance improvement. In [13-15], the SD was used to improve the performance of V-BLAST through joint detection which will provide full receive diversity. Also, it has been adapted to the detection of non-orthogonal STBCs [16],[17].

The complexity of the SD, as shown in [11], is polynomial in the dimension of the lattice, z. Analysis and simulation carried out by [18] and [19] showed that at high and moderate SNRs the average complexity is roughly cubic, $\mathcal{O}(z^3)$, and it doesn't depend on the constellation size. For our application, the dimension of the lattice is $2n_T$, where n_T is the total number of transmit antennas.

The SD used in this paper is well described in [14] for MIMO systems. The decoder starts by searching for a valid point inside a sphere of initial radius and centered at the received point. After that, the radius is reduced to the new valid point and the search is repeated until it converges to the ML solution.

In our work, before applying the SD, the receiver rearranges the received vectors from two time slots into one vector using the *virtual* MIMO model. To illustrate that, let us assume a two group STBC system with

$n_T = 4$ and $n_R = 2$. The received vectors could be rearranged as:

$$\begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2*} \\ y_2^{2*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{12}^* & -h_{11}^* & h_{14}^* & -h_{13}^* \\ h_{22}^* & -h_{21}^* & h_{24}^* & -h_{23}^* \end{bmatrix} \cdot \begin{bmatrix} x_{11} \\ x_{21} \\ x_{12} \\ x_{22} \end{bmatrix} + \begin{bmatrix} v_1^1 \\ v_2^1 \\ v_1^{2*} \\ v_2^{2*} \end{bmatrix} \quad (12)$$

$$\vec{y} = \hat{\mathbf{H}}\vec{x} + \vec{v} \quad (13)$$

where y_m^t and v_m^t are the received signal and the AWGN at time t and at receive antenna m, respectively. Also, x_{pi} is the p^{th} symbol transmitted from group i and * denotes the complex conjugation.

The SD will operate on the received vector in (13) and it will jointly detect the transmitted symbols from all groups resulting in full receive diversity advantage.

5. Results

In the simulation, $(4,4)$, $(4,2)$ and $(8,8)$ MLSTBC systems are considered. Transmitting antennas are grouped so that each layer has 2 antennas resulting in 2 and 4 layers, respectively. The modulation scheme is QPSK and the block length is 2. The simple orthogonal Alamouti code [5] is used as STBC. The spectral efficiency is 4bps/Hz and 8bps/Hz, respectively. Perfect channel knowledge is assumed at the receiver. Channel coefficients over each block are constant but vary independently from one block to another.

5.1. Effect of ordering criteria on the performance of SGIC

The importance of decoding order on the performance of SGIC is illustrated in Figure 2 for the $(4,4)$ case. It is observed that ordering based on the singular value (SV) criteria performs better than Frobenius norm criteria. The SV criteria in (9) take into account the modified channel matrix, which becomes the *effective* channel for decoding the corresponding layer. However, the ordering based on Frobenius norm only considers the original channel matrix neglecting the transformation associated with nulling. The performance of SGIC with no ordering is shown as a reference. By no ordering we refer to the fact that the receiver always detects the first layer irrespective of the channel or SNR condition. Also SGIC with perfect interference cancellation (IC) is shown as a lower bound. Note that even with perfect IC, there is only a little gain achieved. This effect is primarily due to the error floor set by the first detected layer. Even though the second layer achieves receives diversity, the performance is limited by the first layer.

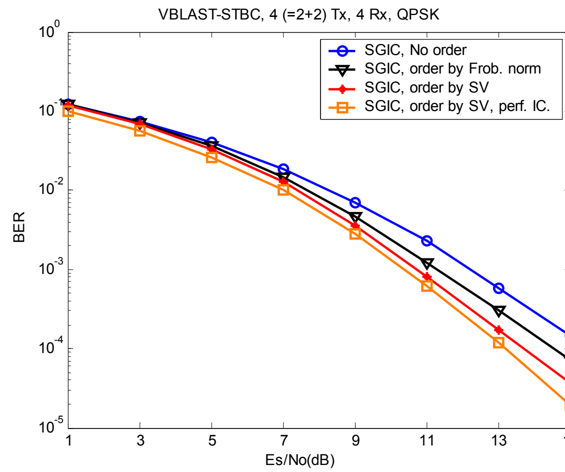

Figure 2: SGIC performance with different ordering criteria.

5.2. Performance comparison of SGIC, PGIC

Figure 3 depicts the performance of SGIC and PGIC for $(4,4)$ system. As expected, at low SNR, PGIC outperforms SGIC, since all the layers have the potential to achieve full receive diversity. However, in practice, the diversity advantage in PGIC is limited by the error propagation. This fact is clearly illustrated by the PGIC with perfect IC performance. At high SNR, SGIC approaches the performance of PGIC. Iterative PGIC can improve performance with diminishing return. For only two layers there is not much improvement for the second iteration Instead of using iterative PGIC, we propose a scheme where in the first stage SGIC is used, while PGIC is used in the second stage and is referred as 'SGIC+PGIC' in Figure 3.

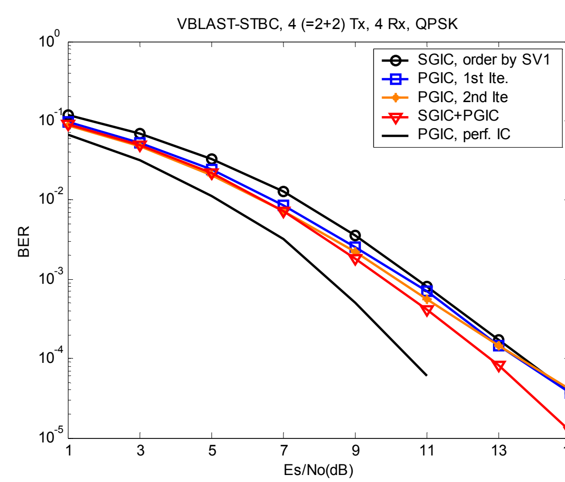

Figure 3: Performance comparison between SGIC and PGIC for $(4,4)$ system.

Note that this scheme outperforms PGIC with two iterations. This is primarily due to fact that in the first stage it is benefited from ordering and in the second stage it cancels interference instead of nulling out. The performance IC algorithms for $(8,8)$ system is also shown in Figure 4. As the number of layers increases, error propagation hurts PGIC more than diversity advantage helps. As a result SGIC outperforms PGIC even with less receive diversity advantage. The advantage of using 'SGIC+PGIC' instead of iterative PGIC is obvious form the figure.

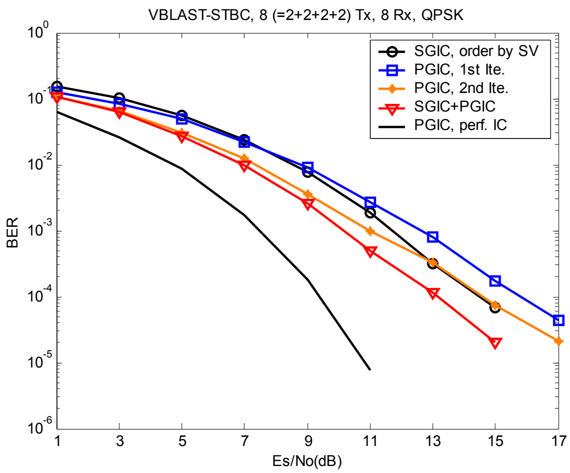

Figure 4: Performance comparison between SGIC and PGIC for $(8,8)$ system.

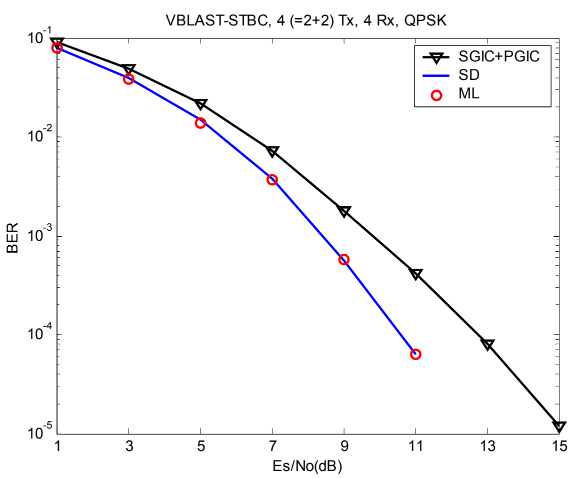

Figure 5: Performance comparison between Sphere decoding and IC algorithms for $(4,4)$.

5.3. Performance analysis of joint detection technique

The optimum decoding algorithm to MLSTBC is the joint detection technique. Sphere decoding (SD) is an efficient way of implementing this technique. Unlike IC algorithms, it does not suffer from error propagation problem. Also it achieves full receive diversity. Due to these two facts, it approaches the performance of maximum likelihood (ML) decoding. The performance of SD is compared against the IC algorithm in Figure 5 for $(4,4)$ system. The diversity advantage achieved by SD is noticeable from the slope of the curve. As an upper bound, the performance of ML decoding is also shown which is very much the same as the performance of SD. The same trend in performance is observed for $(8,8)$ system as shown in Figure 6.

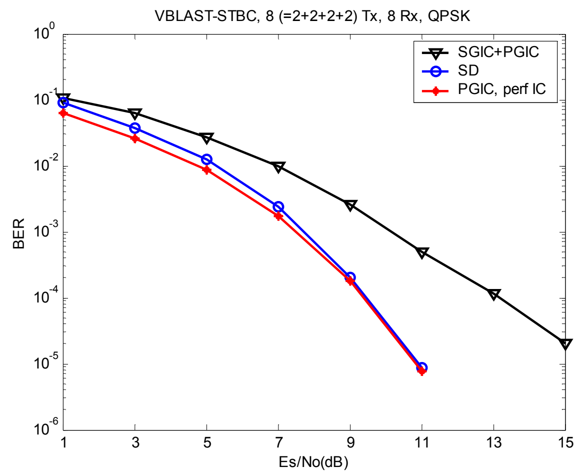

Figure 6: Performance comparison between Sphere decoding and IC algorithms for $(8,8)$.

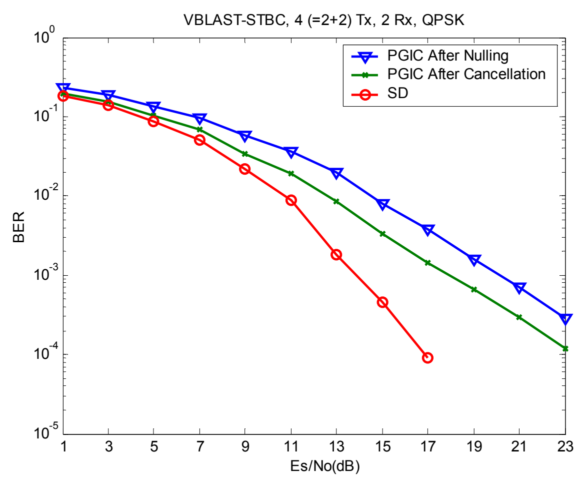

Figure 7: Performance of $(4,2)$ system using virtual MIMO concept.

5.4. Comments on the number of receive antennas

Unlike V-BLAST where $n_R \geq n_T$, the minimum number of receive antennas required in the MLSTBC systems is equal to the number of groups. That is due to the inherent delay in STBC which will transform the $n_R \times n_T$ channel matrix into $(n_R \cdot l) \times n_T$ virtual channel matrix as done in (12). Figure 7 shows the performance of SD and PIC receivers for a $(4,2)$ two layer system.

6. Conclusion

In this paper the decoding algorithms for multi-layered space time block codes were discussed. Two ordering criteria for serial interference cancellation algorithm were studied. The performance of serial and parallel interference cancellation algorithms was compared. Since PGIC has the potential to use full receive diversity it outperforms SGIC for small number of layers. However, for large number of layers, SGIC outperforms PGIC at high SNRs. Iterative PGIC can improve performance with added complexity. The performance of the IC algorithms is limited by the error propagation problem. Since sphere decoding algorithm does not suffer from error propagation problem it outperforms IC algorithms. In fact it achieves the performance of ML decoding and provides full receive antenna diversity with moderate complexity.

7. Reference

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