Multi-Layered Space Frequency Time Codes

Samir Al-Ghadhban
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samir@kfupm.edu.sa
http://faculty.kfupm.edu.sa/EE/samir/

Chairmen: Brian Woerner and R. Michael Buehrer

Members: William Tranter, Sedki Riad and James Holub
Outline

• Introduction and background
• Motivation and contribution
• Multi-layered space time block codes
• Performance of MLSTBC-OFDM
• IQ- space frequency time codes
• Uplink MIMO scheduling
Introduction: Multiple Input Multiple Output (MIMO) Channels

- A MIMO channel is a wireless link between $MT$ transmit and $MR$ receive antennas.
- MIMO channels boost the information capacity of wireless systems by order of magnitude [Telater95][Foschini98].

\[
H(t) = \begin{bmatrix}
    h_{11}(t) & \cdots & h_{1M_T}(t) \\
    \vdots & \ddots & \vdots \\
    h_{M_S,1}(t) & \cdots & h_{M_S,M_T}(t)
\end{bmatrix}
\]
Introduction: Open Loop MIMO Communication Systems

Open Loop MIMO Communication Systems

Transmit Diversity

Spatial Multiplexing

Differential Block Coding [Tar00]
Block Coding [Ala98][Tar99a]
Trellis Coding [Tar98]

D-BLAST [Fos96]
V-BLAST [Wal99]

STBC

STBC Combiner and Detector

MIMO Fading Channel

V-BLAST Detector

MIMO Fading Channel

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Dissertation Scope and Motivation

Bandwidth efficient advances for MIMO systems

- Multi-layered space time codes
- Space frequency time codes
- Uplink scheduling for spatial multiplexing systems

• [Tarokh99] combined array processing and STTC.
• Propose multilayered detection algorithms.
• How does it compare to open loop MIMO systems?
• Performance over MIMO-OFDM.

• Coding for MIMO-OFDM
• MaxDiv is $M_2LM_R$ [Ben Lu 00]
• Full spatial and frequency diversity at much lower number of states
• Interleaving effect on diversity

• Capacity maximizing scheduling algorithms for practical schemes
• Suboptimal scheduling criteria
Our contributions are:

- Developing joint, interference nulling and cancellation, and spatial sequence estimation detection algorithms for multi-layered space time trellis codes.
- Outage capacity study of multi-layered space time block codes and comparison to other open loop MIMO systems.
- A modified sphere decoder for non-rectangular and rotated constellations.
Contributions

• Comparison study of detection algorithms for multi-layered space time block coded OFDM systems.
• A reduced complexity space-frequency-time code, based on IQ-trellis codes, for MIMO-OFDM systems.
• Proposing and comparing scheduling criteria for uplink spatial multiplexing systems.
Publications


Publications


Recent submissions

In the preliminary exam, we presented our work on MLSTTC

- We proposed and compared multi-layered detection algorithms that are based on:
  - Group interference nulling and cancellation algorithms
  - Joint iterative detection
  - Spatial sequence estimation
Multi-Layered Space Time Block Codes [MLSTBC]

- Information theoretic comparison of MLSTBC and other open loop MIMO systems.
- Spectral efficiency MLSTBC detection algorithms.
- Performance evaluation of MLSTBC detection algorithms over MIMO-OFDM systems.
Multi-layered STBC is a single user system that consists of $K$ parallel STBC

- It combines spatial multiplexing with transmit diversity.
- It is a V-BLAST system with STBC on each layer.
How does MLSTBC compare to V-BLAST and STBC?
The discrete received signal over $T$ time slots is

$$Y = HS + V$$

$$= \begin{bmatrix} H_1 & H_2 & \cdots & H_K \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_K \end{bmatrix} + V$$

$S_i$ is the $i^{th}$ STBC. $H_i$ is the $M_R \times N_G$ MIMO matrix from group $i$ to the receiver.

$M_R$: total number of receive antennas

$N_G$: number of transmit antennas per group

$M_T$: total number of transmit antennas
Due to the short code length of STBC, the received signals over $T$ slots are rearranged into a vector 

$$y = \hat{H}x + \eta$$

$$= \begin{bmatrix} \hat{H}_1 & \hat{H}_2 & \cdots & \hat{H}_K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} + \eta$$

$x_i$ is the symbols of the $i^{\text{th}}$ layer.

$\hat{H}_i$ is the $M \cdot T \times N_G$ MIMO matrix from group $i$ to the receiver.
Serial Group Interference Nulling and Cancellation (SGINC)

• **Group interference nulling**: Based on an ordering criterion, assume that the first detected group is the $i^{th}$ group. Then, the algorithm calculates the orthonormal bases of the null space of:

$$H_i = \begin{bmatrix} \hat{H}_1 & \cdots & \hat{H}_{i-1} & \hat{H}_{i+1} & \cdots & \hat{H}_K \end{bmatrix}$$

• Denote the orthonormal bases of the null space of $H_i$ by $N_i$, then the received signal for the $i^{th}$ group after nulling is:

$$\tilde{y}_i = N_i y = \tilde{H}_i x_i + \tilde{\eta}_i$$

Where $\tilde{H}_i$ is the post-processing channel matrix.
SGINC

• **STBC Combiner:** \( \tilde{\mathbf{x}}_i = \tilde{\mathbf{H}}_i^H \tilde{\mathbf{y}}_i \)

• **Group interference cancellation:** After detecting the \( i^{th} \) group, its contribution is subtracted from the received signal and the processing is repeated serially for each group.

• **Ordering** is based on the Frobenius norm (FN) of \( \tilde{\mathbf{H}}_i \), the layer with the maximum FN is detected first.

• **Number of receive antennas** should be greater than or equal to number of layers.
Outage Capacity

- **V-BLAST [Papadias and Foschini 2002]:**
  \[
  C_{ZF_{VBLAST}}^{VBLAST} = K \cdot \min_{i=1,2,...,K} \left\{ \log_2 \left( 1 + \frac{\rho}{K \|W_{ZF,i}\|^2} \right) \right\}
  \]

- **STBC [Sandhu and Paulraj 2000]:**
  \[
  C_{STBC} = r_c \log_2 \left( 1 + \frac{\rho}{N_G} \|H\|_F^2 \right)
  \]

- **MLSTBC**
  \[
  C_{MLSTBC}^{GNIC} = K \cdot \min_{i=1,2,...,K} \left\{ r_c \log_2 \left( 1 + \frac{\rho}{K \cdot N_G} \left( \frac{\|\tilde{H}_i\|_F^2}{T} \right) \right) \right\}
  \]
Comparative Study

- Complementary cumulative distribution function (CCDF)
- Spectral efficiency
- Outage probability
Simulation Setup

- Each STBC in the multi-layered system is Alamouti’s full-rate full-rank code with two transmit antennas.
- STBC used for four transmit antennas is full-rank orthogonal code of rate $\frac{3}{4}$.
- The capacity of the different systems is estimated by generating random complex Gaussian channel realizations from which the instantaneous capacity is calculated and then the capacity probability distribution function (pdf) is approximated.
The key difference between MLSTBC and V-BLAST at same number of antennas is

- MLSTBC has more spatial diversity while V-BLAST has more layers.
- For 4×4 MIMO system, MLSTBC has two layers and each layer has a transmit diversity of two. At the receiver, the first detected layer has a receive diversity of three since it needs one antenna to null out one interfering layer and the rest provide diversity.
- On the other hand, V-BLAST has four layers and no transmit diversity. In addition, the first detected layer has no receive diversity because the algorithm needs three antennas to null out three interfering layers.
CCDF at 4×4 MIMO Channels

At low outage probabilities, MLSTBC supports more capacity than V-BLAST and STBC.
Spectral Efficiency at 4×4 MIMO Channels

- V-BLAST spatial multiplexing gain is parallel to optimal MIMO at high SNR.
- Although, MLSTBC has slower rate of increase in spectral efficiency, it supports more rate at low and moderate SNR and at low outages.
Outage Probability at 4 bps/Hz and over 4×4 MIMO Channels

- MLSTBC is more power efficient than V-BLAST and STBC.
- 5dB far from optimal MIMO at 10^-4 outage.
Therefore, the comparative study shows that at the same number of transmit and receive antennas,

- MLSTBC is more power efficient.
- Also, it is more spectral efficient than V-BLAST at low and moderate SNR and low outage probabilities.
- It is a good candidate for low power wireless data applications.
We also examine the spectral efficiency of PG INC and joint detection algorithms

- **Parallel GINC**

- **Joint detection**
  - A joint detector detects all layers jointly using a maximum likelihood (ML) detector. A practical ML detector that has a cubic complexity at high SNR is the sphere decoder (SD).
Spectral efficiency of MLSTBC detection algorithms

Spectral Efficiency, $K=4$, $M_R=4$, 10% Outage

Rate vs Layers, $M_R=8$, 10% Outage
MLSTBC-OFDM

- Wide bandwidth and high data rates result in frequency selective channels (FSC) which cause ISI.
- OFDM is robust against FSC. It transforms FSC to parallel flat fading channels.
- WLANs such as IEEE 802.11a and Hyperlan2 are based on OFDM.
Architecture

- $N_c$ point FFT ($N_c$ subcarriers)
- Two Layered STBC $k=1,2$

$$
s_{k1} = \begin{bmatrix} s_{k1,1} & s_{k1,2} & \cdots & s_{k1,N_c} \end{bmatrix}^T
$$

$$
s_{k2} = \begin{bmatrix} s_{k2,1} & s_{k2,2} & \cdots & s_{k2,N_c} \end{bmatrix}^T
$$

$$
C_k = \begin{bmatrix} s_{k1} & s_{k2} \\ -s_{k2}^* & s_{k1}^* \end{bmatrix}
$$
Comparison of MLSTBC and V-BLAST over 4x4 MIMO-OFDM, $N_c=64$ and $L=4$

![Graphs showing OFDM SER vs. $E_s/N_0$ for different bit rates and modulation schemes](image_url)
Comparison of MLSTBC detection algorithms

4bps/Hz, $K=2$, $L=4$, $N_c=64$

8bps/Hz, $K=4$, $L=4$, $N_c=64$
MLSTBC-OFDM conclusions

- We showed that the MLSTBC is more power efficient than V-BLAST over MIMO-OFDM channels.
- We compared different multi-layered detection algorithms. The sphere decoder performs very close to the optimal performance since it provides full receive diversity with a cubic complexity at high SNR.
IQ-Space Frequency Time Codes

Bandwidth efficient advances for MIMO systems

Multi-layered space time codes

Space frequency time codes

Uplink scheduling for spatial multiplexing systems

• Full spatial and frequency diversity design at low number of states
• Interleaving effect on diversity
IQ-SFT codes outline

- Background and design criteria
- IQ-SFT code description and performance
- Effect of interleaving
- MLIQSFT codes
Background on SFT codes

- SFT codes apply spatial coding across multiple antennas, frequency coding across OFDM subcarriers, and temporal coding across successive OFDM symbols.
- [Agrawal98]: STTC-OFDM, not optimized for OFDM channels, designed for quasi-static channels.
OFDM Channel Model in the Frequency Domain

\( N_c \) subcarriers

\( L \) taps (FSC length)

Let \( \mathbf{h}_{mn} = [h_0, h_1, \ldots, h_{L-1}]^T \)

The OFDM channel in the frequency domain is \( \mathbf{h}_{mn}^f = \mathbf{F} \mathbf{h}_{mn} \)

\[
\mathbf{F}_{k,l} = \frac{1}{\sqrt{N_c}} \exp \left[ -i \frac{2\pi}{N_c} (k - 1)(l - 1) \right];
\]

\( k = 0, 1, \ldots, N_c - 1 \)

\( l = 0, 1, \ldots, L - 1 \)

Let \( \mathbf{h} \sim N_c (\mathbf{0}, \mathbf{C}_h) \)

The covariance matrix in the frequency domain is \( \mathbf{C}_{h^f} = \mathbf{F} \mathbf{C}_h \mathbf{F}^H \)
Design criteria of SFT codes

• The maximum diversity available in MIMO-OFDM systems is $M_TLM_R$ [Ben Lu 2000].
• The design criterion is to maximize the minimum effective length and break up channel correlation in frequency domain by interleaving.
• To achieve this diversity, the minimum effective length of the SFT code should be equal to at least $M_TL$, which needs large number of states for practical values.
Design criteria of SFT codes

- Our goal in this work is to simplify the design and reduce the number of states required to achieve the full spatial and frequency diversity.
- Our approach is to concatenate trellis coded modulation (TCM) and STBC.
- Spatial diversity is guaranteed by STBC and frequency diversity is provided by TCM.
- We further reduce the number of states of TCM by using IQ-TCM [AlSemari 97].
IQ-TCM [AlSemari97]

- The minimum effective length of TCM is upper bounded by:

\[ l_{\text{min}} \leq \left\lfloor \frac{v}{k} \right\rfloor + 1 \]

2 bps/Hz IQ-16QAM-TCM

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IQ-SFT

Encoder

Decoder

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Advantages of concatenated IQ-TCM-STBC at 2bps/Hz

<table>
<thead>
<tr>
<th>FCS Length</th>
<th>Minimum number of states to achieve full diversity ((M_{TLM_R}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>Tarokh STTC QPSK</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
</tr>
<tr>
<td>3</td>
<td>1024</td>
</tr>
<tr>
<td>4</td>
<td>16384</td>
</tr>
<tr>
<td>5</td>
<td>262144</td>
</tr>
<tr>
<td>6</td>
<td>4194304</td>
</tr>
<tr>
<td>7</td>
<td>67108864</td>
</tr>
</tbody>
</table>
Performance results over 2x1 MIMO-OFDM channels at 2bps/Hz 8-state TCM, $N_c=64$, $W=4$

MT=2, $N_c=64$, $L=2$ and $W=4$

MT=2, $N_c=64$, $L=4$ and $W=4$

MaxSD=2
MaxFD=4

MaxSD=2
MaxFD=2
Interleaving effect over 2x1 MIMO-OFDM channels at 2bps/Hz 8-state TCM

IQ 16QAM TCM-STBC-OFDM (lmin=4) at 2bps/Hz, L=4, Nc=64

IQ 16QAM TCM-STBC-OFDM at 2bps/Hz, L=8, Nc=64

SNR

BER

Uncoded STBC
No Interleaving
W=2
W=4
W=16
W=8

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Effect of interleaving on subcarrier correlation

(a) Correlation No Interleaving at L= 4 paths and Nc=64

(b) Correlation with Interleaving at L= 4 paths and Nc=64 and W= 4
Multi-layered IQ-SFT codes

8x4 MIMO-OFDM
4 layers of IQ-16QAM-SFT
SFT coding conclusions

• Concatenated IQ-TCM-STBC-OFDM achieves full spatial and frequency diversity at much lower complexity than other codes.

• Appropriate block interleaver design is essential to maintain the performance and diversity of the code. Best performance is at $W=L$ and $W=2L$. 
Uplink Scheduling for Multiuser Systems with Spatial Multiplexing

Bandwidth efficient advances for MIMO systems

Multi-layered space time codes

Space frequency time codes

Uplink scheduling for spatial multiplexing systems
Uplink Scheduling for Multiuser Systems with Spatial Multiplexing

• In a multiuser environment, each user has different channel statistics.
• Scheduling transmission to the user with the best channel condition at each time leads to a form of selection diversity known as multiuser diversity.
• In SISO, MaxSNR scheduling maximizes the capacity of the uplink [Kno95] and downlink [Tse97].
Our focus is on

- Scheduling for uplink MIMO system.
- Scheduling and STBC aren’t a good match [Gozali03].
- We focus on scheduling for spatial multiplexing systems selecting a single user at a time and we focus on practical detection algorithms, such as V-BLAST, SMZF and SMSD.
System Model

- Average SNR is assumed to be the same for all users.
- The base station scans the users.
- The user with the best channel condition is allowed to transmit.
- The received signal from user $k$ is:
  \[ y_k = H_k x_k + \eta_k \]
Scheduling Algorithms

• **Optimal MIMO capacity maximizing scheduler**

\[
C_{\text{max}} = \max_{k=1,2,\ldots,K} C_k ; \text{ where } \quad C_k = \log_2 \left( \det \left( I_{M_R} + \frac{\text{SNR}}{M_T} H_k H_k^H \right) \right)
\]

• **MaxSNR scheduler** selects the user with maximum MIMO channel power \( \left( \text{trace}(H_k H_k^H) \right) \)

• **RR:** Round robin scheduling allows each user to transmit in a time-division fashion regardless of their channel condition.
Scheduling Algorithms

• V-BLAST capacity maximizing scheduler

V-BLAST capacity is dominated by the weakest layer [Pap02]

\[
C_{VBLAST}^{ZF} = M_T \cdot \min_{i=1,2,...,M_T} \left\{ \log_2 \left( 1 + \frac{\text{SNR}}{M_T \left\| W_{ZF,i} \right\|^2} \right) \right\}
\]

Thus, the scheduler selects the user with \( \min_{k=1,...,K} \{w_k\} \) where

\[
w_k = \max_{i=1,2,...,M_T} \left\{ \left\| W_{ZF,i}^k \right\|^2 \right\}
\]
Scheduling Algorithms

- **MinES: Minimum Eigenspread**

  \[ k = \arg \min_{k=1,\ldots,K} \left\{ s_k = \frac{\lambda_{\max,k}}{\lambda_{\min,k}} \right\} \]

  \( \lambda_{\max} \) and \( \lambda_{\min} \) are the largest and smallest eigenvalues of \( H_k H_k^H \)

- **MaxMinSV: Maximum Minimum Singularvalue**

  \[ k = \arg \max_{k=1,\ldots,K} \{ \rho_{\min,k} \} \]

  \( \rho_{\min} \) is the smallest singularvalue of \( H_k \)
Simulation results of V-BLAST uplink scheduling

![Graph showing 10% Outage at 4x4 MIMO Channels at 15dB](image1)

![Graph showing K=10 Users, 4x4 QPSK V-BLAST](image2)
Effect of Suboptimal Detection

2x2 MIMO Channels at 15dB

4x4 MIMO Channels at 15dB

Within 1 bps/Hz

Within 2.5 bps/Hz
Scheduling for Spatial Multiplexing with Sphere Decoder

- **K=10 Users, 4x4 MIMO at 8bps/Hz**

  - BER vs SNR graph
  - Graph shows BER decreasing with increasing SNR.
  - Key points: RR, MaxSNR, MaxMinSV, MaxMIMOCapc.

- **10% Outage at 4x4 MIMO Channels and at 15dB**

  - Rate bps/Hz vs Users graph
  - Graph shows rate increasing with increasing number of users.
  - Key points: MaxMIMOCapc, MaxMinSV, MaxSNR, RR.

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Uplink MIMO Scheduling Conclusions

• We found the V-BLAST capacity maximizing scheduler.
• We showed that scheduling based on maximum MIMO capacity doesn’t work well for a V-BLAST system.
• We compared several scheduling algorithms and found that MaxMinSV scheduling performs close to MaxVBLAST capacity scheduler.
• The difference between V-BLAST and SMZF performance is not substantial, especially at low number of antennas and large number of users.
Summary and Conclusions

Bandwidth efficient advances for MIMO systems

Multi-layered space time codes

Space frequency time codes

Uplink scheduling for spatial multiplexing systems

Multi-layered space time codes

Space frequency time codes

Uplink scheduling for spatial multiplexing systems
Acknowledgment, Questions, Comments?
The received vectors over two time periods at the output of the FFT is

\[
\begin{bmatrix}
Y_1^{t_1} & Y_2^{t_1} \\
Y_1^{t_2} & Y_2^{t_2}
\end{bmatrix} =
\begin{bmatrix}
H_{11} & H_{12} & H_{13} & H_{14} \\
H_{21} & H_{22} & H_{23} & H_{24}
\end{bmatrix}
\begin{bmatrix}
s_{11} & -s_{12} \\
s_{12} & s_{11}^* \\
s_{21} & -s_{22}^* \\
s_{22} & s_{21}^*
\end{bmatrix}
+ \begin{bmatrix}
\eta_1^{t_1} & \eta_2^{t_1} \\
\eta_1^{t_2} & \eta_2^{t_2}
\end{bmatrix}
\]

Let \( h_{mn,l} \) be the flat fading complex Gaussian RV in the frequency domain between the \( n^{th} \) TX and the \( m^{th} \) RX at the \( l^{th} \) subcarrier, then the channel matrix for all subcarriers can be arranged as

\[ H_{mn} = \begin{bmatrix}
h_{mn,1} & 0 & \cdots & 0 \\
0 & h_{mn,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & h_{mn,L}
\end{bmatrix} \]
Transmitter

Architecture of one STBC-OFDM group. Each group transmits one layer of information using Alamouti code with two transmit antennas.
Channel Model per Subcarrier

At the $l^{th}$ subcarrier,

$$\begin{bmatrix}
y_{1,1}^l & y_{1,2}^l \\
y_{2,1}^l & y_{2,2}^l
\end{bmatrix} = \begin{bmatrix}
h_{11,l} & h_{12,l} & h_{13,l} & h_{14,l} \\
h_{21,l} & h_{22,l} & h_{23,l} & h_{24,l}
\end{bmatrix} \begin{bmatrix}
s_{11,l} & -s_{12,l} \\
s_{12,l} & s_{11,l} \\
s_{21,l} & -s_{22,l} \\
s_{22,l} & s_{21,l}
\end{bmatrix} + \begin{bmatrix}
\eta_{1,l}^1 \\
\eta_{1,l}^2
\end{bmatrix}$$

Rearranging the received vector over two periods into one received vector

$$\begin{bmatrix}
y_{1,1}^l \\
y_{2,2}^l \\
y_{1,2}^* \\
y_{2,1}^*
\end{bmatrix} = \begin{bmatrix}
h_{11,l} & h_{12,l} & h_{13,l} & h_{14,l} \\
h_{21,l} & h_{22,l} & h_{23,l} & h_{24,l} \\
h_{12,l} & -h_{11,l} & h_{14,l} & -h_{13,l} \\
h_{22,l} & -h_{21,l} & h_{24,l} & -h_{23,l}
\end{bmatrix} \begin{bmatrix}
s_{11,l} \\
s_{12,l} \\
s_{21,l} \\
s_{22,l}
\end{bmatrix} + \begin{bmatrix}
\eta_{1,l}^1 \\
\eta_{1,l}^2 \\
\eta_{2,l}^1 \\
\eta_{2,l}^2
\end{bmatrix}$$

$$\mathbf{y}_l = \begin{bmatrix}
H_{1,l} & H_{2,l}
\end{bmatrix} \mathbf{s}_l + \mathbf{\eta}_l$$

$$\mathbf{y}_l = \mathbf{H}_l \mathbf{s}_l + \mathbf{\eta}_l$$
Multi-layered SFT codes

SGINC with different ordering criteria
Interleaving effect on SFT at L=2.
Advantage of V-BLAST compared to SISO and SIMO systems

10% Outage at 15dB

MaxCapc Scheduling comparison at K=10 Users and at 8bps/Hz
Number of layers Effect on the Capacity at Eight Receive Antennas

10% Outage

0.1% Outage